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ANALYTIC STOCHASTIC REGULARIZATION IN FERMIONIC GAUGE THEORIES

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ABSTRACT

We analyse the influence of the Analytic Stochastic Regularization method in gauge symmetry, evaluating the 1-loop photon propagator correction for spinor QED. Consequences in the non-abelian case are discussed.

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1. INTRODUCTION

Some years ago, Parisi and Wu(1) developed a new, original method to quantize Euclidean Field Theories in the continuum, the so-called "Stochastic Quantization Method" (SQM). One of the most striking contributions offered by SQM was a non-perturbative regularization scheme, which in various aspects makes contact with Speer's method(2) - the Analytic Stochastic Regularization (ASR)(3,4). The evidences were that the method could respect all physical symmetries involved in a given field theory, as the gauge symmetry or supersymmetry, but we have been tempted to see more closely this influence. In recent papers, the method has been used to evaluate gauge symmetry breaking in Scalar QED and QCD at the one loop order(5,6) in $D = 4$. We have been able to extend this analysis to theories containing fermion fields of spin 1/2(7), like Spinor QCD. Some unsuspected cancellations in these calculations bring us with a fortunate finding - at least to lowest order ASR seems to preserve supersymmetry.

In the first section, we outline the basics of SQM in the original sense of (1) and the necessary changes for the inclusion of fermions. In the following section the ASR scheme is quickly reviewed. The third section is devoted to Spinor QED and corresponding "Feynman rules". We calculate the 1-loop correction to the photon propagator and verify transversality. Our conclusions are depicted in the last section.

2. A SURVEY OF STOCHASTIC QUANTIZATION METHOD

The physical motivation of the SQM is the well-known formal analogy between Euclidean Field Theory and Classical Statistical Mechanics. If we write the Euclidean N-point Green function, one can associate the functional measure

$$e^{-S[\phi]/\hbar} \tag{2.1}$$

to the equilibrium Boltzmann distribution of a Statistical system. Parisi and Wu[1] considered this system as performing a stochastic process.

In order to study this evolution, they supplement the classical field with an additional parameter, here called "ficticious time" t. Moreover, the stochastic dynamics may be described by Langevin equation. For the simple case of a boson field with Euclidean action $S[\phi]$, we shall write:

$$\frac{\partial\phi(x,t)}{\partial t} = -\frac{\delta S[\phi]}{\delta\phi(x,t)} + \eta(x,t) \tag{2.2}$$

$\eta(x,t)$ is the (white) noise field, whose correlation functions are given by:

$$\langle\eta(x,t)\rangle = 0 \tag{2.3a}$$

$$\langle\eta_i(x,t)\,\eta_j(x',t')\rangle = 2\delta_{ij}\,\delta(x-x')\,\delta(t-t') \tag{2.3b}$$

It is not difficult to prove that the N-point correlation functions, in the stationary limit (equal and very big ficticious times), reduce to the conventional Euclidean Green Functions. There are several proofs[8] about the equivalence of SQM to the other methods, to all orders of perturbation.

The perturbative approach to Stochastic Quantization consists in:

a) a choice of initial conditions for the above Langevin equation so that we can rewrite it in an integral form;

b) solving the resultant integral equation by iterative procedures, in powers of the coupling constant;

c) making a convenient graphical association, we assign full lines to the Green functions (propagators) and crosses to the noises. Vertices are linked to coupling constants as in usual field theories;

d) N-point correlation functions are easily obtained by joining the graphical iterative expansion of the above step. Crosses are fused in accord with their white noise property, and we need to consider all possible contractions. The graphs so drawn are called stochastic diagrams;

e) in stochastic diagrams, the lines containing fused crosses describe composite ("crossed") propagators.

Let us exemplify things directly with fermionic theories, although some remarks must be needed for sake of correctness. Stochastic Quantization of fermions is not a straightforward matter, because there is no classic analog for anticommuting quantities, like fermions - this fact leads to the non-positivity of operators and ill-defined probability distributions for the field system (in the sense of the corresponding Fokker-Planck equation). The most accepted prescription for dealing with fermions is to introduce kernels in the Langevin equations to guarantee positiveness of the operators[9]:

$$\frac{\partial \phi_i(x,t)}{\partial t} = - \int d^D y \, K_{ij}(x,y) \frac{\delta S[\phi]}{\delta \phi_j(y,t)} + \eta_i(x,t) \, . \qquad (2.4)$$

The noise correlations are also modified, giving

$$\langle \eta_i(x,t)\, \eta_j(x',t') \rangle = 2K_{ij}(x,x')\, \delta(t-t') \quad . \qquad (2.5)$$

Notice that higher point functions are obtained from those by a process very similar to the Wick decomposition.

The Langevin equation for free fermions, whose Euclidean action is

$$S[\psi,\overline{\psi}] = - i \int d^4x \, \overline{\psi}(x)(\not\partial + iM)\, \psi(x) \qquad (2.6)$$

is readily obtained with the use of the fermionic kernel

$$K^F_{ij}(x,y) = (i \not\partial_x + M)_{ij} \, \delta^4(x-y) \qquad (2.7)$$

giving a "bosonised" expression:

$$\frac{\partial \psi}{\partial t} = (\partial^2 + M^2)\, \psi + \vartheta \qquad (2.8)$$

and its conjugate counterpart. ϑ and $\overline{\vartheta}$ are Grassman (anticommuting) noises, obeying the correlations (2.5). With an adequate choice of initial conditions for the above equation, we write its integral expression, with the help of the Greens function (in the momentum space)

$$G_F(k;t-t')_{ij} = \delta_{ij} \exp\left[-(k^2+M^2)(t-t')\right] \theta(t-t') \qquad (2.9)$$

as

$$\psi_i(k,t) = \int dt \, G_F(k;t-t')_{ij} \, \vartheta_j(k,t') \quad . \qquad (2.10)$$

Graphically, we depict this object as a dashed line. The convolution of this function (also called "uncrossed propagator") with the fermionic kernel (2.7) yields a "fermionic" Green function, represented as a full line:

$$\Gamma_{ij}(k;t-t') = (-\not k+M)_{ij} \exp\left[-(k^2+M^2)(t-t')\right] \theta(t-t') \, . \qquad (2.11)$$

3. ANALYTIC STOCHASTIC REGULARIZATION

The stochastic process described by Langevin equation (2.2) is typically Markovian, due to the white noise property (2.3). Breit, Gupta and Zaks[3] had the idea of smearing the ficticious time delta function in (2.3) using a regulator function, parameter dependent. Obviously, when this parameter approaches to zero we recover the unregularized noises. Henceforth:

$$\langle \eta_i(x,t)\,\eta_j(x',t')\rangle = K_{ij}(x,x')\,f_\varepsilon(t-t') \tag{3.1}$$

and

$$\lim_{\varepsilon\to 0} f_\varepsilon(t-t') = 2\,\delta(t-t') \quad . \tag{3.2}$$

This is characteristic of a non-Markovian process, and the noise is non-white.

Following Alfaro[4], we use the particular regulator function:

$$f_\varepsilon(t) = \varepsilon|t|^{\varepsilon-1} \tag{3.3}$$

and get its Fourier transform:

$$f_\varepsilon(w) = \int_{-\infty}^{+\infty} \frac{dt}{2\pi}\, f_\varepsilon(t)\,\exp(-iwt) = 2\,\hat f_\varepsilon |w|^{-\varepsilon} \tag{3.4}$$

with

$$\hat f_\varepsilon = \varepsilon\,\Gamma(\varepsilon)\sin\frac{\pi}{2}(1-\varepsilon) \quad . \tag{3.5}$$

These new correlations enable us to compute the two-point fermion correlation function, whose lowest order contribution gives the so-called "crossed propagator":

$$\Delta_{ij}(k;t,t') = \langle \psi_i(k,t)\,\overline{\psi}_j(-k,t')\rangle \quad . \tag{3.6}$$

Using (3.1), (2.10) and its corresponding conjugate we obtain

$$\Delta_{ij}(k;t,t') = 2\int_0^t dt'' \int_0^{t'} dt'''\, G_{i\ell}(k,t-t'')\,G_{\ell j}(-k,t'-t''')\,f_\varepsilon(t''-t''')$$

$$= \hat f_\varepsilon \frac{(\not k-m)_{ij}}{(k^2+m^2)^{1+\varepsilon}} \int_{-\infty}^{+\infty} \frac{dx}{\pi}\, \frac{e^{-ix(k^2+m^2)(t-t')}\,|x|^{-\varepsilon}}{1+x^2} \tag{3.7}$$

where we use (2.9) and (3.3).

An outstanding feature of the ASR method is that one is led to meromorphic amplitudes for stochastic diagrams constructed upon the rules here exposed. In this case, ultraviolet divergences show up as simple poles in the ε parameter, as in Speer's conventional method[2].

In order to compute stochastic amplitudes associated with gauge theories we need a similar expression for the crossed propagator of the gauge field. We may find, using the "Feynman gauge", a very familiar expression, given by[(6)]:

$$D_{ij}(k;t,t') = \hat{f}_\varepsilon \frac{\delta_{ij}}{(k^2)^{1+\varepsilon}} \int_{-\infty}^{+\infty} \frac{dx}{\pi} \frac{e^{-ixk^2(t-t')}\,|x|^{-\varepsilon}}{1+x^2} \quad . \qquad (3.8)$$

Notice also that the uncrossed propagator, in the same gauge, has the simple form:

$$G_{ij}(k;t-t') = \delta_{ij} \exp\left[-k^2(t-t')\right] \Theta(t-t') \qquad (3.9)$$

commonly used are the graphical conventions, for the fermionic and gauge propagators (crossed and uncrossed) given in fig. I.

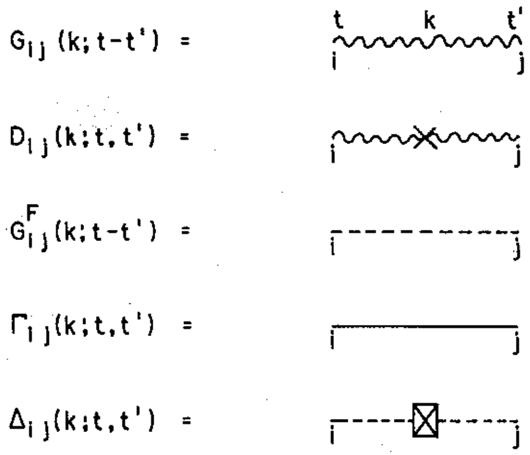

fig.1

4. VACUUM POLARIZATION TENSOR IN SPINOR ELECTRODYNAMICS

We are now ready to consider a non-trivial field theory, like four-dimensional Spinor QED, whose Euclidean action is

$$S[A,\psi,\bar{\psi}] = \int d^4x \left[\frac{1}{4} F_{\mu\nu}F_{\mu\nu} - i\,\bar{\psi}(\not{\partial} - ie\not{A} + iM)\,\psi\right] . \qquad (4.1)$$

These three fields possess Langevin equations, straightforwardly obtained as

$$\dot{\psi}_i(x,t) = -\left[(\not{D} - iM)(\not{D} + iM)\right]_{ij} \psi_j(x,t) + \vartheta_i(x,t) \qquad (4.2)$$

$$\dot{\bar{\psi}}_i(x,t) = -\bar{\psi}_k(x,t) \left[(\not{D}' - iM)^T (\not{D}' + iM)^T\right]_{ji} + \bar{\vartheta}_i(x,t) \qquad (4.3)$$

$$\dot{A}_\mu(x,t) = -\partial_\nu F_{\mu\nu} + e\,\bar{\psi}\gamma_\mu\psi + \eta_\mu(x,t) \qquad (4.4)$$

where we have

$$\not{D}_\mu = -\partial_\mu - ieA_\mu \qquad (4.5)$$

and T stands for matrix transposition. The commuting (η) and anticommuting (ϑ) noises satisfy regularized correlations (3.1).

We are mainly interested in calculate the 1-loop photon propagator correction, in the framework of ASR. In this order, the relevant diagrams to be evaluated are depicted in figure 2. In order to establish whether the ASR preserves gauge invariance, it is necessary to verify transversality of this object, at least for its divergent part. As the one loop ultraviolet divergences occur as poles in ε, the approximation $|x|^{-\varepsilon} \approx 1$ suffices for the crossed propagator expressions. The vertices of this stochastically quantized theory are the same type as that appearing in conventional quantized ones, in the Euclidean space.

Using the standard rules for diagrammatic calculations in SQM[8], we find for the contribution (G-1):

$$(G\text{-}1) = 2e^2 \int_{-\infty}^{t} dt_2' \int_{-\infty}^{t_2'} dt_1' \int \frac{d^4k}{(2\pi)^4}\, G_{\mu\rho}(p;t-t_1')\, G_{\sigma\nu}(p;t-t_2')$$

$$\times\, \mathrm{Tr}\left[\gamma_\mu\, \Delta(p+k;t_1',t_2')\, \gamma_\nu\, \Delta(k;t_1',t_2')\right] \tag{4.6}$$

$$(G\text{-}1) = \frac{\hat{f}_\varepsilon^2\, e^2}{2p^2} \int \frac{d^4k}{(2\pi)^4}$$

$$\times\, \frac{\mathrm{Tr}[\gamma_\mu(-\not{p}-\not{k}+M)\,\gamma_\nu(-\not{k}+M)]}{[(p+k)^2+M^2]^{1+\varepsilon}\,(k^2+M^2)^{1+\varepsilon}\,(p^2+k^2+p.k+M^2)} \,. \tag{4.7}$$

If we take in account the diagrams (G-2) and (G-3) one can find, taking the trace on the integrand, as usual in

loop computations with fermions (details of the algebra are found in the appendix):

$$(G\text{-}2)+(G\text{-}3) = 4e^2 \int_{-\infty}^{t} dt_2' \int_{-\infty}^{t_2'} dt_1' \int \frac{d^4k}{(2\pi)^4}\, G_{\mu\rho}(p;t-t_1')$$

$$\times\, D_{\sigma\nu}(p;t,t_2')\, \mathrm{Tr}\left[\gamma_\mu\, \Delta(k+p;t_1',t_2')\, \gamma_\nu\, \Gamma(k;t_1',t_2')\right] \tag{4.8}$$

$$(G\text{-}2)+(G\text{-}3) = \frac{4\hat{f}_\varepsilon\, e^2}{(p^2)^2} \int \frac{d^4k}{(2\pi)^4}$$

$$\times\, \frac{-\delta_{\mu\nu}(M^2+k^2+k.p) + 2k_\mu k_\nu + p_\nu k_\mu + p_\mu k_\nu}{[(k+p)^2+M^2]^{1+\varepsilon}\,(p^2+k^2+k.p+M^2)} \,. \tag{4.9}$$

These integrals are very difficult to evaluate in a closed form. Although we may obtain exact results in two dimensions, it seems more interesting to use an expansion, taking off the divergent and perhaps finite terms of the expressions. The cornerstone of our method is the analiticity of the polarization tensor for large values of the mass.

Hence, one rescales the loop momentum $k \to Mk$ and expand the problematic integrand in powers of p/M until the order that furnishes divergent pieces, i.e., isolates the simple poles. Further terms in this expansion are finite.

The above integrals, calculated by this way, read (see appendix):

$$(G-1) \;=\; -\frac{\delta_{\mu\nu}}{32\,p^2\,\pi^2\,\varepsilon} \tag{4.10}$$

$$(G-2)+(G-3) = \frac{7\,\delta_{\mu\nu}}{48\,p^2\,\pi^2\,\varepsilon} - \frac{p_\mu\,p_\nu}{12\,\pi^2(p^2)^2\,\varepsilon}\,. \tag{4.11}$$

For the polarization tensor itself, we have (divergent piece):

$$\pi^{(div)}_{\mu\nu}(p) \;=\; \frac{11\,p^2\,\delta_{\mu\nu}}{96\,\pi^2\,\varepsilon} - \frac{p_\mu p_\nu}{12\,\pi^2\,\varepsilon} \tag{4.12}$$

which is non-transverse due to double crossed diagram (G-1), which is half the necessary value - $2(G-1)+(G-2)+(G-3)$ is transverse - in the sense of dimensional regularization with analytically continued dimension $D = 4-2\varepsilon$. The other two diagrams are equal to the corresponding dimensionally regularized ones. This is also true for 3 and 4 point functions in QCD, at one loop order.

5. CONCLUSIONS

These results are in apparent contradiction with those obtained by Gavela and Huffel[10], by using a different type of regulator[11]. They showed the maintenance of gauge invariance,

a result already expected from the naive observation that Stochastic Regularization, due to ficticious time delta function smearing, do not break any physical (space-time or internal) symmetry of the theory. We think there is an over-simplification in this reasoning, because ASR gives a non-gauge invariant counterterm for the gauge field.

However, an exciting observation seems to restore the faithfulness of the method in a completely different context. Based in previous works[5,6], where the ASR was used for scalar electrodynamics and QCD, we found that the (G-1) contribution (4.10) is minus twice the corresponding bosonic value. This would indicate that ASR is a reasonable method to regularize supersymmetric theories, because the problematic bosonic and fermionic contributions cancels in some supermultiplets.

In a recent paper[7], we consider Spinor QCD and propose the usefulness of the method for two classes of supersymmetric models, namely:

- The coupling of a gauge field with a supersymmetric matter multiplet (two bosonic charged fields and one Dirac field);

- The case of N=1 Supersymmetric Yang-Mills theory with one Majorana fermion in the adjoint representation. The scalar matter contribution is given by the non-abelian self-interaction with a combinatorial factor of two.

We have to draw also an important conclusion, from the detail of the computation. In one loop diagram with one cross, the result is the same as in dimensional regularization. As we verified, all results are the same in the non-abelian case. That is why the conclusions above were drawn in this case.

We expect for the future a more rigorous proof of these matters, i.e., that there is no breaking of gauge invariance and supersymmetry induced by ASR prescription.

APPENDIX

In this appendix, we show the intermediate steps necessary to find the stochastic amplitudes associated with the diagrams (G-2) + (G-3) . There are four topologically equivalent diagrams need to be considered. In order to integrate over the ficticious internal times, we make

$$t_1 < t_2 < t'_1 = t'_2 = \text{finite ficticious time} \quad ,$$

because we start our Langevin process with $t = -\infty$.

Using the "Feynman rules" (3.10), we rewrite (4.8) as $(e = 1)$:

$$(G-2) + (G-3) = 4\hat{f}_\varepsilon^2 \int_{-\infty}^{t} dt'_2 \int_{-\infty}^{t'_2} dt'_1 \int \frac{d^4x}{(2\pi)^4} \int_{-\infty}^{+\infty} \frac{dx_1}{\pi} \frac{e^{-ix_1 p^2 (t-t'_2)}}{1+x_1^2}$$
$$\times \int_{-\infty}^{+\infty} \frac{dx_2}{\pi} \frac{e^{-ix_2[k+p)^2+M^2](t'_2-t'_1)}}{1+x_2^2}$$
$$\times \frac{e^{-p^2(t-t'_1)-(k^2+M^2)(t'_2-t'_1)} \; \mathrm{Tr}[\gamma_\mu(-\not{k}-\not{p}+M)\,\gamma_\nu(-\not{k}+M)]}{(p^2)^{1+\varepsilon}\,[(k+p)^2+M^2]^{1+\varepsilon}} \quad . \tag{A.1}$$

The trace in the above expression is very easy to compute using the Euclidean Clifford algebra

$$\left\{\gamma_\mu , \gamma_\nu\right\} = -2\delta_{\mu\nu} \tag{A.2}$$

and a simple algebra gives (4.9) as well.

Due to a symmetry among the diagrams, we may take $k \to -k-p$ and proceed as prescribed. Expanding in powers of the external momenta we have:

$$(G-2) + (G-3) =$$

$$= \frac{4\hat{f}_\varepsilon^2 M^2}{(p^2)^2} \int \frac{d^4k}{(2\pi)^4} \frac{-\delta_{\mu\nu}(1+k^2+k_\lambda p_\lambda) + 2k_\mu k_\nu + \left(\frac{p}{M}\right)_\nu k_\mu + \left(\frac{p}{M}\right)_\mu k_\nu}{(k^2+1)^{2+\varepsilon}}$$

$$\otimes \left[1 - \frac{k_\rho\left(\frac{p}{M}\right)_\rho + \left(\frac{p}{M}\right)^2}{k^2+1} - \frac{k_\rho k_\sigma\left(\frac{p}{M}\right)_\rho\left(\frac{p}{M}\right)_\sigma}{(k^2+1)^2}\right] \tag{A.3}$$

which gives nothing less than twelve integrals to be evaluated, with the help of the simple formula:

$$\int \frac{d^Dk}{(2\pi)^D} \frac{(k^2)^n}{(k+1)^{m+\varepsilon}} = \frac{1}{(2\pi)^D} \frac{\Gamma(D/2+n)(m-n+D/2+\varepsilon)}{\Gamma(D/2)\,\Gamma(m+\varepsilon)} \tag{A.4}$$

and their sum gives (4.11).

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FIGURE CAPTIONS

Fig. 1 - Graphical conventions for the Feynman rules.

Fig. 2 - QED 2-point function with one and two internal crossed lines.

fig. 02