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LONG-DISTANCE BEHAVIOR OF QUARK FORM FACTOR IN QCD

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LONG–DISTANCE BEHAVIOR OF QUARK FORM FACTOR IN QCD

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ABSTRACT

We study the infrared behavior of the color singlet quark form factor in perturbative QCD. We express the non–leading fourth–order contributions in terms of an infrared anomalous dimension, which characterizes the renormalization group equation describing the long–distance behavior of the form factor. We discuss the important features of this function and comment on its behavior at high energy.

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I. INTRODUCTION

We are concerned with dominant and subdominant infrared divergences occurring in processes involving a color singlet quark–antiquark pair. These arise in the study of many important high–energy reactions, like the Drell–Yan and the electron–positron annihilation processes. In order to describe the asymptotic behavior in this domain, we shall derive a renormalization–group equation in a form suggested sometime ago[1] in connection with the process of quark scattering by an external potential. This approach has been recently employed[2], with the help of more general results obtained to two–loop order, to describe the infrared behavior of the quark form factor in the space–like region.

The question which we address here is the derivation of an explicit expression for the anomalous dimension in the renormalization–group equation describing the infrared properties of the form factor in the time–like region. We consider in what follows the reaction:

$$\gamma^* \rightarrow q + \bar{q} \tag{1}$$

which defines the color singlet quark–antiquark form factor in the time–like region.

Our approach takes advantage of the fact that the infrared singularities characterizing this form factor cancel[3] when combined in the cross section σ with the corresponding ones resulting from the reaction:

$$\gamma^* \rightarrow q + \bar{q} + \text{soft gluons} \tag{2}$$

where soft gluon production up to a maximum energy Δ is included.

The calculation simplifies considerably if we make use of the quantity $\Delta\, d\sigma/d\Delta$ evaluated to order g^4 in reference 4. In section II we derive a connection relating it to the infrared singularities occurring in process (2). We then find to this order a completely explicit expression for the infrared singular contributions, which is expressible in terms of dilogarithmic and threelogarithmic functions[5]. In the high energy domain $s \gg m^2$, where $\sqrt{s}$ denotes the invariant energy of the massive $q-\bar{q}$ pair, these functions yield contributions which behave respectively like $\ln^2(s/m^2)$ and $\ln^3(s/m^2)$. Yet, in the final result these cancel out, leaving a dominant contribution which behaves only like $\ln(s/m^2)$ in this domain.

A physical reason for this behavior is given in section III. Here we also present the renormalization group equation describing the long distance behavior of the form factor in the time–like region. We show that apart from the renormalization group beta function, the non–abelian effects can be expressed in terms of an effective infrared anomalous dimension γ_A. By analytical continuation of the corresponding explicit expression for γ_A, we establish a connection with the results previously obtained in the space–like region.

II. RESULTS OF THE CALCULATIONS

In the process of calculating the contributions from the Feynman diagrams, two Casimir operators $C_F \equiv \mathrm{Tr}\, t^a t^a$ and $C_A \delta_{cd} \equiv f_{abc} f_{abd}$ appear, where t_a are the representation matrices for the fermions and f_{abc} are the structure constants associated with the Yang–Mills theory. The fourth order terms are proportional to C_F^2 or $C_F C_A$, the special case of QED being obtained by setting $C_F = 1$ and $C_A = 0$. Since QED results are well known[6], we concentrate in what follows on the terms proportional to $C_F C_A$. We work in the Feynmann gauge and use consistently dimensional regularization[7], in a space–time dimension $d = 4+\eta$. In order to find the infrared behavior, we consider graphs with eikonalized quarks and, to set up the notation, present in more detail the diagrams

shown in Fig. 1.

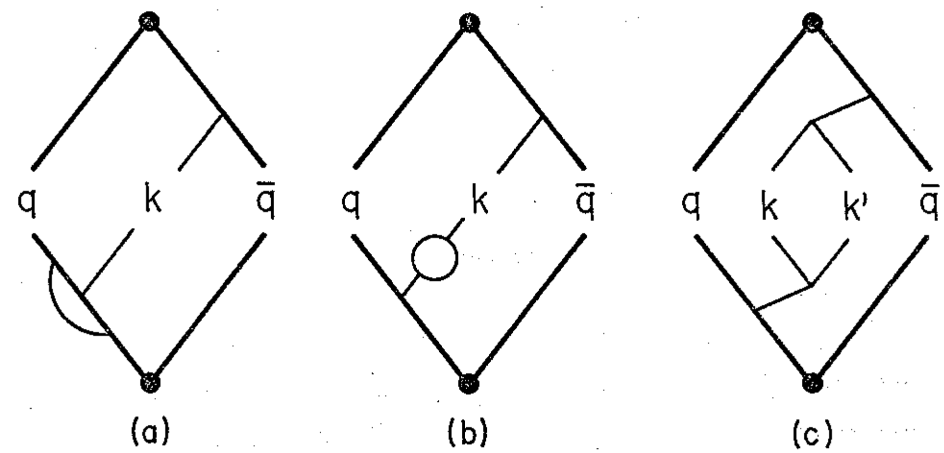

Fig. 1 – Fourth–order „renormalization–group" type diagrams.

The upper side of the graphs represent a contribution to the complex conjugate part of the amplitude, and a sum over all other diagrams with similar topology is to be understood. Thick lines are quarks (antiquarks) while thin lines denote gluons. The black blob represents the point where the virtual photon in eq. (2) produces a pair quark antiquark, with momenta q and $\bar{q}$, respectively. We begin by considering the diagram (1a) and denote by x the cosine of the angle between $\vec{k}$ and $\vec{q}$. Adding the complex conjugate part we obtain the following contribution:

$$G_{1a} = \frac{g^4 C_F C_A}{(2\pi)^4} \, \frac{2^{-\eta}\,\pi^{-\eta/2}}{\Gamma(1+\eta/2)} \, \frac{1}{\eta} \int_{-1}^{1} dx(1-x^2)^{\eta/2} \left[1 - \frac{1}{1-\beta x}\right] \cdot$$

$$\cdot \left[\frac{\pi^{-\eta/2}\ \Gamma(1+\eta/2)\ \Gamma(1-\eta)}{1+\eta} \frac{\Delta^{2\eta}}{2\eta} - \frac{\Delta^{\eta}\mu^{\eta}}{\eta} \right] \tag{3}$$

Here β is the speed of the quark in the rest frame of the antiquark and is related to s/m^2 by the expression

$$1 - \beta^2 = \left[\frac{s}{2m^2} - 1\right]^{-2} \tag{4}$$

In order to obtain eq. (3) we have renormalized the ultraviolet divergences of the subdiagram by substracting the pole part times μ^{η}, which corresponds to using a renormalized coupling constant $\alpha_s \equiv g^2/4\pi$ with dimension of (length)$^{\eta}$. Expanding the integrand in powers of η and performing the x integration we find an expression of the form:

$$G_{1a} = \left[\frac{\alpha_s}{\pi}\right]^2 C_F C_A \frac{P_1(\eta)}{\eta} \left[P_2(\eta) \frac{\Delta^{2\eta}}{2\eta} - \frac{\Delta^{\eta}\mu^{\eta}}{\eta}\right] \tag{5}$$

where $P_1(\eta)$ and $P_2(\eta)$ are polynomial functions of η:

$$P_1(\eta) = \left\{-1 + \left[\ln 2 + \tfrac{1}{2}\ln \pi - \tfrac{\gamma}{2}\right]\eta\right\} B(\beta) + \tag{6}$$

$$+ \left\{2\ln 2 - 2 - \frac{1}{2\beta}\left[\ln\frac{\beta^2-1}{\beta^2}\ln\frac{1+\beta}{1-\beta} - \mathrm{Li}_2\left[\frac{1+\beta}{1-\beta}\right] + \mathrm{Li}_2\left[\frac{1-\beta}{1+\beta}\right]\right]\right\}\eta + \cdots$$

and

$$P_2(\eta) = 1 - \left[1 + \tfrac{1}{2}\ln \pi - \tfrac{\gamma}{2}\right]\eta + \cdots \tag{7}$$

In expression (6) $B(\beta)$ represents the bremsstrahlung probability function:

$$B(\beta) = \frac{1}{\beta}\ln\left[\frac{1+\beta}{1-\beta}\right] - 2 \tag{8}$$

Furthermore γ denotes the Euler constant and Li_2 stands for the dilogarithmic function[5]:

$$\mathrm{Li}_2(z) = -\int_0^z \ln(1-x)\,\frac{dx}{x} \tag{9}$$

Proceeding in a similar manner, we obtain corresponding to the graph (1b) a contribution of the form:

$$G_{1b} = \left[\frac{\alpha_s}{\pi}\right]^2 C_F C_A \frac{\Delta^{\eta}\mu^{\eta}}{\eta^2} P_3(\eta) \tag{10}$$

where the polynomial $P_3(\eta)$ is given by:

$$P_3(\eta) = \frac{5}{6}\left[1 + \left[-\ln 2 - \tfrac{1}{2}\ln \pi + \tfrac{1}{2} + \tfrac{\gamma}{2}\right]\eta\right] B(\beta) + \tag{11}$$

$$+ \left\{\frac{5}{3} - \frac{5}{3}\ln 2 + \frac{5}{12\beta}\left[\ln\frac{\beta^2-1}{\beta^2}\ln\frac{1+\beta}{1-\beta} - \mathrm{Li}_2\left[\frac{1+\beta}{1-\beta}\right] + \mathrm{Li}_2\left[\frac{1-\beta}{1+\beta}\right]\right]\right\}\eta + \cdots$$

Analogously, we obtain from diagram (1c) a result of the form:

$$G_{1c} = \left[\frac{\alpha_s}{\pi}\right]^2 C_F C_A \frac{\Delta^{2\eta}}{2\eta^2} P_4(\eta) \tag{12}$$

where the polynomial $P_4(\eta)$ can be expanded as:

$$P_4(\eta) \;=\; -\tfrac{5}{6}\,B(\beta) + \mathcal{O}(\eta) \qquad (13)$$

We now turn to the consideration of the remaining fourth order diagrams which yield contributions with color factor $C_F C_A$. Typical graphs are shown in Fig. 2.

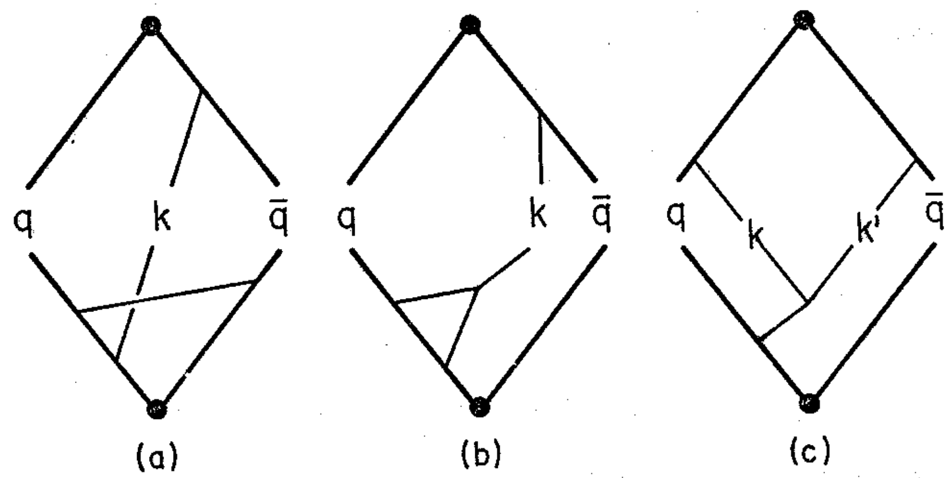

Fig. 2 – Examples of QED and nonabelian type of bremsstrahlung diagrams.

Due to the antisymmetry of the 3–gluon vertex, it turns out that graphs like (2b) are actually ultraviolet convergent in the Feynman gauge. Furthermore, although these graphs individually yield cubic and quadratic infrared divergences, these contributions cancel in the sum of all relevant fourth order diagrams. Consequently, the infrared contribution associated with the above graphs has the general form:

$$G_2 \;=\; \left[\frac{\alpha_s}{\pi}\right]^2 C_F C_A \,\frac{\Delta^{2\eta}}{2\,\eta}\, P_5(\eta) \qquad (14)$$

where $P_5(\eta)$ is a polynomial function of η whose coefficients depend on the parameter β.

We are now in a position to determine to order α_s^2 the infrared divergent contributions G_{in} corresponding to reaction (2). With the help of equations (5–14) one finds:

$$G_{in} \;=\; \left[\frac{\alpha_s}{\pi}\right]^2 C_F C_A \left\{\frac{11}{12\eta^2}\,B(\beta) + \right. \qquad (15)$$

$$\left. + \frac{1}{2\eta}\left[P_1(0)P_2'(0) - P_1'(0) + 2P_3'(0) + P_4'(0) + P_5(0)\right]\right\}$$

where

$$P_\ell(\eta) \;=\; P_\ell(0) + \eta\, P_\ell'(0) + \mathcal{O}(\eta^2) \qquad (16)$$

In order to evaluate the non–leading infrared contributions we need to know, in addition to the functions $P_1(\eta)$, $P_2(\eta)$ and $P_3(\eta)$ already described, also $P_4'(0)$ and $P_5(0)$. Since a direct attempt to calculate these functions is very laborious, we will instead relate the right–hand side of the above equations to the quantity $\Delta\, d\sigma/d\Delta$ which was studied in reference 4. To this end we remark, with the help of eqs. (5–14), that

$$\Delta\,\frac{d\sigma}{d\Delta} \;=\; \left[\frac{\alpha_s}{\pi}\right]^2 C_F C_A \left\{-\frac{11}{6}\ln(\Delta)B(\beta) + \right. \qquad (17)$$

$$\left. +\left[P_1(0)P_2'(0) + P_3'(0) + P_4'(0) + P_5(0)\right]\right\}$$

Now we recall that the explicit form for $\Delta\, d\sigma/d\Delta$ is given by the expressions (14a) and

(14b) in reference 4. [We take this opportunity to mention that there is a correction in the coefficient multiplying the bremsstrahlung function $B(\beta)$ appearing in eq. (12) of this reference. This has the effect of replacing the factor $\pi^2/4 + 53/18$, which occurs multiplying $B(\beta)$ in eq. (14a), by $\pi^2/3 + 119/36$]. Therefore, comparing eqs. (15) and (17) given above, we see that in order to determine the non–leading infrared contributions all is needed is to calculate, in addition, the part involving $P_3'(0) - P_1'(0)$. It is easy to see, using eqs. (6) and (11), that this yields:

$$P_3'(0) - P_1'(0) = \frac{11}{3} - \frac{11}{3}\ln 2 + \frac{1}{12}[5 - 22\ln 2 - 11\ln\pi + 11\gamma]B(\beta) + $$
$$+ \frac{11}{12\beta}\left[\ln\frac{\beta^2-1}{\beta^2}\ln\frac{1+\beta}{1-\beta} - \mathrm{Li}_2\left[\frac{1+\beta}{1-\beta}\right] + \mathrm{Li}_2\left[\frac{1-\beta}{1+\beta}\right]\right] \quad (18)$$

By substituting this contribution, together with the one obtained from (17) into eq. (15), we finally find:

$$G_{in} = \left[\frac{\alpha_s}{\pi}\right]^2 C_F C_A \left\{\frac{11}{12\eta^2} B(\beta) + \frac{1}{\eta} G(\beta)\right\} \quad (19)$$

where

$$G(\beta) = \frac{7}{2} - \frac{11}{6}\ln 2 - \frac{3\beta+1}{24\beta}\pi^2 + \left[\frac{1}{2\beta^2} - \frac{5}{24\beta} - \frac{3}{4}\right]\mathrm{Li}_2\left[-\frac{2\beta}{1-\beta}\right] + $$
$$+ \left[\frac{\pi^2}{4}\left[\frac{2}{3} - \frac{1}{\beta}\right] + \frac{67}{36} - \frac{11}{12}\ln 2\right]B(\beta) + \quad (20)$$
$$+ \frac{11}{24\beta}\left[\ln\frac{\beta^2-1}{\beta^2}\ln\frac{1+\beta}{1-\beta} - \mathrm{Li}_2\left[\frac{1+\beta}{1-\beta}\right] + \mathrm{Li}_2\left[\frac{1-\beta}{1+\beta}\right]\right] + $$
$$+ \left[\frac{1}{2\beta^2} + \frac{5}{24\beta} - \frac{3}{4}\right]\mathrm{Li}_2\left[\frac{2\beta}{1+\beta}\right] - \frac{1}{2\beta^2}\mathrm{Li}_3(1) + \frac{1}{2}\mathcal{O}(1-\beta)$$

Here Li_3 denotes the threelogarithmic function[5]:

$$\mathrm{Li}_3(z) = \int_0^z \mathrm{Li}_2(x)\frac{dx}{x} \quad (21)$$

and $\mathcal{O}(1-\beta)$ represents a contribution expressed in terms of dilogarithmic and threelogarithmic functions which vanishes as $\beta \to 1$. It is given explicitly by eq. (14b) of reference 4 as follows:

$$\mathcal{O}(1-\beta) = \frac{1}{2\beta}\left[\frac{1-\beta}{\beta}\mathrm{Li}_3\left[\frac{1+\beta}{1-\beta}\right] + \frac{1+\beta}{\beta}\mathrm{Li}_3\left[\frac{1-\beta}{1+\beta}\right]\right] + \frac{1}{2}\frac{1-\beta^2}{\beta^2}\ln^2\frac{1+\beta}{1-\beta} - $$
$$- \frac{1}{4\beta}\ln\frac{1+\beta}{1-\beta}\left[\frac{1-\beta}{\beta}\mathrm{Li}_2\left[\frac{1+\beta}{1-\beta}\right] - \frac{1+\beta}{\beta}\mathrm{Li}_2\left[\frac{1-\beta}{1+\beta}\right]\right] + \frac{1+\beta}{2\beta}\mathrm{Li}_2\left[\frac{1-\beta}{1+\beta}\right] + $$
$$+ \frac{1-\beta}{2\beta}\left[\ln\frac{1-\beta}{\beta}\ln\frac{1+\beta}{1-\beta} + \mathrm{Li}_2\left[\frac{1-\beta}{1+\beta}\right] + \frac{1}{2}\ln^2\frac{1+\beta}{1-\beta} - \frac{\pi^2}{2}\right] + $$
$$+ \frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\left[\frac{1+\beta}{2}\ln\frac{1+\beta}{\beta} - \ln 2\right] \quad (22)$$

It is interesting to remark in the above equations, that individual contributions which yield at high energies $\ln^3(1-\beta)$ and $\ln^2(1-\beta)$ terms, are suppressed by a factor $(1-\beta)$. Indeed, using the properties of the polylogarithmic functions[5], we find in the high energy limit that the dominant contributions to $G(\beta)$ are given by:

$$G(\beta) \cong \left[\frac{\pi^2}{12} - \frac{67}{36}\right]\ln(1-\beta) \quad , \qquad \beta \to 1 \quad (23)$$

The physical reason for this behavior will be given in the next section.

III. DISCUSSION

We now turn to the analysis of the infrared quark–antiquark form factor F which characterizes the virtual soft gluon exchanges in the amplitude associated with reaction (1). In general, this form factor will be a complex function in the time–like region. Our method, which makes use of the fact that the infrared singularities cancel in the inclusive cross section, determines only the real part of F. However, it is precisely this part of the form factor which contributes to the cross section and is therefore of physical interest.

Typical examples of fourth order graphs are illustrated in Fig. 3.

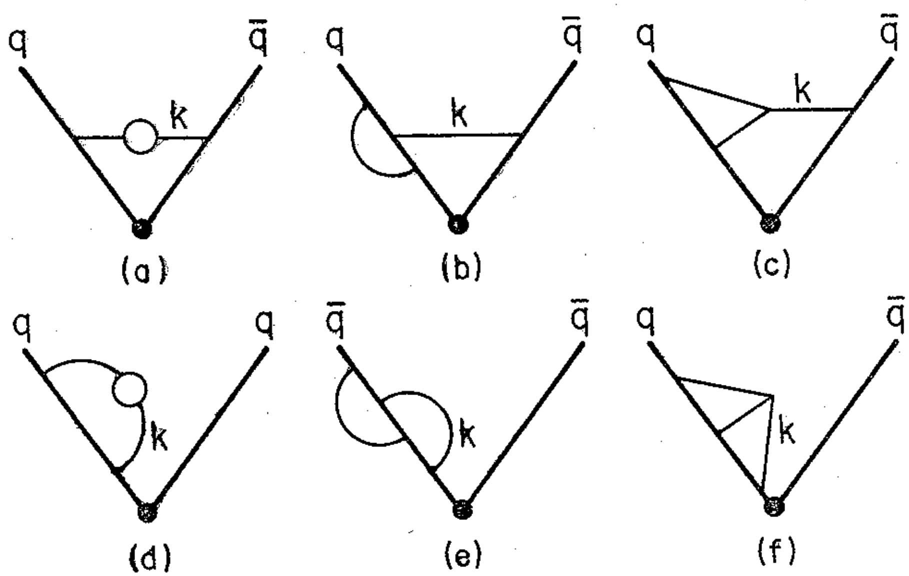

Fig. 3 – Feynman diagrams contributing to the $q\bar{q}$ form factor.

The infrared form factor F, which is defined so that it includes exclusively the infrared divergent corrections, has an expansion of the form:

$$F(\beta,\alpha_s,\eta) = 1 + \frac{\alpha_s}{\pi} f_{11} \frac{1}{\eta} + \left[\frac{\alpha_s}{\pi}\right]^2 \left[f_{22} \frac{1}{\eta^2} + f_{21} \frac{1}{\eta}\right] + \cdots \tag{24}$$

where the factors $f_{\ell m}$ are functions of the parameter β. In particular, f_{11} is given by:

$$f_{11}(\beta) = -\frac{C_F}{2} B(\beta) \tag{25}$$

Furthermore, we find from eq. (19) that:

$$f_{22}(\beta) = -C_F C_A \frac{11}{24} B(\beta) \tag{26}$$

and

$$f_{21}(\beta) = -\frac{C_F C_A}{2} G(\beta) \tag{27}$$

It is convenient in what follows to express, via eq. (4), β as a function of s/m^2 and to define $t \equiv -\eta^{-1}$. Then, it is straightforward to verify that the form factor $F(s/m^2,g^2,t)$ satisfies the renormalization group equation:

$$\left[-\frac{\partial}{\partial t} + \beta_A(g) \frac{\partial}{\partial g} + g^2 \frac{C_F}{8\pi^2} B(s/m^2) + \gamma_A(s/m^2,g^2)\right] F(s/m^2,g^2,t) = 0 \tag{28}$$

where $\beta_A(g)$ is the renormalization group function:

$$\beta_A(g) = -\frac{11}{48\pi^2} C_A g^3 + \cdots \tag{29}$$

and $\gamma_A(s/m^2,g^2)$ denotes the infrared anomalous dimension given by:

$$\gamma_A(s/m^2,g^2) = g^4 \frac{C_F C_A}{32\pi^4} G\left[\frac{s}{m^2}\right] + \cdots \tag{30}$$

It is important to notice here, with the help of eqs. (4) and (23), that in the high energy domain γ_A behaves to fourth order as follows:

$$\gamma_A(s/m^2,g^2) = \frac{g^4}{16\pi^4} C_F C_A \left[\frac{67}{36} - \frac{\pi^2}{12}\right] \ln\left[\frac{s}{m^2}\right] \quad , \quad s \gg m^2 \ , \tag{31}$$

The solution of the renormalization group equation (28), sums up all the long distance effects which contribute to the infrared form factor $F(s/m^2,g^2,t)$. It generalizes the well known QED result[6], where the parameters β_A, γ_A vanish and the infrared singularities exponentiate in a simple way. Using the standard technique employed in solving the Callan–Symanzik equation[8], together with the boundary condition which follows from the from (24):

$$F[s/m^2,g^2,t{=}0] = 1 \tag{32}$$

we obtain for the infrared form factor the following expression:

$$F[s/m^2,g^2,t] = \exp \int_0^t \left\{ \frac{C_F}{8\pi^2} B\left[\frac{s}{m^2}\right] g^2(t') + \gamma_A\left[\frac{s}{m^2}, g^2(t')\right] \right\} dt' \tag{33}$$

where $g^2(t)$ represents the effective coupling constant given by:

$$g^2(t) = g^2\left[1 + \frac{11}{24\pi^2} C_A g^2 t\right]^{-1} \tag{34}$$

Before we proceed let us remark that the expression for the form factor in the space–like region, which is a real function, can be connected by analytical continuation to the corresponding one in the time–like region. To this end we make the replacement $s \to Q^2$, where Q^2 is the square of the 4–momentum transfer in the quark scattering process. In general, due to the appearance of the threelogarithmic functions $Li_3(s/m^2)$ in γ_A, via eq. (22), this continuation will yield additional real contributions. However, in the high energy domain where the leading contribution behaves only like $\ln(s/m^2)$, the continuation is very simple. Indeed, in this case the corresponding expression for γ_A can be obtained directly by replacing s with Q^2 in eq. (31). This agrees with the results previously obtained[1,2] in the space–like region.

The relevant features which characterize the non–abelian theory are the appearance of the running coupling constant and that of the infrared anomalous dimension γ_A given by eq. (30). Furthermore, as we have noticed following eq. (20), it is striking that although $G(s/m^2)$ contains individual contributions which behave at high energies like $\ln^3(s/m^2)$ and $\ln^2(s/m^2)$, these cancel out.

These features can be more clearly understood in physical gauges[9]. For instance, in the axial gauge, the η–dependent part of the renormalized gluon self–energy shown in figures (3a) and (3d) is given by:

$$\Pi_{\mu\nu} = \frac{11}{24\pi^2} C_A g^2 \eta^{-1} \left[\left[\frac{k^2}{\mu^2}\right]^{\eta/2} - 1\right] (\delta_{\mu\nu} k^2 - k_\mu k_\nu) \tag{35}$$

which provides for the factor $\frac{11}{24\pi^2} C_A g^2 t$ connected with the running coupling constant $g^2(t)$.

In order to understand the reason why the infrared anomalous dimension γ_A should behave to order g^4 only like $\ln(s/m^2)$ at high energies, let us recall that in physical gauges the mass singularities are connected with configurations of gluons nearly collinear to a given quark line. The leading singularities result when all gluons are simultaneously parallel with the quark line. Hence for our purpose it is sufficient to concentrate on the diagrams shown in figures (3d), (3e) and (3f).

In order to estimate the behaviour of the leading contributions associated with graph (3d), we use the expression (35) for the gluon self–energy subdiagram and perform the d–dimensional integration over the gluon momenta k. Using the Feynman parametrization and denoting by x the cosine of the angle between $\vec{k}$ and $\vec{q}$, we are led in the limit $\beta \to 1$ to integrals of the form:

$$I(\alpha) = \int_0^1 d\tau\tau^{-1+\eta+2\alpha} \int_{-1}^1 dx \frac{(1-x^2)^{1+\eta/2+\alpha}}{(1-\beta x)^2} \tag{36}$$

where τ denotes a Feynman parameter and α is given, from the first bracket in eq. (35), respectively by $\eta/2$ or zero. The mass singularities appear by expanding the integrand in powers of η and performing the x integration. This yields a factor involving $(\eta/2 + \alpha)\ln(1-x^2)$ which after integration over x produces an $(\eta/2+\alpha)\ln^2(1-\beta)$ mass singularity coming from the region where the gluon momenta become collinear with that of the fermion. Performing also the τ integration, we see that the ensuing infrared singularity $(\eta + 2\alpha)^{-1}$ is promoted to a mass singularity behaving like $\ln^2(1-\beta)$ as $\beta \to 1$. However the coefficient multiplying it is α–independent, and consequently this contribution will cancel in the difference of the terms appearing in eq. (35).

We now turn to the leading mass singularities associated with graphs (3e) and (3f). A power counting analysis, similar to that discussed in the second work of reference 9, shows that for fixed values of the magnitude of the gluon momenta, these diagrams can yield single logarithmic mass singularities. However, when these momenta become

vanishing,there will appear in addition single and double infrared poles. By the mechanism described following eq. (36), one of these may be promoted into an additional mass singularity. In this way, individual graphs like (3e) or (3f) can yield single infrared divergent contributions which are multiplied by double mass singularities. Let us denote by $\Gamma^A_\lambda(q,k)$ the part of the total quark–gluon vertex which is proportional to C_A. It corresponds to the sum of the subdiagrams shown in these graphs which are obtained by opening the gluon line denoted by k. In the axial gauge, this vertex satisfies the simple Ward identity:

$$k_\lambda \Gamma^A_\lambda(q,k) = 0 \tag{37}$$

which implies the cancellation of the dominant and most of the sub–dominant infrared divergences. Consequently, the double mass singularity which is associated with the promotion of an infrared divergence will similarly cancel out.

These considerations can be extended to higher order in perturbative QCD. We then obtain that, as a consequence of the Ward identity, the leading and many non–leading powers of $\ln s/m^2$ will cancel out in the high–energy limit. One might suppose these cancellations to be sufficient to ensure that the infrared anomalous dimensions γ_A is linear in $\ln s/m^2$, to arbitrary orders in perturbation theory[2]. However, other possibilities could arise in connection with the previously discussed mechanism for the promotion of infrared divergences into mass singularities. Since this important feature is often missed out in the literature, we have presented in the Appendix an illustrative example to $(g^2)^{N+1}$ order. We find that also in this case, the infrared divergent contributions proportional to $\ln^P(s)$ cancel out when $p > 1$. Due to these circumstances, and in view of the relevance of the infrared anomalous dimension to the investigation of the Wilson loops, we believe that the above conjecture deserves further study.

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APPENDIX

Here we will discuss the contributions resulting in the Feynman gauge form the graph shown in Fig. 4.

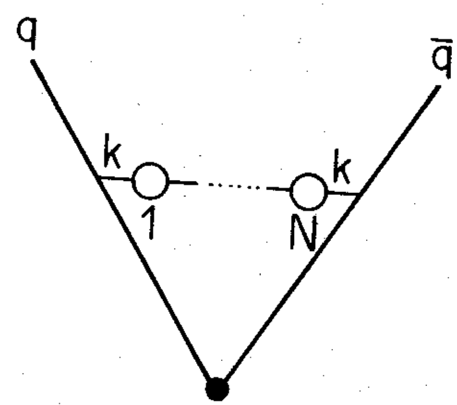

Fig. 4 – Feyman Diagram for the N–gluon loops contributing to the $q\bar{q}$ infrared form factor.

The renormalization gluon (and ghost) self–energy loop yields the contribution:

$$\Pi_{\mu\nu}(k) = \frac{g^2 \ \pi^2}{(2\pi)^4} C_A (\delta_{\mu\nu} k^2 - k_\mu k_\nu) \eta^{-1} \left[H(\eta) \left[\frac{k^2}{\mu^2}\right]^{\eta/2} - H(0) \right] \qquad (A.1)$$

where:

$$H(\eta) = 2^{-\eta} \pi^{-\eta/2} \Gamma(1-\eta/2) \Big[(8+4\eta) B(2+\eta/2, 2+\eta/2) + (2-\eta) B(1+\eta/2, 1+\eta/2) \Big] \qquad (A.2)$$

and $B(x,y)$ denotes the Euler beta function[10].

In order to determine the contribution to the infrared form factor we need to evaluate the infrared singular part of the integral:

$$I(\eta,N) = (g^2)^{N+1} \frac{(q\cdot\bar{q})\,\mu^{-\eta}}{(2\pi)^{4+\eta}\, i} \int d^{4+\eta}k \, \frac{1}{k^2} \, \frac{1}{k^2-2q\cdot k} \, \frac{1}{k^2+2\bar{q}\cdot k} \,\cdot$$

$$\cdot \frac{1}{\eta^N} \left[H(\eta) \left[\frac{k^2}{\mu^2}\right]^{\eta/2} - H(0) \right]^N \qquad \text{(A.3)}$$

A typical term in the above integrand contains the factor $\left[\frac{k^2}{\mu^2}\right]^{\alpha}$, where $\alpha = r\,\eta/2$ with $r \le N$. To calculate its contribution we combine denominators using the parametrization:

$$\frac{1}{A^{1-\alpha}\, BC} = \frac{\Gamma(3-\alpha)}{\Gamma(1-\alpha)} \int_0^1 x(1-x)^{-\alpha}\, dx \int_0^1 dy \left[A + (C-A)x + (B-C)xy \right]^{\alpha-3} \qquad \text{(A.4)}$$

We next perform the $4+\eta$ dimensional k integration and obtain for the corresponding part of the integral a contribution proportional to:

$$J(r,\eta) = \frac{\Gamma(1-(r+1)\eta/2)}{\Gamma(1-r\,\eta/2)} \left[\frac{m}{\mu}\right]^{(r+1)\eta} \frac{q\cdot\bar{q}}{m^2} H(\eta)]^r \,\cdot \qquad \text{(A.5)}$$

$$\cdot \int_0^1 dx\, x^{-1+(r+1)\eta} (1-x)^{-r\,\eta/2} \int_0^1 dy \left[1 + 2y(1-y) \left[\frac{q\cdot\bar{q}}{m^2} - 1\right]\right]^{-1+(r+1)\eta/2}$$

In terms of $J(r,\eta)$, the integral $I(\eta,N)$ can be expressed as:

$$I(\eta,N) = \frac{(g^2)^{N+1}\, \pi^{2+\eta/2}}{(2\,\pi)^{4+\eta}} \frac{1}{\eta^N} \sum_{r=0}^{N} (-1)^r \left[\begin{matrix} N \\ r \end{matrix}\right] [H(0)]^{N-r} J(r,\eta) \qquad \text{(A.6)}$$

Let us determine the asymptotic behavior of the infrared singular part of $I(\eta,N)$ at high energies. To this end, we note from (A.5) that the infrared singularity in $J(r,\eta)$ comes entirely from the x–integration which yields[10] $B((r+1)\eta\,,\, 1-r\,\eta/2)$. The asymptotic behavior $J(r,\eta)$ results when performing the y–integration, from the regions $y \to 0$ and $y \to 1$. Expanding the integrand in powers of η we then obtain as $s \to \infty$ the following expression

$$J(r,\eta) = \frac{2\Gamma(1-(r+1)\,\eta/2)\Gamma(1+(r+1)\eta)}{\Gamma(1+(r+2)\,\eta/2)} H^r(\eta)\,\cdot$$

$$\cdot \frac{1}{[(r+1)\eta]^2} \sum_{p=1}^{N+1} \frac{[(r+1)\eta]^p}{p\,!} \left[\ln^p(\sqrt{s}/\mu) - \ln^p(m/\mu) \right] + \cdots \qquad \text{(A.7)}$$

where we have indicated explicitly only those terms which contribute to the infrared divergent part of $I(\eta,N)$.

Consider now the infrared singular contributions to $I(\eta,N)$ which are proportional to $\ln^p(s)$ for $1 \le p \le N+1$. It is then straightforward to verify, with the help of the algebraic identity[10].

$$\sum_{r=0}^{N} (-1)^r \left[\begin{matrix} N \\ r \end{matrix}\right] r^{n-1} = 0 \qquad [N \ge n \ge 1] \qquad \text{(A.8)}$$

that the corresponding coefficients vanish identically, with the important exception of the case when $p = 1$. This behavior is connected with the fact that for $p > 1$ (A.7) depends upon μ and a physical answer should be independent of this parameter.

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