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SURFACE TENSION IN FIELD THEORY AT FINITE TEMPERATURE: SEMICLASSICAL FERMIONIC PLUS BOSONIC CONTRIBUTIONS

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ABSTRACT

We compute, within the one loop approximation, the temperature dependent surface tension for a model involving a scalar field coupled to a fermionic field. The fermionic and bosonic contribution have been computed in this approximation by evaluating explicitly the determinants in the presence of a domain wall.

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I. INTRODUCTION

It has been suggested[(1-5)] that the analysis of the Free Energies associated to different topological defects constitutes an important artifact in order to analyze the phase diagram of field theoretic models.

At the classical level the Free Energies associated to these defects give their total mass and are positive. Quantum--thermal fluctuations however, can change this picture for certain ranges of the coupling constants and the temperature[(1-5)]. For high enough temperatures it can be shown, in the semiclassical approximation, that the Free Energy of certain defects vanish even in the region of weak coupling constants. In this situation the system represented by this Field Thoery is expected to go to a condensate of such defects, representing a new phase.

In earlier papers[(1-3)], we proposed that the restauration of spontaneously broken discrete symmetries by thermal effects can be viewed as a condensation of wall-like defects which yield a background state of zero mean field value. That is an alternative view to the Effective Potential approach[(6)] to the same problem. In teh latter view, the background over which the quantum vacuum is constructed is always thought to be a constant value field (the lowest energy solution to the classical equation of motion) and it is shown that above a certain critical temperature is driven from the non zero value (representing a breakdown of the symmetry) to a zero value which means restoration of symmetry.

In (3) we studied the one loop Free Energy of wall--like defects in the model of a Higgs field interacting with a fermion field by a Yukawa coupling. There, the fermionic determinant was calculated by quadrating it and using a high temperature expansion[2] due to Weinberg[7].

In the present paper we return to this problem by calculating the fermionic determinant without any approximation. A better care is also given to the finite renormalization parts, which improve the determination of the mass of the soliton-wall, even at zero temperature.

II. THE MODEL

We shall use the imaginary-time formalism to study the system at finite temperature. So it is convenient to write the model in the ν-dimensional Euclidean space. The Lagrangean density is

$$L = \frac{1}{2}(\partial_\mu\varphi)^2 - \overline{\psi}\not\partial\psi + \frac{\lambda}{4}\left(\varphi^2 - \frac{m^2}{\lambda}\right)^2 - ig\overline{\psi}\psi\varphi \qquad (2.1)$$

where λ and m^2 are positive constants, the index μ runs from 1 to ν ($\nu = 2$ or 4), $\overline{\psi}$ stands for $\psi^+\gamma^0$ and $\not\partial$ for $\gamma^\mu\partial_\mu$.

The Euclidean Dirac matrices γ^μ are defined in terms of the usual Minkowsky ones by

$$\gamma^\mu = (\vec{\gamma}, -i\gamma^0) \quad ; \quad \mu = 1, \ldots, \nu$$

and satisfy the anticomutation rules

$$\left\{\gamma^\mu, \gamma^\rho\right\} = -2\,\delta^{\mu\rho} \quad .$$

The renormalized Lagrangian of the model can be written as

$$L = \frac{1}{2}(1+A)(\partial_\mu\varphi)^2 + \frac{1}{2}(-m^2+B)\varphi^2 + \frac{1}{4}(\lambda+C)\varphi^4 -$$

$$- (1+F)\,\overline{\psi}\not\partial\psi - i(g+G)\,\overline{\psi}\psi\varphi + \left(\frac{m^4}{4\lambda} + H\right) \qquad (2.2)$$

where A, B, C, F, G and H are the counterterms necessary to render the physical quantities finites.

As usual, to implement the loop expansion around a classical solution of the field equations we separate the fields in the form

$$\begin{aligned} \varphi &= \varphi_c + \Phi \\ \psi &= \psi \end{aligned} \qquad (2.3)$$

where a factor $\hbar^{\frac{1}{2}}$ is assumed multiplying Φ and ψ. Similarly, the contribution to the counterterms coming from the one loop contributions carry a factor of $\hbar$.

By substituting (2.3) in (2.2) and using the equation of motion of φ_{class}, the renormalized Lagrangian density turns out to be

$$L = L_c + L_0 + L_{int} \tag{2.4}$$

where

$$L_c = \frac{1}{2}(\partial_\mu \varphi_c)^2 + \frac{\lambda}{4}\left(\varphi_c^2 - \frac{m^2}{\lambda}\right)^2 \tag{2.5a}$$

$$L_0 = \frac{1}{2}\Phi(-\Box + 3\lambda\varphi_c^2 - m^2)\Phi - \bar{\psi}(\partial\!\!\!/ + ig\varphi_c)\psi \tag{2.5b}$$

$$\begin{aligned} L_{int} = {} & \frac{A}{2}(\partial_\mu\varphi_c)^2 + \frac{B}{2}\varphi_c^2 + \frac{C}{4}\varphi_c^4 + \lambda\varphi_c\Phi^3 + \frac{\lambda}{4}\Phi^4 - \\ & - ig\bar{\psi}\psi\Phi + (-A\Box\varphi_c + B\varphi_c + C\varphi_c)\Phi + \\ & + \frac{A}{2}(\partial_\mu\Phi)^2 + \left(\frac{B}{2} + \frac{3}{2}C\varphi_c^2\right)\Phi^2 + F\bar{\psi}\partial\!\!\!/\psi - \\ & - iG\varphi_c\bar{\psi}\psi + C\varphi_c\Phi^3 - iG\bar{\psi}\psi\Phi + \frac{C}{4}\Phi^4 + H \; . \end{aligned} \tag{2.5c}$$

Of course, at semiclassical level we need to keep only the terms up to order $\hbar$. Thus, the relevant contributions in (2.5c) are the three first ones, with the counterterms A, B and C evaluated at the one loop approximation. To fix these counterterms we will impose

the renormalization conditions:

$$\tilde{\Gamma}^{(1)} = 0 \tag{2.6a}$$

$$\tilde{\Gamma}^{(2)}(p^2=0) = 2m^2 \tag{2.6b}$$

$$\left.\frac{d\tilde{\Gamma}^{(2)}}{dp^2}\right|_{p^2=0} = 1 \tag{2.6c}$$

for the 1PI functions calculated up to one loop order in the vacuum sector, i.e., $\varphi_c = \varphi_v = m/\sqrt{\lambda}$. These calculations are carried out in the appendix and the results are listed therein.

III. DOMAIN WALL SURFACE TENSION

Let us now calculate the surface tension of a domain wall-like defect at finite temperature. In a ν-dimensional space-time this topological defect can be represented by a solution to the classical equation of motion. If x_L is the coordinate perpendicular to the plane of the wall (we call it the longitudinal coordinate) we have

$$\varphi_w = \frac{m}{\sqrt{\lambda}}\tanh\left(\frac{m}{\sqrt{2}}x_L\right) \quad . \tag{3.1}$$

The surface tension of this wall-like defect is defined as

$$\tau_w = \frac{1}{L^{\nu-2}}\left\{F(\varphi_w) - F(\varphi_v)\right\} \tag{3.2}$$

where $L^{\nu-2}$ is the "area" of the wall and $F(\varphi_c)$ is the free energy associated to the classical field configuration φ_c. In terms of the corresponding partition function $Z(\varphi_c)$, we have

$$\tau_w = -\frac{1}{\beta L^{\nu-2}} \ln \frac{Z(\varphi_w)}{Z(\varphi_v)} \quad . \tag{3.3}$$

Up to the one loop order $Z(\varphi_c)$ is given by

$$Z(\varphi_c) = \exp\left\{-S_c(\varphi_c) - S_{cT}(\varphi_c)\right\} \times$$

$$\oint D\Phi\, e^{-S_{bil}(\Phi)} \oint D\bar{\psi}\, D\psi\, e^{-S_{bil}(\bar{\psi},\psi)} \tag{3.4}$$

where

$$S_c(\varphi_c) = \int d^\nu x \left\{\frac{1}{2}(\partial_\mu \varphi_c)^2 + \frac{\lambda}{4}\left(\varphi_c^2 - \frac{m^2}{\lambda}\right)^2\right\} \tag{3.5}$$

$$S_{cT}(\varphi_c) = \int d^\nu x \left\{\frac{1}{2} A (\partial_\mu \varphi_c)^2 + \frac{1}{2} B \varphi_c^2 + \frac{C}{4}\varphi_c^4\right\} \tag{3.6}$$

$$S_{bil}(\Phi) = \int d^\nu x\, \Phi\, \hat{\Omega}_B(\varphi_c)\Phi \tag{3.7}$$

$$S_{bil}(\bar{\psi},\psi) = \int d^\nu x\, \bar{\psi}\, \hat{\Omega}_F(\varphi_c)\psi \tag{3.8}$$

with

$$\hat{\Omega}_B(\varphi_c) = -\Box + 3\lambda\varphi_c^2 - m^2 \tag{3.9}$$

$$\hat{\Omega}_F(\varphi_c) = -(\not{\partial} + i g \varphi_c) \tag{3.10}$$

The integration in the Euclidean time extends from zero to $\beta = 1/T$ and the functional integration in the bosonic (fermionic) field assumes periodic (antiperiodic) boundary conditions in the Euclidean time. By formally doing these integrations, the surface tension of the wall may be written as

$$\tau_w = \tau_c + \tau_{cT} + \tau_B + \tau_F \tag{3.11}$$

where

$$\tau_c = M_s = \frac{2\sqrt{2}\, m^3}{3\lambda} \tag{3.12}$$

$$\tau_{cT} = \frac{\sqrt{2}\, m^3}{3\lambda} A - \frac{\sqrt{2}\, m}{\lambda} B - \frac{2\sqrt{2}\, m^3}{3\lambda^2} C \tag{3.13}$$

and

$$\tau_B = \frac{1}{2\beta L^{\nu-2}}\left\{\mathrm{tr}\,\ln \hat{\Omega}_B(\varphi_w) - \mathrm{tr}\,\ln \hat{\Omega}_B(\varphi_v)\right\} \tag{3.14}$$

$$\tau_F = \frac{1}{\beta L^{\nu-2}}\left\{\mathrm{tr}\,\ln \hat{\Omega}_F(\varphi_w) - \mathrm{tr}\,\ln \hat{\Omega}_F(\varphi_v)\right\} \tag{3.15}$$

with

$$\hat{\Omega}_B(\varphi_v) = -\Box + 2m^2 \tag{3.16a}$$

$$\hat{\Omega}_B(\varphi_w) = -\Box + 2m^2 - 3m^2\,\mathrm{sech}^2\left(\frac{m}{\sqrt{2}}x_L\right) \tag{3.16b}$$

$$\hat{\Omega}_F(\varphi_v) = -\not{\partial} - I\,g\frac{m}{\sqrt{\lambda}} \tag{3.17a}$$

$$\hat{\Omega}_F(\varphi_w) = -\not{\partial} - I\,g\frac{m}{\sqrt{\lambda}}\tanh\left(\frac{m}{\sqrt{2}}x_L\right) \tag{3.17b}$$

In writing (3.14) and (3.15) we used the identity $\det\hat{O} = \exp\{\mathrm{tr}\,\ell n\,\hat{O}\}$.

A. The bosonic contribution.

Let us compute (3.14), i.e., the bosonic contributions to the surface tension. The vacuum sector is trivial and the eigenvalues of the operator $\hat{\Omega}_B(\varphi_v)$ are

$$\Omega_B(\varphi_v) = \omega_n^2 + \vec{k}^2 + 2m^2 \tag{3.18}$$

where $\beta\omega_n = 2n\pi$ $(n = 0, \pm1, \pm2, \ldots)$ and $\vec{k}$ is a continuum of $(\nu-1)$-dimensional momenta.

For the soliton sector, i.e., for the wall-like defect we are led to the well known Poschl-Teller problem - see, for instance, ref. (8) - in the longitudinal direction. Therefore, the eigenvalues of $\hat{\Omega}_B(\varphi_w)$ are

$$\Omega_B(\varphi_w) = \begin{cases} \omega_n^2 + \vec{k}_\parallel^2 \\ \omega_n^2 + \vec{k}_\parallel^2 + \frac{3}{2}m^2 \\ \omega_n^2 + \vec{k}_\parallel^2 + k_L^2 + 2m^2 \end{cases} \tag{3.19}$$

where $\beta\omega_n = 2n\pi$, $\vec{k}_\parallel$ is a continuum of $(\nu-2)$-dimensional momenta parallel to the wall and k_L is the longitudinal momentum and the phase-shift is

$$\delta(k_L) = 2\left\{\pi - \sum_{\ell=0,1}\arctan\left[\frac{\sqrt{2}\,k_L}{(\ell+1)\,m}\right]\right\}. \tag{3.20}$$

Using (3.18)-(3.20) we get for (3.14)

$$\tau_B = \frac{1}{2\beta L^{\nu-2}}\left\{\sum_{n=-\infty}^{\infty}\int\frac{L^{\nu-2}d^{\nu-2}k}{(2\pi)^{\nu-2}}\right\}\Bigg\{\ell n(\omega_n^2+\vec{k}_\parallel^2) + $$
$$+ \ell n(\omega_n^2+\vec{k}_\parallel^2+\tfrac{3}{2}m^2) + \int\frac{L\,dk_L}{2\pi}\left(1-\frac{1}{L}\,\delta(k_L)\,\frac{d}{dk_L}\right).$$
$$.\ \ell n(\omega_n^2+\vec{k}_\parallel^2+k_L^2+2m^2) - \int\frac{L\,dk_L}{2\pi}\,\ell n(\omega_n^2+\vec{k}_\parallel^2+k_L^2+2m^2)\Bigg\}.$$

Substituting (3.20) into the last expression and using the well-known identity[6]

$$\sum_{n=-\infty}^{\infty}\ell n\left(\omega^2+\frac{4\pi^2n^2}{\beta^2}\right) = \beta\omega + 2\,\ell n\,(1-e^{-\beta\omega}) + I \quad ,$$

where I is a ω independent infinite, we obtain

$$\tau_B = \int \frac{d^{\nu-2}k_\parallel}{(2\pi)^{\nu-2}} \left\{ \frac{1}{2}\sqrt{\vec{k}_\parallel^2} + \frac{1}{2}\sqrt{\vec{k}_\parallel^2 + \frac{3}{2}m^2} + \right.$$

$$+ \frac{1}{\beta}\ln\left(1 - e^{-\beta\sqrt{\vec{k}_\parallel^2}}\right) + \frac{1}{\beta}\ln\left(1 - e^{-\beta\sqrt{\vec{k}_\parallel^2 + \frac{3}{2}m^2}}\right) -$$

$$- \frac{3\sqrt{2m^2}}{2\pi} + \sqrt{2m^2}\int \frac{dk_L}{2\pi}\left(\frac{1}{2k_L^2 + m^2} + \frac{1}{k_L^2 + 2m^2}\right) \times$$

$$\left. \times \left[\sqrt{\vec{k}_\parallel^2 + k_L^2 + 2m^2} + \frac{2}{\beta}\ln\left(1 - e^{-\beta\sqrt{k^2 + k_L^2 + 2m^2}}\right)\right]\right\} \qquad (3.21)$$

B. The fermionic contribution

Let us now calculate (3.15), i.e., the fermionic contribution to the surface tension. For the vacuum sector, charge conjugation, allow us to write

$$\det(\not\partial + i\,g\,\varphi_v I) = \det^{\frac{1}{2}}\left[(-\Box + g^2\varphi_v^2)\,I\right] \quad . \qquad (3.22)$$

where I is the identity matrix. Thus we obtain

$$\mathrm{tr}\,\ln\hat{\Omega}_F(\varphi_v) = \frac{\nu}{2}\,\mathrm{tr}\,\ln\hat{\Omega}_{BF}(\varphi_v) \qquad (3.23a)$$

where

$$\hat{\Omega}_{BF}(\varphi_v) = -\Box + g^2\varphi_v^2 \qquad (3.23b)$$

whose eigenvalues are

$$\Omega_{BF}(\varphi_v) = \bar{\omega}_n^2 + \vec{k}^2 + g^2\varphi_v^2 \qquad (3.24)$$

where $\beta\bar{\omega}_n = (2n+1)\pi$ $(n = 0, \pm1, \pm2, \ldots)$ and $\vec{k}$ is a continuum of $(\nu-1)$-dimensional momenta.

For the soliton sector, i.e., for the domain wall-like defect, the choice

$$\gamma_L = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and the use of charge conjugation invariance allow us to write

$$\det(\not\partial + ig\varphi_w I) = \det^{\nu/4}\hat{\Omega}_{BF}^{(+)}(\varphi_w)\,.\,\det^{\nu/4}\hat{\Omega}_{BF}^{(-)}(\varphi_w) \qquad (3.25)$$

where

$$\Omega_{BF}^{(\pm)}(\varphi_w) = -\Box + g^2\varphi_w^2 \pm g\frac{\partial\varphi_w}{\partial x_L} \quad . \qquad (3.26a)$$

Therefore we obtain

$$\ln\det(\not\partial + ig\varphi_w I) = \mathrm{tr}\,\ln(\not\partial + ig\varphi_w I) =$$

$$= \frac{\nu}{4}\,\mathrm{tr}\,\ln\hat{\Omega}_{BF}^{(+)}(\varphi_w) + \frac{\nu}{4}\,\mathrm{tr}\,\ln\hat{\Omega}_{BF}^{(-)}(\varphi_w) \quad . \qquad (3.26b)$$

By substituting (3.1) in (3.26) and defining the parameter s as

$$S^2 = \frac{2g^2}{\lambda} \tag{3.27}$$

we obtain

$$\hat{\Omega}^{(\pm)}_{BF}(\varphi_w) = -\Box + \frac{m^2}{2}\left[S^2 - S(S\pm 1)\ \mathrm{sech}^2\left(\frac{m}{\sqrt{2}}x_L\right)\right] . \tag{3.28}$$

Since λ and g are independent constants, $S \equiv (2g^2/\lambda)^{\frac{1}{2}}$ is an arbitrary positive constant. The eigenvalues of (3.28) are easily obtained, but the corresponding ones of $\det \Omega_{BF}(\varphi_w)$ are very length and complicated expressions.

To get insight into this problem without facing these technical difficulties we choose to evaluate it for S an integer positive number. In this case the associated Poschl--Teller problem has a discrete plus a nonreflexive continuum spectra[8] (like the bosonic contribution above). The results for $\Omega^{(\pm)}_{BF}(\varphi_w)$ are:

$$\Omega^{(+)}_{BF}(\ell,S) = \bar{\omega}_n^2 + \vec{k}_\parallel^2 + m^2\ell(S-\tfrac{\ell}{2}) \quad ; \quad \ell = 0,1,2,\dots,S-1$$

$$\Omega^{(+)}_{BF}(k_L,S) = \bar{\omega}_n^2 + \vec{k}_\parallel^2 + k_L^2 + \tfrac{1}{2}m^2S^2 \tag{3.29}$$

with the phase shift:

$$\delta^{(+)}_S(k_L) = S\pi - 2\sum_{\ell=0}^{S-1}\arctan\left[\frac{\sqrt{2}\,k_L}{m(\ell+1)}\right]$$

$$\Omega^{(-)}_{BF}(\ell,S) = \bar{\omega}_n^2 + \vec{k}_\parallel^2 + m^2(\ell+1)\left(S-\frac{\ell+1}{2}\right) \quad ; \ \ell = 0,1,2,\dots,S-2$$

$$\Omega^{(-)}_{BF}(k_L,S) = \bar{\omega}_n^2 + \vec{k}_\parallel^2 + k_L^2 + \tfrac{1}{2}m^2S^2 \tag{2.30}$$

with the phase shift:

$$\delta^{(-)}_S(k_L) = (S-1)\pi - 2\sum_{\ell=0}^{S-2}\arctan\left[\frac{\sqrt{2}\,k_L}{m(\ell+1)}\right] .$$

Collecting the above results and using the following identity

$$\sum_{n=-\infty}^{\infty}\ell n\left(\frac{(2n-1)^2\pi^2}{\beta^2}+\omega^2\right) = \omega\beta + 2\,\ell n\left(1+e^{-\beta\omega}\right) + I ,$$

we get, after some manipulations paralelling the bosonic case,

$$\tau_F = -\frac{\nu}{4}\int\frac{d^{\nu-2}k_\parallel}{(2\pi)^{\nu-2}}\Bigg\{\sqrt{\vec{k}_\parallel^2} + 2\sum_{\ell=1}^{S-1}\sqrt{\vec{k}_\parallel^2 + m^2\ell(S-\tfrac{\ell}{2})} +$$

$$+\frac{2}{\beta}\ell n\left(1+e^{-\beta\sqrt{\vec{k}_\parallel^2}}\right) + \frac{4}{\beta}\sum_{\ell=1}^{S-1}\ell n\left(1+e^{-\beta\sqrt{\vec{k}_\parallel^2+m^2\ell(S-\frac{\ell}{2})}}\right) -$$

$$-\frac{\sqrt{2m^2}\,S^2}{2\pi} - \sqrt{2}\,m\int\frac{dk_L}{2\pi}\Bigg[S\frac{\sqrt{\vec{k}_\parallel^2+k_L^2+\frac{1}{2}m^2S^2}}{k_L^2+\frac{1}{2}m^2\ell^2} +$$

$$+2\sum_{\ell=1}^{S-1}\ell\frac{\sqrt{\vec{k}_\parallel^2+k_L^2+\frac{1}{2}m^2S^2}}{k_L^2+\frac{1}{2}m^2\ell^2} + \frac{2}{\beta}S\frac{\ell n\left(1+e^{-\beta\sqrt{\vec{k}_\parallel^2+k_L^2+\frac{1}{2}m^2S^2}}\right)}{k_L^2+\frac{1}{2}m^2S^2} +$$

$$+ \frac{4}{\beta}\sum_{\ell=1}^{S-1} \ell \, \frac{\ln\left(1 + e^{-\beta\sqrt{\vec{k}_{\parallel}^{2} + k_L^2 + \frac{1}{2} m^2 S^2}}\right)}{k_L^2 + \frac{1}{2} m^2 \ell^2} \Bigg]\Bigg\} \; . \qquad (3.31)$$

IV. CANCELLATION OF DIVERGENCIES AND QUANTUM CORRECTIONS

We shall now analyze the renormalized expression (3.11) for the surface tension. We first separate the zero temperature contributions to τ_B and τ_F and look at the cancellation of their divergencies against the τ_{cT} contributions.

A. Bosonic zero-temperature corrections

Collecting together the temperature independent contribution to the expression (3.21) and the bosonic ones to (3.13) (look at formulas (A-6) to (A-8) in the appendix) we obtain for $\epsilon_B = \tau_B(0) + \tau_{cT_B}$:

$$\epsilon_B(\nu) = \int \frac{d^{\nu-2} k_{\parallel}}{(2\pi)^{\nu-2}} \left\{ \frac{1}{2}\sqrt{\vec{k}_{\parallel}} + \frac{1}{2}\sqrt{\vec{k}_{\parallel} + \frac{3}{2} m^2} - \frac{3\sqrt{2m^2}}{2\pi} + \right.$$

$$\left. - \sqrt{2}\, m \int \frac{dk_L}{2\pi}\left[\frac{1}{2}\, \frac{1}{k_L^2 + \frac{m^2}{2}} + \frac{1}{k_L^2 + 2m^2}\right] \sqrt{\vec{k}^2 + k_L^2 + 2m^2} \right\} +$$

$$+ 3\sqrt{2}\, m \int \frac{d^{\nu}k}{(2\pi)^{\nu}} \left\{ \frac{1}{k^2 + 2m^2} + \frac{m^2}{(k^2+2m^2)^2} + \right.$$

$$\left. + 2\, \frac{4-\nu}{\nu}\, \frac{m^4}{(k^2+2m^2)^3} - \frac{16}{\nu}\, \frac{m^6}{(k^2+2m^2)^4} \right\} \; . \qquad (4.1)$$

From now on we will take $\nu = 4$. After regularizing the integrations above with convenient cut-off in k_L and $|\vec{k}_{\parallel}|$ we get

$$\epsilon_B(4) = -\left(\frac{1}{\sqrt{3}} - \frac{1}{\pi}\right) \frac{\sqrt{2}\, m^3}{26\pi} - \frac{\sqrt{2}\, m^3}{32\pi^2} \; . \qquad (4.2)$$

The last term in (4.2) was separated to allow a comparation of ϵ_B to related results in the literature[1,9,10]; it comes from finite contributions to the counterterms: A, B and C.

B. Fermionic corrections

By extracting the temperature independent contributions to (3.31) and the fermionic ones to (3.13) (look at formulas (A-6) to (A-8) in the apppendix) we get, for $\epsilon_F = \tau_F(0) + \tau_{cT_F}$:

$$\frac{4}{\nu}\, \epsilon_F(\nu) = \int \frac{d^{\nu-2} k_{\parallel}}{(2\pi)^{\nu-2}} \left\{ -\sqrt{\vec{k}_{\parallel}^{2}} - 2\sum_{\ell=1}^{S-1} \sqrt{\vec{k}_{\parallel}^{2} + m^2 \ell \left(S - \frac{\ell}{2}\right)} + \right.$$

$$\left. + \frac{\sqrt{2m^2}\, S^2}{2\pi} \right\} +$$

$$+\sqrt{2m^2}\int \frac{d^{\nu-1}k}{(2\pi)^{\nu-1}}\left\{S\frac{\sqrt{k_L^2+\vec{k}_\parallel^2+\frac{m^2S^2}{2}}}{k_L^2+\frac{m^2S^2}{2}}+\sum_{\ell=1}^{S-1}2\ell\frac{\sqrt{k_L^2+\vec{k}_\parallel^2+\frac{m^2S^2}{2}}}{k_L^2+\frac{m^2\ell^2}{2}}\right\}+$$

$$-2\sqrt{2m^2}\,S^2\int\frac{d^\nu k}{(2\pi)^\nu}\left\{\frac{1}{k^2+\frac{m^2S^2}{2}}+\left(\frac{\nu-2}{3\nu}-\frac{5}{6}S^2\right)\frac{m^2}{\left(k^2+\frac{m^2S^2}{2}\right)^2}+\right.$$

$$\left.+\frac{5-\nu}{3\nu}S^2\frac{m^4}{\left(k^2+\frac{m^2S^2}{2}\right)^3}-\frac{2S^4}{3\nu}\frac{m^6}{\left(k^2+\frac{m^2S^2}{2}\right)^4}\right\}. \qquad (4.3)$$

For $\nu=4$ the fermionic contribution to the wave function renormalization counterterm is infinite and is essential to cancel the divergencies. After performing the integrations in (4.3), with cut-offs for k_L and $|\vec{k}_\parallel|$, we get

$$\epsilon_F = \frac{\sqrt{2}\,m^2}{32\pi^2} \qquad \text{for} \qquad S=1\ (\lambda=2g^2) \qquad (4.4)$$

$$\epsilon_F = -\frac{\sqrt{2}\,m^3}{32\pi^2}\left(\frac{44}{3}-\frac{8\pi}{\sqrt{3}}\right) \qquad \text{for} \qquad S=2\ \left(\lambda=\frac{g^2}{2}\right). \qquad (4.5)$$

V. THE THERMAL EFFECTS

A. The bosonic contributions

From the result (3.21), the one loop thermal effects, appearing in the bosonic determinant, may be written

in the form

$$\tau_B(\beta) = \frac{1}{\beta}\int\frac{d^{\nu-2}k_\parallel}{(2\pi)^{\nu-2}}\left[b_0(\beta)+b_1(\beta)\right] \qquad (5.1)$$

where

$$b_0(\beta) = \ln\left[1-e^{-\beta\sqrt{k_\parallel^2}}\right]-2\sqrt{2}\,m\int\frac{dk_L}{2\pi}\frac{1}{k_L^2+2m^2}\ln\left[1-e^{-\beta\sqrt{k_\parallel^2+k_L^2+2m^2}}\right] \qquad (5.2)$$

$$b_1(\beta) = \ln\left[1-e^{-\beta\sqrt{k_\parallel^2+\frac{3}{2}m^2}}\right]-2\sqrt{2}\,m\int\frac{dk_L}{2\pi}\frac{1}{2k_L^2+m^2}\ln\left[1-e^{-\beta\sqrt{k_\parallel^2+k_L^2+2m^2}}\right]$$

These expressions were obtained in ref. (2).

In the high-temperature limit, the bosonic contribution to the domain wall surface tension ($\nu=4$) is given by

$$\tau_B(\beta) = -\frac{\sqrt{2}\,m}{4\beta^2}+O\left(\frac{1}{\beta}\right). \qquad (5.3)$$

B. Fermionic contributions

The one loop thermal effect appearing in the fermionic determinant may be extracted from (3.31). For the particular value $S=1$ ($\lambda=2g^2$) it has the form

$$\tau_F^{(1)}(\beta) = -\frac{1}{\beta}\int\frac{d^{\nu-2}k_\parallel}{(2\pi)^{\nu-2}}f_0^{(1)}(\beta) \qquad (5.4)$$

where

$$f_0^{(1)}(\beta) = \ell n \left[1 + e^{-\beta\sqrt{k_\parallel^2}} \right] +$$

$$+ 2\sqrt{2}\, m \int \frac{dk_L}{2\pi} \frac{1}{2k_L^2 + m^2} \ell n \left[1 + e^{-\beta\sqrt{k^2 + k_L^2 + \frac{1}{2} m^2}} \right] . \qquad (5.5)$$

For $S = 2 \quad (2\lambda = g^2)$ it is given by

$$\tau_F^{(2)}(\beta) = -\frac{1}{\beta} \int \frac{d^{\nu-2} k_\parallel}{(2\pi)^{\nu-2}} \left[f_0^{(2)}(\beta) + 2 f_1^{(2)}(\beta) \right] \qquad (5.6)$$

where

$$f_0^{(2)}(\beta) = \ell n \left[1 + e^{-\beta\sqrt{k_\parallel^2}} \right] + 2\sqrt{2}\, m \int \frac{dk_L}{2\pi} \frac{1}{2k_L^2 + m^2} \ell n \left[1 + e^{-\beta\sqrt{k_\parallel^2 + k_L^2 + 2m^2}} \right]$$

$$(5.7)$$

$$f_1^{(2)}(\beta) = \ell n \left[1 + e^{-\beta\sqrt{k_\parallel^2 + \frac{3}{2} m^2}} \right] + 2\sqrt{2}\, m \int \frac{dk_L}{2\pi} \frac{1}{k_L^2 + 2m^2} \ell n \left[1 + e^{-\beta\sqrt{k_\parallel^2 + k_L^2 + 2m^2}} \right] .$$

In the high-temperature limit, the fermionic contribution to the domain wall surface tensions $(\nu = 4)$ is given by

$$\tau_F(\beta) = -\frac{\sqrt{2}\, m}{12\beta^2} + O\left(\frac{1}{\beta}\right) \quad ; \quad S = 1 \ (\lambda = 2g^2)$$

$$(5.8)$$

$$\tau_F(\beta) = -\frac{\sqrt{2}\, m}{3\beta^2} + O\left(\frac{1}{\beta}\right) \quad ; \quad S = 2 \ (2\lambda = g^2) \quad .$$

Expressions (5.4) to (5.7) are exact. The $\tau_F(\beta)$ was previously calculated in ref. (3) by doing a high temperature approximation to the fermionic determinant. There, we used the fact that among all the one-loop graphs appearing in the expansion of $\ell n \det \hat{O}_F$ around the vacuum, the leading contribution, (αT^2), comes from the graph with only one propagator.

VI. CONCLUSIONS

In this paper we have computed, within the one-loop approximation, the surface tension associated to domain walls when scalar fields are coupled to Fermionic Fields. In this context, it is interesting to implement a careful renormalization of the surface tension. We have checked explicitly that the usual perturbative counterterms suffices to render the expressions finite even at finite temperature and in the presence of a topological defect. After a careful renormalization we obtained for the zero temperature surface tension

$$\varepsilon(T=0) = \frac{2\sqrt{2}\, m^3}{3\lambda} - \frac{\sqrt{2}\, m^3}{32\pi^2} \left(\frac{2\pi}{\sqrt{3}} - 1 \right) +$$

$$+ \frac{\sqrt{2}\, m^3}{32\pi^2} \begin{Bmatrix} 1 & \text{for} \quad \lambda = 2g^2 \\ -\left(\frac{44}{3} - \frac{8\pi}{\sqrt{3}} \right) & \text{for} \quad \lambda = \frac{g^2}{2} \end{Bmatrix} \qquad (6.1)$$

In (6.1), the first term is the classical result, the second is the bosonic contribution and the last one is the fermionic contribution that was computed for two values of λ. As pointed out in chapter IV, the bosonic contribution differs from previous ones presented in the literature[1,9,10].

The dependence of the surface tension with the temperature can be infered from expressions (3.31) and (5.5). At high temperatures, one gets for the leading contribution (in powers of T),

$$\varepsilon(T) = \varepsilon(T=0) - m\alpha T^2 \qquad (6.2)$$

where $\varepsilon(T=0)$ is the zero temperature surface tension and α a positive parameter that depends on the ratio of the coupling constant $\frac{\lambda}{g^2}$. We have computed α explicitly for $\lambda = 2g^2$ and $\lambda = \frac{g^2}{2}$ obtaining $\alpha = \frac{\sqrt{2}}{3}$ and $\alpha = \frac{\sqrt{2}\ 7}{12}$ respectively.

The first conclusion to be drawn from our results is that quantum effects seems to lower the surface tension. That means that the cost for introducing a domain wall in the system, when quantum effects are considered, is lower than the classical prediction. It is even conceivable that for a certain range of the physical parameters (m,g,λ) the surface tension vanishes. (For the model studied here from (6.1), $\lambda \sim 10^2$ and consequently out of the range of validity of the approximation).

The vanishing of the surface tension signals a new phase of the theory[5]. In this new phase condensation of domain walls occurs and consequently symmetry restoration. This is precisely what happens when the temperature is high enough. In fact there is a critical temperature fo which

$$\varepsilon(T_c) = 0 \quad .$$

From expression (4.2) it follows that

$$T_c = \sqrt{\frac{\varepsilon(T=0)}{m\alpha\left(\frac{\lambda}{g^2}\right)}} \quad .$$

For $\lambda = \frac{g^2}{2}$ we obtain explicitly

$$T_c^2 = \frac{24\ m^2/\lambda}{7}\left[\frac{1}{3} - \frac{\lambda}{32\pi^2}\left(\frac{1}{\sqrt{3}} - \frac{1}{\pi}\right)\right] .$$

in the $\lambda \to 0$ limit we obtain the result of ref. (1).

APPENDIX

In this appendix we present the calculation of the counterterms A, B and C to the one loop order. From (2.5) given in the text, in the vacuum sector L_c vanishes and L_0 reduces to

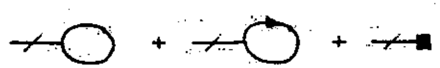
$$L_0 = \frac{1}{2}\,\phi(-\Box + 2m^2)\phi - \bar{\psi}(\not\partial + iM)\psi \tag{A-1}$$

where $M = mg/\sqrt{\lambda}$. The relevant L_{int} for our purpose is

$$L_{int} = m\sqrt{\lambda}\,\phi^3 + \frac{\lambda}{4}\,\phi^4 - i\,g\,\bar{\psi}\psi\phi + \frac{m}{\sqrt{\lambda}}\left(B + \frac{m^2}{\lambda}\,C\right)\phi +$$

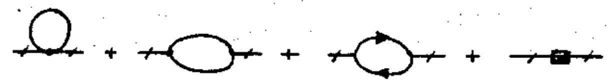
$$- \frac{A}{2}\,\phi\Box\phi + \frac{1}{2}\left(B + 3\,\frac{m^2}{\lambda}\,C\right)\phi^2 \quad . \tag{A-2}$$

The one loop contributions to $\Gamma^{(1)}$ and $\Sigma^{(2)}$ at zero temperature, are:

$\bar{\Gamma}^{(1)} =$ 

$$= -\frac{m}{\sqrt{\lambda}}\left\{3\lambda\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{k^2+2m^2} - \nu g^2\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{k^2+M^2} + B + \frac{m^2}{\lambda}\,C\right\} \tag{A-3}$$

and

$\Gamma^{(2)} =$  $+ (p^2+2m^2)$

$$= -3\lambda\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{k^2+2m^2} + 18m^2\lambda\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+2m^2)[(p+k)^2+2m^2]} +$$

$$+ \nu g^2\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{p.k + k^2 - M^2}{(k^2+M^2)[(p+k)^2+M^2]} - \left(Ap^2 + B + 3\frac{m^2}{\lambda}\,C\right) + (p^2+2m^2). \tag{A-4}$$

By introducing the renormalization conditions (2.6) we obtain:

$$C = 9\lambda^2\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+2m^2)^2} - \nu g^4\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+M^2)^2} \tag{A-5}$$

$$B = -3\lambda\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+2m^2)} - 9\lambda m^2\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+2m^2)^2} +$$

$$+ \nu g^2\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{k^2+M^2} + \nu g^2 M^2\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+M^2)^2} \tag{A-6}$$

and

$$A = \frac{4-\nu}{\nu}\,18m^2\lambda\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+2m^2)^3} - \frac{144m^4\lambda}{\nu}\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+2m^2)^4} +$$

$$+ (2-\nu)g^2\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+M^2)^2} + (\nu-5)\,2g^2M^2\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+M^2)^3} +$$

$$+ 8\,g^2M^4\int\frac{d^\nu k}{(2\pi)^\nu}\,\frac{1}{(k^2+M^2)^4} \quad . \tag{A-7}$$

The only divergent contribution in one loop to A comes from the fermionic loop in four-dimensions.

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