UNIVERSIDADE DE SÃO PAULO

# PUBLICAÇÕES

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

IFUSP/P-702

## HADRON INTERFEROMETRY FOR EXPANDING SOURCES

**Y. Hama**
Instituto de Física, Universidade de São Paulo



Março/1988

HADRON INTERFEROMETRY FOR EXPANDING SOURCES*

Y. HAMA

Instituto de Física, Universidade de São Paulo
São Paulo, BRASIL

ABSTRACT: Hambury-Brown-Twiss effect is discussed for high-energy hadron-hadron or nucleus-nucleus collisions, where the particle emitting sources are typically in rapid expansion. On the light of the general results, we examine the recently obtained heavy-ion data and propose procedures for analyzing new data.

## 1. INTRODUCTION

In previous papers[1,2] we have studied some effects of the source expansion on the identical particle correlation and shown that these effects are absolutely non trivial, so that in order to extract correct information on the space-time structure of the particle-emitting source, we have inevitably to analyze the data by taking these effects into account. Related discussions have also been done by other authors[3-6], by using different models. Here, I would like to focus our attention to the recently obtained heavy-ion data[7] and show how the above mentioned effects deform the apparency of the phenomena and how more meaningful results could be obtained, by analyzing the new data in somewhat more complete way. The plan of presentation is to start giving a short account of the main effects of the source expansion (Sec.2). The question of where the correlation curve intercepts the vertical axis is closely related to the experimental uncertainties and shall be examined in Sec.3. In Sec.4, we make comments on the recently obtained heavy-ion data[7] in the light of our analysis. Conclusions are then drawn in the final Section.

* Work supported by FAPESP, Brazil.

Talk presented to Int.Conf. on Physics and Astrophysics of Quark-Gluon Plasma, Bombay, Feb.8-12,1988.

## 2. EXPANDING SOURCE

Consider a source which expands[1,2] along x-direction as shown in Fig.1 and let us suppose we are measuring two-identical-boson correlation at 90° The correlation function may be written

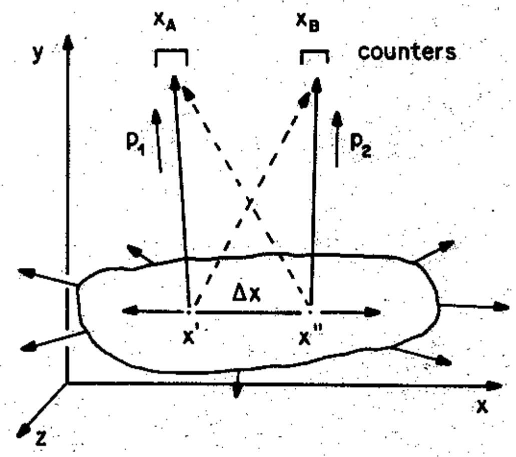


Fig.1: Illustration of a linearly expanding source, with counters placed at $\sim 90°$. Two representative source points x' and x" are also shown.

$$C(p_1,p_2)=\frac{P(p_1,p_2)}{P(p_1)P(p_2)} = 1 + \langle \cos(\Delta p^\mu \Delta x_\mu) \rangle , \qquad (1)$$

where

$$\begin{cases} P(p_i) = \text{Probability of finding a particle with four-momentum } p_i \\ P(p_1,p_2) = \text{Probability of simultaneously finding 2 particles} \\ \qquad \text{of four-momenta } p_1 \text{ and } p_2 \text{ respectively,} \end{cases}$$

$$\begin{cases} \Delta p^\mu \equiv (E_2-E_1, \vec{p}_2-\vec{p}_1), \\ \Delta x^\mu \equiv (\Delta t, \Delta\vec{r}) = (t'-t'', \vec{r}\,'-\vec{r}\,''), \end{cases} \qquad (2)$$

and $\langle\ \rangle$ means average over the space-time points. In the <u>static case</u>, it follows from eq.(1) that $\Delta r \Delta p \sim 1$, so a measurement of the correlation width directly gives us the radius $\Delta r \sim 1/\Delta p$ of the particle emitting system. If the <u>source</u> is <u>in expansion</u>, however, such a simple relation is not valid and the very meaning of radius itself becomes vague. We may then ask i)which dimension are we measuring? and ii)which is the meaning of $1/\Delta p$ in the new situation?

Assume that each small volume $\Delta V$ of the source emits <u>isotropically</u> in its proper frame with a momentum distribution

$$E\frac{dn}{d\vec{p}} \propto f(E_0) = f(u_\mu p^\mu) = f(m_T ch(y-\alpha)), \qquad (3)$$

where $\alpha$ is the rapidity of $\Delta V$ and $y$ is the rapidity of an emitted particle. Suppose we are measuring the correlation in $\Delta p_x$. Then, when averaging $\cos(\Delta p^\mu \Delta x_\mu)$, the contributions become negligible if

$$f(m_T ch(y-\alpha'))\, f(m_T ch(y-\alpha'')) \ll f^2(m_T). \qquad (4)$$

For example, if $f(E_0)=\exp(-\beta E_0)$, we have, at $y=\alpha'$ ($\alpha'-\alpha''=\Delta\alpha$), $\exp(-\beta m_T(ch\Delta\alpha-1)) \ll 1$. So, we can define an <u>effective rapidity width</u> by

$$\beta m_T(ch\Delta\alpha-1) \simeq 1. \qquad (5)$$

or $$\Delta\alpha \simeq \ln\left[1+\frac{1}{\beta m_T}+\sqrt{\frac{1}{\beta^2 m_T^2}+\frac{2}{\beta m_T}}\right]. \qquad (6)$$

Then, <u>the relevant size in the correlation measurement is precisely the one which corresponds to the effective width $\Delta\alpha$ defined above</u>. It also becomes clear from eq.(6) and Table I that $\Delta\alpha$ is $p_T$ dependent. Now, an important feature of some realistic models such as the hydrodynamic one (and some other ones[3,5]) is a strong correlation between $\alpha$(or $\vec{v}$) and the space-time point

$$v = \frac{e^{2\alpha}-1}{e^{2\alpha}+1} \simeq \frac{x}{t}. \qquad (7)$$

Table I: $p_T$ dependence of $\Delta\alpha$ ($\beta^{-1}=m_\pi$).

| $p_T$ (GeV) | $\Delta\alpha$ |
|---|---|
| 0.1 | 1.20 |
| 0.2 | 1.03 |
| 0.4 | 0.79 |
| 1.0 | 0.52 |
| $\infty$ | 0 |

This correlation implies a <u>shrinking of the effective source size as $p_T$ increases</u>. But, then it is a nice tool for studying the space-time structure of a source, for it allows us to look at different rapidity windows

Let us now turn to the transverse $\Delta p$ correlation. Assuming the average longitudinal momentum of the pair is $p_x \simeq 0$,

a) If <u>$\Delta p_x \simeq 0$, $\Delta p_y \simeq 0$ and $p_z \neq 0$</u>, then $\Delta E \simeq 0$. We have in this case $\Delta p^\mu \Delta x_\mu \simeq -\Delta p_z \Delta z$. Thus, as expected

$$\frac{1}{\Delta p_z} \simeq \langle \Delta z\rangle \simeq \sqrt{\langle \Delta z^2\rangle}. \qquad (8)$$

b) If <u>$\Delta p_x \simeq 0$, $\Delta p_y \neq 0$ and $\Delta p_z \simeq 0$</u>, then $E \neq 0$. We have in this case $\Delta p^\mu \Delta x_\mu \simeq \Delta E \Delta t - \Delta p_y \Delta y$. Thus,

$$\langle(\Delta p^\mu \Delta x_\mu)^2\rangle \simeq \Delta p_y^2\left[\left(\frac{\Delta E}{\Delta p_y}\right)^2\langle \Delta t^2\rangle + \langle \Delta y^2\rangle\right].$$

So, $$\frac{1}{\Delta p_y} \simeq \sqrt{\langle \Delta y^2\rangle + \left(\frac{\Delta E}{\Delta p_y}\right)^2\langle \Delta t^2\rangle} > \sqrt{\langle \Delta y^2\rangle}. \qquad (9)$$

i.e., if there is an emission time fluctuation, the correlation in $\Delta p_y$ decreases and the apparent size increases. The latter is actually the same effect which has long been studied by Kopylov and Podgoretsky[8] with a source made of independent oscillators, but such a time fluctuation constitutes a basic feature of more realistic models of expanding sources (see a $\tau$(T)-=const. curve in Fig.2). Besides, if we are dealing with q-g plasma, the transition should occur in a finite time interval. In Ref.(2) we have studied such a system by using a hydrodynamical model, wherein a free expansion is assumed during the

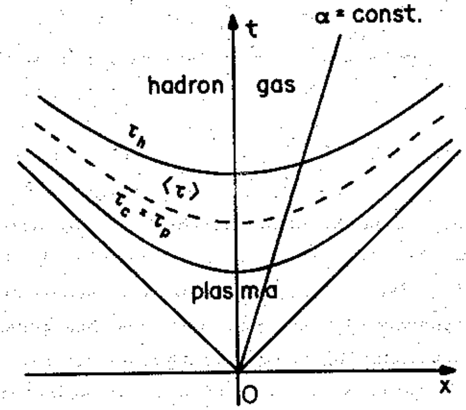


Fig.2: Plasma in expansion undergoing a phase transition at $T=T_C$. Hadron emission occurs in the region $\tau_p < \tau < \tau_h$.

transition (as the pressure = const. there). We could, then reproduce the ISR hadron-hadron data[9] as shown by examples in Fig.3. A point of interest in the present context is that the proper time interval for the transition turns out to be rather long, which we estimate to be ~10 fm at the ISR.

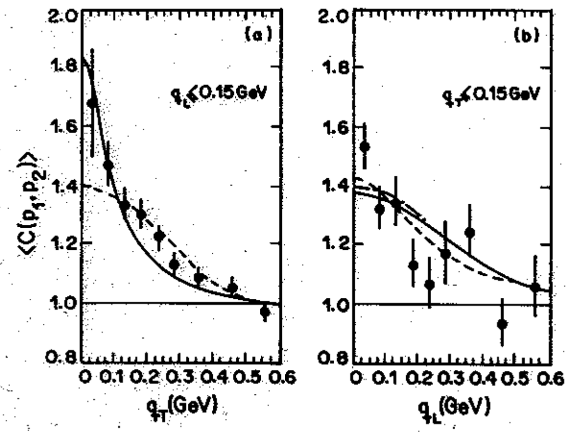


Fig.3: Estimate of $\pi^+\pi^+$ correlation (continuous curves[2)]) in pp collisions is compared with the data[9)]. The broken curves are the fits given in Ref.9).

## 3. INTERCEPTS OF THE CORRELATION CURVES

Let us next discuss a point that, although has nothing to do with the source flow, nevertheless is crucial to getting a correct information about it. Often, chaoticity parameter $\lambda$ is introduced in order to account for the fact that the intercept of $C(p_1,p_2)$ is $<2$. Whether the source is chaotic or not is certainly an important question which merits an investigation. However, one should not forget that in practice when $C(p_1,p_2)$ is measured as a function of $\Delta p_i$, the other components $\Delta p_j$ $(j\neq i)$ are never $=0$. One may think that this is only a small factor, but actually it may change the results quite a lot as it appears in Fig.3, where we have adopted a totally chaotic source, with the experimental errors taken into account. It becomes patent that in any meaningful attempt of determining the chaoticity parameter $\lambda$, the experimental uncertainties have to be taken into account. Clearly, it affects also the correlation width.

## 4. HEAVY-ION DATA[7)]

In a recent experiment, NA35 Collaboration has measured $\pi^-\pi^-$ correlations in 200 GeV/nucl. $^{16}$O+Au central collisions. The data have been fitted with two different parametrization, but I shall limit myself to the Gaussian one, namely

$$C(\Delta p_T,\Delta p_L)=A\left[1+\lambda\exp\left(-\frac{\Delta p_T^2R_T^2}{2}-\frac{\Delta p_L^2R_L^2}{2}\right)\right], \qquad (10)$$

where $R_T(R_L)$ is the apparent transverse(longitudinal) source radius (see Table II).What is intriguing in their data is the large transverse radius when compared with the well known $^{16}$O radius. Even if one takes the surrounding Au nucleons into account, the true radius $R_{To}$ hardly exceeds $\sim$4 fm. Transverse expansion may occur, but it is known to be small. An alternative interpretation would be the emission-time fluctuation. From eqs.(8) and (9), it follows

$$R_T^2\simeq\langle\Delta z^2\rangle+\left[\langle\Delta y^2\rangle+\left(\frac{\Delta E}{\Delta p_y}\right)^2\langle\Delta t^2\rangle\right]\simeq R_{To}^2+\langle\Delta t^2\rangle, \qquad (11)$$

where $(\Delta E/\Delta p_y)^2\sim 1$. Then, assuming $R_{To}\sim 4$ fm, we obtain $R_T\simeq 8$ fm, if $\sqrt{\langle\Delta t^2\rangle}\simeq 7$ fm, which is quite reasonable as discussed before.

As for $\lambda_{exp}$, it is sensibly $<1$ and decreases as one goes away from the central y region. Now, the third column of Table II, cal-

Table II: Some of NA35 fits to eq.(10), together with the computed (see the text) and the measured intercepts.

| y-interval ($y_{cm}$) | $R_T$ (fm)<br>$R_L$ (fm)<br>$\lambda$ | $\exp\left[-\frac{\langle\lvert\Delta p_L\rvert\rangle^2R_L^2}{2}\right]$ | $C(\Delta p_T=0)$ |
|---|---|---|---|
| $2<y<3$<br>($y_{cm}=2.5$) | 8.1±1.6<br>5.6+1.2/−0.8<br>0.77±0.19 | 0.38 | 1.5 |
| $1<y<2$<br>($y_{cm}=1.5$) | 3.8±0.5<br>4.0±0.8<br>0.34+0.09/−0.06 | 0.60 | ? |

culated with the quoted uncertainties ( $\Delta p_L < 0.1$ GeV), shows that the central-rapidity data are consistent with $\lambda = 1$. It is not excluded that $\lambda < 1$ in other rapidity intervals, although $\lambda \gg \lambda_{exp}$.

## 5. CONCLUSIONS

In short, i)HBT effect determines the longitudinal size $\Delta x$ which corresponds to a characteristic rapidity width $\Delta\alpha(p_T)$; ii) $\Delta x$ shrinks with $p_T$, suggesting data analysis in different $p_T$ windows; iii) $1/\Delta p_z \simeq \langle \Delta z \rangle$ and $1/\Delta p_y \simeq \sqrt{\langle \Delta y^2 \rangle + (\Delta E/\Delta p_y)^2 \langle \Delta t^2 \rangle}$. Thus, it would be desirable to analyze the data by separating $\Delta p_y$ from $\Delta p_z$; iv) $R_T$ may be much smaller than the published values; v) $\lambda$ is overestimated, because of the experimental uncertainties in $\Delta p_i$.

## REFERENCES:


1) Hama,Y. and Padula,S.S., "Bose-Einstein Correlation in Landau's Model" in "Hadronic Matter in Collision(LESIP II)", eds.: Carruthers,P. and Strottman,D., World Scientific, pg 63(1986).
2) Hama,Y. and Padula,S.S., "Bose-Einstein Correlation of Particles Produced by Expanding Sources", preprint IFUSP/P-673 (Univ. de São Paulo); submitted for publication on Phys.Rev.
3) Anderson,B. and Hofman,W., Phys.Lett.169B,364(1986).
4) Makhlin,A.N. and Sinyukov,Yu.M., preprints ITP-86-27E (Kiev, 1986); ITP-87-64E (Kiev,1987).
5) Kolehmainen,K. and Gyulassy,M., Phys.Lett.180B,203(1986).
6) Pratt,S., Phys.Rev.D33,72(1986); Phys.Rev.D33,1314(1986).
7) NA35 Collab., Humanic,T.J., "Pion Interferometry with Ultrarelativistic Heavy-Ion Collisions from the NA35 Experiment", Quark Matter 1987 Conference, Nordkirchen, West Germany(Aug.1987).
8) Kopylov,G.I. and Podgoretsky,M.I., Sov.J.Nucl.Phys.19,215(1974).
9) Axial-Field-Spectrometer Collab.,Akesson,T. et al., Phys.Lett. 129B,269(1983); 155B,128(1985).