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ABSTRACT

We discuss the interaction of 2-dimensional bosons with an electromagnetic field, quantizing the system via Dirac procedure. A quantum theory is obtained. However, due to a mass generated for the bosonic field, there is an incompatibility between the chiral constraint and Lorentz symmetry, unless an "a priori undetermined" parameter a is fixed.

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1. INTRODUCTION

Frequently, symmetries do not survive the quantization procedure. A major example of such anomalous conservation law is given by chiral gauge theories[1], where gauge invariance is broken at the quantum level. In the case of 2-dimensional spinor QED and QCD, there have been several proposals[2][3], cancelling anomalies introducing Wess-Zumino terms[4]. Consistent quantization of the chiral Schwinger model has been achieved by Jackiw and Rajaraman[5], who showed the existence of a one parameter family of chiral Schwinger models. Girotti, Rothe and Rothe[6] quantized the bosonized theory with the Wess-Zumino term via Dirac procedure[7] for constrained systems. Thus QED_2 has been shown to depend on the value of Jackiw-Rajaraman parameter, and may be presented as two types of theories, with either two or four constraints.

The nonabelian case was considered in references [8] and [9]. In the former reference, the author based his discussion on Wittens[10] bosonization prescription using Coleman's arguments[11]. In the latter work, a "first principle" derivation of the effective action involving the WZ term is given, and thereafter Dirac quantization procedure was applied. As in the abelian case there were two cases, involving either 2 or 4 constraints, the latter in a distinguished value of the JR's a-parameter.

In the present paper we discuss the coupling of chiral bosons to an abelian gauge field.

Chiral bosons have been recently studied by a number of authors[12][13]. It has been consistently quantized[14] using a non-local Lagrangean and a local commutation rule, or equivalently a local Lagrangean but a non-local commutation rule.

It was also shown that its quantization presents the same problems as those found in string theories[15].

In section 2 we consider the coupling to a gauge field, writing the (classically) gauge invariant action, and the constraints. In section 3, we derive the constrained algebra, and quantize the theory via Dirac Brackets. For a particular value of the arbitrary parameter <u>a</u> we see that further constraints arise. In section 4 we briefly review the equations of motion, and discuss the full solution of the theory. Section 5 concludes the paper.

2. CHIRAL SCALAR QED_2

A pure left moving bosonic field $\varphi(x_0-x_1)$ may be described by the Lagrangean

$$\mathbf{L} = -\frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi \qquad (2.1)$$

with the constraint

$$\partial_-\varphi = 0$$

which may be canonically quantized, as far as we are able to deal with the second class constraint

$$\Omega(x) = \pi(x) - \varphi'(x) = 0 \quad . \qquad (2.2)$$

Some authors proposed to substitute the above constraint by the first class constraint[12]

$$L(x) = \Omega(x)^2 \qquad (2.3)$$

which is dealt by the action[16]

$$\mathbf{L} = -\frac{1}{2}\sqrt{g}\,g^{\alpha\beta}\partial_\alpha\varphi\,\partial_\beta\varphi \qquad (2.4)$$

with the gravity field truncated as

$$\sqrt{g}\,g^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ 1 & -\lambda^{--} \end{pmatrix} \qquad (2.5)$$

which leaves us with the expression

$$\mathbf{L} = -\partial_+\varphi\,\partial_-\varphi + \frac{1}{2}\lambda^{--}(\partial_-\varphi)^2 \qquad (2.6)$$

where the doubly self dual field λ^{--} realizes the first class constraint. A description of the model by means of the second class constraint (2.2) is feasible using Dirac method. The constraint satisfies

$$\{\Omega(x)\,,\,\Omega(y)\} \;=\; -\,2\delta(x-y) \qquad (2.7)$$

which determines a second class system (although the number of constraints seems to be odd[7], going to a discretized version of space one sees that an antisymmetric matrix emerges[17]).

Thus one derives the commutator[14]

$$[\varphi(x)\,,\,\varphi(y)] \;=\; -\frac{i}{2}\,\varepsilon(x-y) \quad . \qquad (2.8)$$

The interaction of the above model with a gauge field was proposed in [12]. We write the Lagrange density

$$L = \partial_+\varphi\,\partial_-\varphi + A_-\partial_+\varphi - A_+\partial_-\varphi - \frac{1}{4}F_{\mu\nu} + \frac{1}{2}\,a\,A^\mu A_\mu \qquad (2.10)$$

where the last term has an arbitrary parameter <u>a</u> related to the gauge symmetry breaking, and should be put in, to take into account the renormalization arbitrariness.

3. DIRAC QUANTIZATION OF THE MODEL

The primary constraints of the theory appear as follows: the left moving character of the bosonic field requires

$$\Omega_1(x) \;=\; \pi(x) - \varphi'(x) \;=\; 0 \quad ; \qquad (3.1)$$

and the absence of the time derivative of $A_0(x)$ in the action which implies

$$\Omega_2(x) \;=\; \pi_0(x) \;=\; 0 \qquad (3.2)$$

The momentum conjugate to $A_1(x)$ is

$$\pi_1(x) \;=\; F_{10}(x) \qquad (3.3)$$

and the Hamiltonian is readily computable

$$H = \int dx^1 \Big\{ \frac{1}{2}\,\pi_1(x)^2 + A_0(x)\,\partial_1\pi_1(x) + \frac{1}{2}\,\pi(x)^2 + \frac{1}{2}\,\varphi'(x)^2 + $$
$$+ \pi(x)\,A_1(x) - \varphi'(x)\,A_0(x) - \frac{1}{2}a\,A_0^2 + \frac{1}{2}a\,A_1^2 \Big\} \; . \qquad (3.4)$$

The Poisson brackecs are given by their usual expressions, namely

$$\Big\{\varphi(x)\,,\,\pi(x)\Big\} \;=\; \delta(x^1 - y^1) \qquad (3.5a)$$

$$\left\{A^1(x)\,,\,\pi_1(x)\right\} = \delta(x^1-y^1) \tag{3.5b}$$

$$\left\{A_0(x)\,,\,\pi_0(x)\right\} = \delta(x^1-y^1) \quad . \tag{3.5c}$$

Further constraints arise from the time conservation of $\Omega_1(x)$ and $\Omega_2(x)$, and are given, respectively by

$$\Omega_3(x) = \{\Omega_1(x),H\} =$$

$$= \frac{d}{dx^1}\left\{\varphi'(x) - \pi(x) - A_0(x) - A_1(x)\right\} \tag{3.6a}$$

and

$$\Omega_4(x) = \{\Omega_2(x),H\} =$$

$$= -\partial_1\pi_1(x) + \varphi'(x) + a\,A_0(x) \tag{3.6b}$$

In order to use Dirac quantization procedure[7] compute the constraints' Poisson bracket matrix

$$Q_{ij}(x,y) = \left\{\Omega_i(x)\,,\,\Omega_j(y)\right\} =$$

$$= \begin{bmatrix} -2\delta'(x-y) & 0 & -2\delta''(x-y) & \delta'(x-y) \\ 0 & 0 & -\delta'(x-y) & -a\,\delta(x-y) \\ 2\delta''(x-y) & -\delta'(x-y) & 2\delta'''(x-y) & 0 \\ 0 & a\,\delta'(x-y) & 0 & 0 \end{bmatrix} \tag{3.7a}$$

whose inverse is given by

$$Q^{-1}(x,y) =$$

$$= \frac{1}{1+4a}\begin{bmatrix} a^2\,\varepsilon(x-y) & 2a\delta(x-y) & \frac{a}{2}(1+2a)|x-y| & \frac{1}{2}(1+2a)\,\varepsilon(x-y) \\ -2a\delta(x-y) & -2\delta'(x-y) & -\frac{1}{2}(1+2a)\,\varepsilon(x-y) & 2\delta(x-y) \\ -\frac{a}{2}(1+2a)|x-y| & -\frac{1}{2}(1+2a)\varepsilon(x-y) & -\frac{a^2}{2}(x-y)^2\varepsilon(x-y) & -a|x-y| \\ \frac{1}{2}(1+2a)\varepsilon(x-y) & -2\delta(x-y) & a|x-y| & \varepsilon(x-y) \end{bmatrix} \tag{3.7b}$$

With these results we compute all relevant equal time commutators readily[7]. Nonzero commutators are

$$[\varphi(x)\,,\,\varphi(y)] = -\frac{2ia}{1+4a}\,\delta(x-y) \tag{3.8a}$$

$$[\varphi(x)\,,\,\varphi(y)] = -\frac{ia}{1+4a}\,\varepsilon(x-y) \tag{3.8b}$$

$$[\varphi(x)\,,\,A_0(y)] = -\frac{i}{1+4a}\,\delta(x-y) \tag{3.8c}$$

$$[\varphi(x)\,,\,\pi_1(y)] = -\frac{ia/2}{1+4a}\,\varepsilon(x-y) \tag{3.8d}$$

$$[\varphi(x)\,,\,A_1(y)] = \frac{i}{1+4a}\,\delta(x-y) \tag{3.8e}$$

$$[A_0(x)\,,A_1(y)] = \frac{2i}{1+4a}\,\delta(x-y) \tag{3.8f}$$

$$[\pi_1(x), A_1(y)] = i \frac{1+2a}{1+4a} \delta(x-y) \tag{3.8g}$$

$$[\pi_1(x), \pi_1(y)] = i \frac{a^2}{1+4a} \varepsilon(x-y) \tag{3.8h}$$

Positive definiteness requires (see 3.8a) that either $a < -\frac{1}{4}$ or $a > 0$. Limiting cases have to be studied separately. In particular, if $a = -\frac{1}{4}$ the determinant of Q is zero, and the system is not second class. Further constraints arise. Time conservation of Ω_3 gives

$$\{\Omega_3(x), H\} = -\Omega_3'(x) + \Omega_5(x) \tag{3.9}$$

where

$$\Omega_5(x) = \pi_1'(x) \tag{3.10}$$

whereas conservation of Ω_4 implies

$$\Omega_6(x) = -a A_1'(x) = -\frac{1}{4} A_1'(x) \quad . \tag{3.11}$$

These constraints render the theory second class. However, a nontrivial result is hardly forseeable, with so many constraints.

4. EQUATIONS OF MOTION

The pair of Euler equations derived from (2.10) for φ and A_μ are

$$\Box\varphi + \varepsilon_{\mu\nu}\partial^\mu A^\nu = 0 \tag{4.1}$$

$$-\partial^\mu F_{\mu\nu} - \varepsilon_{\mu\nu}\partial^\mu\varphi - a A_\nu = 0 \quad . \tag{4.2}$$

The solution to the first equation is the expression of the gauge field in terms of φ and an arbitrary gauge excitation

$$A_\mu = -\varepsilon_{\mu\nu}\partial^\nu\varphi + \partial_\mu h \tag{4.3}$$

which substituted in the second equation implies

$$-\varepsilon_{\mu\nu}\partial^\nu(-\partial^2\varphi + (a-1)\varphi) - a\,\partial_\nu h = 0 \quad . \tag{4.4}$$

Thus φ is a massive field, with mass $(a-1)$.

However, since φ is a purely left moving field, it must be massless otherwise Lorentz invariance is broken. Therefore we take $a = 1$ (in this case φ is massless) and h is a trivial constant. The gauge field is constituted only by gauge excitations.

5. CONCLUSIONS

We analysed the coupling of left moving bosons to an abelian gauge field. The resulting theory has been quantized using the Dirac formalism, treating the left moving character of the bosonic field as well as gauge invariance in terms of constraints. Moreover a mass term for the gauge field was used in the action, in order to take into account the non-gauge invariance implied by the left moving bosonic field. Thus the theory depends on a parameter a which is the coefficient of the mass term.

All commutators came out after some computation. However, the field equations implies a mass for the bosonic field, which is incompatible with a Lorentz invariant left moving boson. In order that this mass vanishes, the arbitrary parameter takes a fixed value, and the gauge field turns out to be a pure gauge excitation. The theory is (almost) trivial.

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