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EXCITED STATES IN STOCHASTIC ELECTRODYNAMICS

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ABSTRACT

We show that the set of Wigner functions associated with the excited states of the harmonic oscillator constitute a complete set of functions over the phase space. An arbitrary probability distribution can be expanded in terms of these Wigner functions. By studying the time evolution, according to Stochastic Electrodynamics, of the expansion coefficients, we are able to separate explicitly the contributions of the radiative reaction and the vacuum field to the Einstein-A coefficients for this system. We also supply a simple semiclassical explanation of the Weisskopf-Heitler phenomenon in resonance fluorescence.

## 1) Introduction

Stochastic Electrodynamics<sup>(1,2)</sup> (SED) is a revival of Panck's second theory. Since it treats the electromagnetic field as a c-number, it may be considered as the oldest semiclassical radiation theory. Reviews<sup>(3,4)</sup> of such theories have generally concentrated on the version of Jaynes, in which there is no vacuum field. The conclusion has been reached that semiclassical theories are inadequate, because of their inability to explain various phenomena, including the coincidence counting of the "photons" in an atomic cascade<sup>(5)</sup>, the "anticorrelation" in two channels of a beam splitter<sup>(6)</sup>, and the "antibunching" observed in resonance fluorescence<sup>(7)</sup>. Recently it has been demonstrated, however, that an optical theory based on SED is capable of explaining all of these phenomena<sup>(8)</sup>, and hence this conclusion is incorrect.

Although the modern revival of SED preceded Jaynes' theory, it has received little attention in the literature. This is probably because, in its original conception, it was overambitious. It treated the electron as a classical point particle, and set out to explain the whole of nonrelativistic Quantum Mechanics (Q.M.) through the interaction of such a particle with the zeropoint electromagnetic field. The programme was remarkably successful for just one quantum system -the harmonic oscillator- but thirty years study of non linear systems has given only one useful result -a correct estimate of the size of the hydrogen atom<sup>(9)</sup>. We have to recognize that, in the absence of new "classical" ideas, which could include, for example, a stochastic version of the electron-positron field, a more modest version of SED should be used. This would continue to insist on the reality of the zeropoint electromagnetic field, and hence on its representation as a c-number. However, it would describe the motion of electrons inside atoms through the nonrelativistic Schrödinger equation. This more modest version of SED would be only semiclassical and not classical.

At this point we may note that a certain convergence has occurred between Quantum Electrodynamics in the optical region and SED. Thirty years ago there was little recognition in Quantum Electrodynamics that the zeropoint field was anything other than a source of "virtual photons". Now, with the emergence of quantum optics, especially the study of the atomic equations of motion

in their Heisenberg form<sup>(3)</sup>, the vacuum field has a much more "real" status. The idea that the ground state of an atom represents some kind of balance between radiative reaction and vacuum field effects certainly originated in SED, but it is now being proposed as a respectable quantum-mechanical description. Parallel with this is the idea that "spontaneous" emission, which Einstein, its discoverer, always considered so unsatisfactory, is actually stimulated by the zeropoint field. We shall show, in this article, that, by returning to the harmonic oscillator we can make more precise the contributions respectively of the radiative reaction and vacuum field to Einstein's A coefficients. This may also cast some light on the general problem of how frictional forces may be included in the quantum formalism.

Our treatment will also show more clearly the status of "negative probability" in Physics. We shall argue that non positive definite distributions can enter into the description of physical systems only as a calculating device. They cannot be properly interpreted as distributions of probability and only those linear combinations which have the property of positive definiteness may be said to describe real physical states. This has a radical consequence which has been recognized in SED long ago<sup>(10)</sup>: no excited state of the harmonic oscillator is a real physical state, and hence a statement that a given atom "is" in a given excited state may not be correct.

One argument<sup>(11)</sup> in favour of "negative probabilities" involves the classical diffusion of a particle between two reflecting walls; the eigenfunctions of the diffusion operator are not positive definite. Since nobody questions that, in this classical process, the particle is always in some definite place, we infer that these eigenfunctions cannot, on their own, represent physical states of the diffusing particle. We shall show that the Wigner functions of the excited states of the harmonic oscillator play a similar role with respect to the Fokker-Planck operator, which gives the time evolution of an arbitrary distribution in phase space. None of these functions, except that representing the ground state, are, however, positive definite. We infer that only certain linear combinations of these excited-state distributions, that is certain mixtures of the first kind, represent real physical states.

## II) Completeness of the excited states in phase space.

A hamiltonian  $H$ , for instance a one dimensional hermitian, operator, may, in certain cases, possess a complete set of excited states  $\phi_n(x)$  such that

$$(2.1) \quad H \phi_n(x) = E_n \phi_n(x)$$

where  $E_n$  are the corresponding energy levels ( $n=0,1,2 \dots$ ) and the eigenfunctions  $\phi_n(x)$  satisfy the completeness relation

$$(2.2) \quad \sum_{n=0}^{\infty} \phi_n^*(x) \phi_n(y) = \delta(x-y)$$

The time evolution, in Quantum Mechanics, of the hamiltonian system specified by  $H$  is then given by:

$$(2.3) \quad \psi(x,t) = \sum_{n=0}^{\infty} a_n \phi_n(x) \exp(-iE_n t/\hbar),$$

where

$$(2.4) \quad a_n = \int_{-\infty}^{\infty} dx \phi_n^*(x) \psi(x,0)$$

Consider now the Wigner function  $W(x,p,t)$  associated with  $\psi(x,t)$

$$(2.5) \quad W(x,p,t) \equiv \int_{-\infty}^{\infty} \frac{dy}{\pi\hbar} \psi^*(x+y,t) \psi(x-y,t) \exp(2ipy/\hbar).$$

The time evolution of  $W(x,p,t)$  is

$$(2.6) \quad W(x,p,t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_m^* a_n W_{m/n}(x,p) \exp[i(E_m - E_n)t/\hbar],$$

where  $a_m^* a_n$  are constant and

$$(2.7) \quad W_{m/n}(x,p) = \int_{-\infty}^{\infty} \frac{dy}{\pi\hbar} \phi_m^*(x+y) \phi_n(x-y) \exp(2ipy/\hbar).$$

Real physical systems do not evolve in time according to (2.6); all states, except the ground state ( $n=0$ ), undergo decays by spontaneous emission of radiation. We propose to study such processes also through the evolution of  $W(x,p,t)$ , substituting for (2.6) the more general relation:

$$(2.8) \quad W(x,p,t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{m/n}(t) W_{m/n}(x,p).$$

The explicit expressions for  $C_{m/n}(t)$  will be obtained below, for the case of free and forced harmonic oscillator, within the realm of SED.

In order to be sure that such an expansion is possible, we need a completeness property for the set  $\{W_{m/n}\}$ . This is guaranteed through the following theorem.

Theorem 1: The set of functions  $W_{m/n}(x,p)$  is complete and orthogonal over the space  $(x,p)$ .

Proof: According to the definition (2.5) we can write

$$(2.9) \quad \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} W_{m/n}^*(x,p) W_{m/n}(y,q) = \\ = (\pi\hbar)^{-2} \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta \exp\left[\frac{2i}{\hbar}(q\eta - p\xi)\right] \sum_{m=0}^{\infty} \phi_m(x+\xi) \phi_m^*(y+\eta) \cdot \\ \cdot \sum_{n=0}^{\infty} \phi_n(y-\eta) \phi_n^*(x-\xi) = \frac{\delta(x-y)\delta(q-p)}{2\pi\hbar},$$

where we have used (2.2) and the Fourier integral representation of the Dirac delta function.

This establishes the property of completeness. The property of orthogonality is obtained in a similar manner and we give only the result

$$(2.10) \quad \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dp W_{m/n}^*(x,p) W_{r/s}(x,p) = \frac{\delta_{mr} \delta_{ns}}{2\pi\hbar}.$$

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In the following two sections we shall consider the special case of the harmonic oscillator, for which the hamiltonian is

$$(2.11) \quad H = -\frac{\hbar^2}{2m_0} \frac{\partial^2}{\partial x^2} + \frac{1}{2} m_0 \omega_0^2 x^2$$

where  $m_0$  is the mass of the particle and  $\omega_0$  the frequency. Then the excited state wave functions can be expressed as

$$(2.12) \quad \phi_n(x) = (2^n n!)^{-1/2} \left( \frac{m_0 \omega_0}{\pi \hbar} \right)^{1/4} H_n \left( x \sqrt{\frac{m_0 \omega_0}{\hbar}} \right) \exp\left( -\frac{m_0 \omega_0 x^2}{2\hbar} \right),$$

where  $H_n$  are Hermite polynomials.

In this case each function of the complete set of Wigner functions  $W_{mn}(x,p)$ , defined in (2.7), can be calculated explicitly giving<sup>(12)</sup>

$$(2.13) \quad W_{mn}(x,p) = \frac{(-1)^m}{\pi \hbar} z^{\frac{n-m}{2}} \left( \frac{m!}{n!} \right)^{1/2} \exp\left( -\frac{m_0 \omega_0 x^2}{\hbar} - \frac{p^2}{m_0 \omega_0 \hbar} \right).$$

$$\cdot \left( x \sqrt{\frac{m_0 \omega_0}{\hbar}} - \frac{ip}{\sqrt{\hbar m_0 \omega_0}} \right)^{n-m} L_{n-m}^m \left( \frac{2m_0 \omega_0 x^2}{\hbar} + \frac{2p^2}{m_0 \omega_0 \hbar} \right)$$

if  $m \leq n$ . Here  $L_n^\alpha$  are associated Laguerre polynomials. In the case  $m > n$  we get a similar result, that is, we must interchange  $m$  with  $n$  everywhere and also replace  $i$  by  $-i$ .

We shall see that for studying transitions induced by thermal radiation, the diagonal subset of  $\{W_{mn}\}$  is sufficient. The functions of this subset, which may be termed "Wigner functions of the excited states", are given by

$$(2.14) \quad W_{nn}(x,p) = \frac{(-1)^n}{\pi \hbar} L_n(z) \exp(-z) \equiv U_n(z),$$

where  $z = m_0 \omega_0 x^2 / \hbar + p^2 / m_0 \omega_0 \hbar$  and  $L_n \equiv L_n^0$  is a Laguerre polynomial.

In this case it is possible to prove another completeness result.

Theorem 2: The set of functions  $U_n(z)$  is complete and orthogonal in the sense:

$$(2.15) \quad \int_0^\infty dz U_m(z) U_n(z) = \frac{\delta_{mn}}{2\pi^2 \hbar^2}$$

and

$$(2.16) \quad \sum_{n=0}^\infty U_n(z) U_n(z') = \frac{\delta(z-z')}{2\pi^2 \hbar^2}$$

These results follow directly from the standard orthogonality and completeness properties of the Laguerre polynomials.

### III) The Einstein A and B coefficients in SED

There has been a great deal of controversy about the respective roles of the vacuum field and radiative reaction in the spontaneous emission of radiation. Milonni<sup>(3)</sup> has shown, by making a suitable ordering of the operators in the second-quantized formalism, that the contribution of either of these fields may be reduced formally to zero. Such a treatment suggests that there is no real difference between the vacuum and radiative reaction fields.

In SED the electromagnetic field, instead of being an operator, is a c-number. A product of fields is independent of its ordering, and "field commutators" are just a coded way of specifying the various autocorrelations of the zeropoint field. We can therefore make an unambiguous distinction between the vacuum fields and the radiation reaction.

The motion of a harmonic oscillator in SED is given by a Langevin-type equation

$$(3.1) \quad m_0 \ddot{x} = -m_0 \omega_0^2 x - m_0 z \dot{x} + e E_x(t) \quad ; \quad \gamma \equiv \frac{e^2 \omega_0^2}{3 m_0 c^3} ,$$

where the term  $-z \dot{x}$  in the equation of motion has its origin in the radiation reaction force  $ze^2 \ddot{x}/3c^3$ . The random force  $e E_x(t)$  is generated by the x component of the fluctuating electromagnetic field, that is,  $E_x(t)$  is a gaussian stochastic process with a spectral density given by:

$$(3.2) \quad I_0(\omega) = \int_{-\infty}^{\infty} d\tau \langle E_x(t) E_x(t+\tau) \rangle \exp(i\omega\tau) .$$

Such a system differs from a Brownian harmonic oscillator in only one respect; the noise  $E_x(t)$  is not white, so that  $I_0$  is not a constant function of  $\omega$ . If we choose for  $I_0(\omega)$  the spectral density of the zeropoint field

$$(3.3) \quad I_0(\omega) = \frac{1}{2} \omega^3 / 6\pi^2 c^3$$

then, because of the linearity of (3.1),  $x(t)$  is also a gaussian process. In particular the stationary phase-space distribution is the ground state distribution given by equation (2.14)

$$(3.4) \quad W(x,p) = U_0(z) = \frac{\exp(-z)}{\pi \hbar}$$

In order to understand the role of excited states in SED, we now consider the time-dependent case. It is convenient to treat "spontaneous" emissions along with those stimulated by incident radiation. This means considering a general spectral density  $I(\omega)$  instead of just the zeropoint field  $I_0(\omega)$ . However, because the zeropoint field is not directly observable we write

$$(3.5) \quad I(\omega) = I_0(\omega) + I_{obs}(\omega)$$

where  $I_{obs}(\omega)$  is the observable part of the incident spectrum. The phase-space distribution  $W(x,p,t)$  satisfies the Fokker-Planck equation<sup>(10)</sup>:

$$(3.6) \quad \frac{\partial W}{\partial t} = \gamma \frac{\partial}{\partial \xi} (\xi W) + \gamma \frac{\partial}{\partial \eta} (\eta W) + D \left( \frac{\partial^2 W}{\partial \xi^2} + \frac{\partial^2 W}{\partial \eta^2} \right) ,$$

where  $\xi$  and  $\eta$  are constants of the deterministic motion, given by

$$\xi = x \sqrt{\frac{m_0 \omega_0}{\hbar}} \cos(\omega_0 t) - \frac{p}{\sqrt{m_0 \omega_0 \hbar}} \sin(\omega_0 t)$$

$$\eta = x \sqrt{\frac{m_0 \omega_0}{\hbar}} \sin(\omega_0 t) + \frac{p}{\sqrt{m_0 \omega_0 \hbar}} \cos(\omega_0 t)$$

and

$$(3.7) \quad D = \frac{(\pi e)^2}{m_0 \omega_0 \hbar} I(\omega_0)$$

is the diffusion coefficient. Because  $\xi^2 + \eta^2 = z$  it is easy to show that

the stationary solution of (3.6) is  $W = U_0(z) = \frac{\chi\psi(z)}{\pi\hbar}$  in the particular case in which  $D = D_0 \equiv \frac{(\pi e)^2 I_0(\omega_0)}{m_0 \omega_0 \hbar} = \gamma/2$

A further transformation is convenient

$$(3.8) \quad \xi = \sqrt{z} \cos\varphi \quad ; \quad \eta = \sqrt{z} \sin\varphi \quad ,$$

where  $z$  is the same as before (see 2.14). Then (3.6) becomes

$$(3.9) \quad \frac{\partial W}{\partial t} = 2\gamma \left( z \frac{\partial W}{\partial z} + W \right) + D \left( 4z \frac{\partial^2 W}{\partial z^2} + 4 \frac{\partial W}{\partial z} + \frac{1}{z} \frac{\partial^2 W}{\partial \varphi^2} \right)$$

We observe that a system which, at  $t=0$ , is in equilibrium with the zeropoint spectrum ( $W = U_0(z)$ ), and which, for  $t > 0$ , interacts with any field of thermal (that is incoherent) radiation, is always homogeneous with respect to the variable  $\varphi$ . We may therefore, for such a system, discard the derivative with respect to  $\varphi$  in (3.9), and consider  $W$  as a function of  $z$  and  $t$  only. Then using theorem 2 of the previous section, we can put

$$(3.10) \quad W(z,t) = \sum_{n=0}^{\infty} C_n(t) U_n(z)$$

where  $U_n(z)$  was defined before in (2.14) and

$$(3.11) \quad C_n(t) = 2\pi^2 \hbar^2 \int_0^{\infty} z U_n(z) W(z,t)$$

The time evolution of the coefficients  $C_n(t)$  can be obtained by substituting (3.10) into (3.9). Using the recurrence relation between the Laguerre polynomials  $L_n^{\alpha}(z)$  we get:

$$(3.12) \quad \sum_{n=0}^{\infty} \dot{C}_n(t) U_n(z) = \sum_{n=0}^{\infty} C_n(t) U_n(z) \cdot$$

$$\cdot \left\{ \gamma \left[ -n \delta_{n,n+1} + \delta_{n,n} + (n+1) \delta_{n,n-1} \right] + \right. \\ \left. + 2D \left[ n \delta_{n,n-1} - (2n+1) \delta_{n,n} + (n+1) \delta_{n,n-1} \right] \right\}$$

Since the functions  $U_n(z)$  are orthogonal we must have

$$(3.13) \quad \dot{C}_n(t) = \left\{ n(-\gamma + 2D) \delta_{n,n+1} + [\gamma - 2D(2n+1)] \delta_{n,n} + \right. \\ \left. + (\gamma + 2D)(n+1) \delta_{n,n-1} \right\} C_n(t) \equiv \\ \equiv (P_{n,n+1} + P_{n,n} + P_{n,n-1}) C_n(t)$$

With the above definition of  $P_{n,n+1}$ ,  $P_{n,n}$  and  $P_{n,n-1}$  we can interpret these as "transition probabilities", per unit time, from the mathematical (not physical) states  $n$  to  $n+1$ ,  $n$  to  $n$  and  $n$  to  $n-1$  respectively. The explicit expressions for these "transition probabilities" are

$$(3.14) \quad P_{n,n+1} = (n+1)(2D - \gamma) \\ P_{n,n} = \gamma - (2n+1)2D \\ P_{n,n-1} = n(2D + \gamma)$$

Here, for instance in the expressions for  $P_{n,n+1}$  and  $P_{n,n-1}$ , we can easily identify the role of radiation reaction due to the presence of  $\gamma$ . In a "downwards transition", like  $P_{n,n-1}$ , we have a positive contribution of radiation reaction as is expected on physical grounds. However in a "upwards transition", like  $P_{n,n+1}$ , we obtain a negative contribution of radiation reaction.

To obtain the Einstein A and B coefficients we substitute (3.5) in the expression for  $D$  given in (3.7). Then

$$(3.15) \quad P_{n,n+1} = B_{n,n+1} I_{obs}(\omega_0) + A_{n,n+1} \\ P_{n,n-1} = B_{n,n-1} I_{obs}(\omega_0) + A_{n,n-1}$$

where

$$(3.16) \quad B_{n,n+1} = \frac{2(n+1)\pi^2 e^2}{\hbar m_0 \omega_0} ; \quad A_{n,n+1} = 0$$

and

$$(3.17) \quad B_{n,n-1} = \frac{2n\pi^2 e^2}{\hbar m_0 \omega_0} ; \quad A_{n,n-1} = \frac{2n e^2 \omega_0^2}{3 m_0 c^3}$$

These results are in full agreement with the formal calculations of Quantum Electrodynamics. In particular, the coefficient  $A_{n,n+1}$  of "spontaneous absorption" is zero and we now see that this is because of a balance between the effects of radiation reaction and vacuum fields ( $\gamma = 2D_0$  as was seen before). Also the coefficient  $A_{n,n-1}$  of "spontaneous emission" has equal contributions coming from these two sources ( $\gamma$  and  $2D_0$ ). The equality between  $B_{n,n+1}$  and  $B_{n+1,n}$  also follows from (3.16) and (3.17).

We note that these conclusions agree with those reached by Milonni<sup>(13)</sup>, though his arguments had a more quantum-mechanical flavour. We also note that one of these results, equation (3.17) for  $A_{n,n-1}$ , is a rather old result in SED<sup>(14)</sup>.

We stress again, however, that the "states" given by the "distributions"  $U_n(z)$  are, in SED, nothing more than a convenient complete set of functions. Their role with respect the Fokker-Planck equation (3.9), is very similar to that of the functions

$$(3.18) \quad V_n(x) = \cos(nx) ; \quad n = 0, 1, 2, \dots$$

With respect to the diffusion problem

$$(3.19) \quad \frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} ; \quad \left( \frac{\partial V}{\partial x} = 0 \text{ at } x = 0, \pi \right)$$

None of the solutions (3.18) derived from (3.19) represents by itself a physical solution, with the sole exception of the "ground state",  $n=0$ . We could describe the solution

$$(3.20) \quad V(x,t) = 1 + \alpha \cos(x) \exp(-t)$$

as a decay of the "mixture" of states  $n=0$  and  $n=1$  into the state  $n=0$  as  $t$  tends to infinity. But we would not, in this context, allow such language to obscure the fact that such a mixture may be a physical state only for those values of  $\alpha$ , namely  $-1 < \alpha < 1$ , for which  $V$  is positive definite. The same considerations apply to the excited states of the harmonic oscillator, and also, possibly, to the excited states of atoms. The state  $n=1$  is represented by the Wigner function (see 2.14)

$$(3.21) \quad W_{11}(x,p) = \frac{(2z-1) \exp(-z)}{\pi \hbar}$$

which is negative for  $z < 1/2$ . If we consider mixtures of this with the ground state  $W_{00}$

$$(3.22) \quad W(x,p) = \frac{W_{00}(z) + \alpha W_{11}(z)}{1 + \alpha}$$

then such states corresponds to physical states only if  $0 < \alpha < 1$ .

The language we are warning against is, of course, strongly entrenched in the literature. It is now becoming fashionable to describe resonance fluorescence as the radiation emitted by an atom which is oscillating coherently between two states. We shall show in the following section that such language is no more justified than in the case we have just been studying.



## IV) Resonance fluorescence in SED

Resonance fluorescence occurs when an atom is driven by a coherent electromagnetic field whose frequency is close to one of its transition frequencies. It was predicted, long before sufficiently coherent sources became available, that the radiation emitted should have the linewidth of the driving field, rather than the natural linewidth of the transition<sup>(16,17)</sup>. Although the prediction was based in Quantum Electrodynamics, it has always been considered somewhat paradoxical, though it is now well verified experimentally.

In SED this Weisskopf-Heitler phenomenon receives a very natural explanation. A driven harmonic oscillator has a Langevin-type equation

$$(4.1) \quad \ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = \frac{e}{m_0} [E_x(t) - F \sin(\omega t)]$$

In the absence of the zeropoint field  $E_x(t)$  we have the deterministic solution

$$(4.2) \quad x_d = \text{Re} \left[ \frac{ie F \exp(i\omega t)}{m_0(\omega_0^2 - \omega^2 + 2i\gamma\omega)} \right]$$

In the absence of  $F$  we have the zeropoint stationary solution  $x_z$  which is wellknown<sup>(1,2)</sup> in SED. Then the steady-state solution of (4.1) is

$$(4.3) \quad x = x_d + x_z$$

Now the radiation emitted by this system at a given frequency is obtained from the autocorrelation of  $\ddot{x}$ , that is

$$(4.4) \quad K(\tau) = \langle \ddot{x}(t) \ddot{x}(t+\tau) \rangle$$

Because  $x_d$  and  $x_z$  are independent we have

$$(4.5) \quad K(\tau) = \langle \ddot{x}_d(t) \ddot{x}_d(t+\tau) \rangle + \langle \ddot{x}_z(t) \ddot{x}_z(t+\tau) \rangle$$

The first term on the right hand side represents radiation at the single frequency  $\omega$  (which is the frequency of the deterministic motion  $x_d$ ), while the second term represents a line of width  $\gamma$  centred on the frequency  $\omega_0$  as is wellknown in SED<sup>(1,2)</sup>. But we saw in the previous section that the ground state of the oscillator, which is represented by the stochastic process  $x_z$ , is an equilibrium state in which the radiation emitted at a given frequency is equal, on average, to the radiation absorbed at that frequency. Hence only the sharp line, at frequency  $\omega$  of the deterministic force, is observed.

The phase-space distribution in the steady state, characterized by (4.3), is easily obtained. Since  $x_z$  is independent of  $x_d$ , the joint distribution of  $x_z$  and  $p_z = m_0 \dot{x}_z$  is simply the ground state, given by (3.4):

$$(4.6) \quad W(x_z, p_z) = \frac{1}{\pi \hbar} \exp\left(-\frac{m_0 \omega_0}{\hbar} x_z^2 - \frac{p_z^2}{m_0 \omega_0 \hbar}\right)$$

Hence the distribution of  $(x, p) = (x_d + x_z, p_d + p_z)$  is<sup>(18)</sup>

$$(4.7) \quad W(x, p) = \frac{1}{\pi \hbar} \exp\left[-\frac{m_0 \omega_0}{\hbar} (x - x_d)^2 - \frac{(p - p_d)^2}{m_0 \omega_0 \hbar}\right]$$

According to theorem 1 of the section II we may write this as

$$(4.8) \quad W(x, p, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{mn}(t) W_{mn}(x, p)$$

where

$$(4.9) \quad C_{mn}(t) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dp W_{mn}^*(x, p) W(x, p, t)$$

These expansion coefficients may be obtained most easily by using the expression (2.7) for  $W_{mn}(x, p)$ , together with the generating functions for the Hermite polynomials. The result is

$$(4.10) \quad C_{mn}(t) = \left[ \frac{\sigma(t)^{m+n}}{m! n!} \right]^{1/2} \exp(-\sigma(t) + i(m-n)[\omega t + \delta(t)])$$

where

$$(4.11) \sqrt{\sigma(t)} \exp[i\delta(t)] = \left[ \sqrt{\frac{m_0 \omega_0}{2\hbar}} X_d(t) + i \sqrt{\frac{m_0}{2\omega_0 \hbar}} \dot{X}_d(t) \right] \exp(-i\omega t),$$

and  $X_d(t)$  is given by (4.2). In general  $\sigma$  and  $\delta$  both have a small periodic part, but for the case of resonance they take the constant values

$$(4.12) \quad \sigma = \frac{e^2 F^2}{8 m_0 \gamma^2 \omega_0 \hbar}, \quad \delta = 0 \quad (\omega = \omega_0)$$

For this resonance case, (4.8) with (4.10) is readily recognized as the quantum mechanical coherent state<sup>(19)</sup>; it corresponds to the state (2.6) with the coefficients

$$(4.13) \quad a_n = \left( \frac{\sigma^n}{n!} \right)^{1/2} \exp(-\sigma/2)$$

This is remarkable because such a state has no formal radiative corrections, whereas the SED description corresponds to that of Quantum Electrodynamics rather than to Q.M.. Just as the ground state of Q.M. represents a balance between radiative reaction and zero point field, so does the coherent state represent a balance between these two fields and the driving field. Because of the time variation in  $\sigma$  and  $\delta$ , such an identification is not so simple for the general, off-resonance case, so (4.7), or equivalently (4.10) and (4.11), give an SED generalization of the coherent state, by the inclusion of the radiative correction.

#### V) Conclusion

By a study of the harmonic oscillator in SED, we have shown that, for this system at least, the only role of the excited states is to provide us with a suitable complete set of Wigner functions in terms of which an arbitrary phase-space probability distribution may be expanded. Consideration of both spontaneous emission and resonance fluorescence leads us to the conclusion that these states have no independent physical existence.

There are, of course, many elementary facts of atomic physics which can not be described by a harmonic oscillator model; for example the Ritz combination principle for line spectra, which gave birth to the excited-state concept, remains unexplained in the completely classical version of SED. That is why we have argued, in the introductory section, for a new semiclassical version of SED.

Nevertheless, we think our results show that it is necessary to treat the excited states with some care. In quantum theory, and in SED, a harmonic oscillator driven by a coherent field at resonance does not alternate between its various excited states. The coefficients  $a_n \exp(-iE_n t/\hbar)$  of (2.3) or  $C_{mn}(t)$  given by (4.10) have time independent modulus; it is their varying phases which give rise to the emission of radiation at the frequency of the driving field. To be sure, quantum theory asserts that a measurement of the energy causes a collapse of  $\Psi$ , leading to a discontinuous change in  $a_n$ . Furthermore, current descriptions of, for example, the "shelved-state" experiment<sup>(20)</sup> interpret as a "measurement" the intervention caused by a second laser operating at a different resonant frequency. However in view of many acknowledged difficulties of quantum theory of measurement, especially the non local features associated with the collapse of the wave function, we think it is premature to conclude that this is the only possible explanation of such experiments. Another possible explanation may come from Stochastic Optics (a branch of SED) which has already given a completely local description for many phenomena which it was previously thought displayed "quantum nonlocality". We therefore believe that further study of SED, especially in its semiclassical version, is justified.

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References

- (1) T.H. Boyer; Phys. Rev. D11, 809 (1975). See also T.H. Boyer in Foundations of Radiation theory and quantum electrodynamics, A.O. Barut (ed.), Plenum, N.Y. (1980).
- (2) L. de la Peña in Stochastic Processes Applied to Physics and other Related Fields, ed. B. Gomez, S.M. Moore, A.M. Rodriguez-Vargas and A. Rueda (World Scientific, Singapore, 1982), p. 428.
- (3) P.W. Milonni; Phys. Lett. 25C, 1 (1976).
- (4) L. Mandel; Prog. Optics 13, 27 (1976).
- (5) A. Aspect, P. Grangier and G. Roger, Phys. Rev. Lett. 49, 91 (1982).
- (6) P. Grangier, G. Roger and A. Aspect; Europhysics Letters 1, 173 (1986).
- (7) M. Dagenais and L. Mandel; Phys. Rev. A18, 2217 (1978).
- (8) T.W. Marshall and E. Santos; Europhys. Lett., 3, 293 (1987); Ann. N.Y. Acad. Sci., 480, 400 (1986); Proc. 2nd Int. Symp. "Foundations of Quantum Mechanics", ed. M. Namiki, Y. Ohmuki, Y. Murayama and S. Nomura, Physical Society of Japan, 1986, p. 66; "Stochastic Optics: A Reaffirmation of the Wave Nature of Light", to be published in Foundations of Physics.
- (9) T.H. Boyer, Phys. Rev. D11, 790 (1975).
- (10) T.W. Marshall; Proc. Roy. Soc. 276A, 475 (1963).
- (11) R.P. Feynman; "Quantum Implications Essays in honour of David Bohm", ed. Routledge and Kegan Paul (1987) p. 235.
- (12) I.S. Gradshteyn and I.M. Ryzhik, Table of Integrals Series and Products, Acad. Press N.Y. (1965) p. 838, formula 7.377.
- (13) P.W. Milonni; Am. J. Phys. 52, 340 (1984).
- (14) T.W. Marshall, Izv. Vuz, Fizika, 11, 34 (1968) (see English translation "Soviet Physics Izvestia").
- (15) P.L. Knight and P.W. Milonni; Phys. Lett. 66C, 21 (1980).

- (16) V. Weisskopf; Ann. Phys. 9, 23 (1931).
- (17) W. Heitler; "The Quantum theory of Radiation" (Oxford Univ. Press, 1954) pp. 348-353.
- (18) H.M. França and M.T. Thomaz; Phys. Rev. D31, 1337 (1985).
- (19) L.I. Schiff; "Quantum Mechanics" (McGraw-Hill Book Company, N.Y.), 1965, p. 74.
- (20) Richard G. Brewer in Proc. 2nd Int. Symp. "Foundation of Quantum Mechanics" (see ref. (8)).