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ABSTRACT

We perfom a qualitative analysis of the results of an Sp(1,R) calculation for ^{8}Be, in a deformed harmonic oscillator basis. The model basis of states is given by the angular momentum projection of deformed phonon states, determined by the method of variation after projection. These deformed phonons are associated to giant monopole and quadrupole resonances.

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INTRODUCTION

During the past decade a significant progress has been made in our understanding of collective phenomena in microscopic terms, (Rowe 1985). The microscopic models show what kinds of collective motions are compatible with the many nucleon structure of the nucleus. It exposes the relationships between the various kinds of collective motions, notably rotations and quadrupole and monopole vibrations, and it reveals what shell model configurations are necessary for a microscopic calculation of collective states, that is, expresses the collective model as a submodel of the nuclear shell model.

The symplectic collective model (SCM), is a microscopic collective model constructed to describe rotations and quadrupole and monopole vibrations of nuclei. The SCM is an algebraic model where the collective operators are a basis of the sp(3,R) algebra and are expressed in microscopic terms. The collective subspace is identified with an irreducible representation (IR) space of Sp(3,R) which, in turn, is a subspace of the spherical harmonic oscillator shell model space. The collective hamiltonian is identified with the restriction of the nuclear many–body hamiltonian to the collective subspace.

A basis of the sp(3,R) algebra (Rowe 1985), is given by the six cartesian components of the quadrupole tensor

$$\hat{Q}_{\mu\nu} = \sum_{i=1}^{A-1} \hat{x}_{\mu i}\, \hat{x}_{\nu i} \,, \qquad (1.1\text{–a})$$

the six generators of monopole and quadrupole deformations

$$\hat{D}_{\mu\nu} = \sum_{i=1}^{A-1} \frac{(\hat{x}_{\mu i}\; \hat{p}_{\nu i} + \hat{p}_{\mu i}\; \hat{x}_{\nu i})}{2\hbar} \qquad (1.1\text{–b})$$

the angular momentum operators

$$\hat{L}_{\mu\nu} = \sum_{i=1}^{A-1} \frac{(\hat{x}_{\mu i}\ \hat{p}_{\nu i} - \hat{x}_{\nu i}\ \hat{p}_{\mu i})}{\hbar} \tag{1.1–c}$$

$$\hat{L}_{\sigma} = \frac{1}{2} \sum_{\mu\nu} \epsilon_{\sigma\mu\nu} \hat{L}_{\mu\nu}$$

and the six cartesian components of the quadrupole tensor of the momentum

$$\hat{K}_{\mu\nu} = \sum_{i=1}^{A-1} \hat{p}_{\mu i}\, \hat{p}_{\nu i} \quad . \tag{1.1–d}$$

In eqs. (1.1), the $\hat{x}_{\mu i}$ are a set of Jacobi coordinates and $\hat{p}_{\mu i}$ its associated canonical momenta, where μ indicates spatial directions and i is a "Jacobi" particle index. The construction of a basis of states in a Sp(3,R) IR space is easier in Fock space, (Rowe 1985). We define the annihilation and creation operators of oscillator quanta,

$$\hat{a}_{\mu i}(b_0) = \frac{1}{\sqrt{2}} \left[\frac{\hat{x}_{\mu i}}{b_0} + \frac{i\ \hat{p}_{\mu i}}{b_0} \hbar \right] \tag{1.2–a}$$

$$\hat{a}^{+}_{\mu i}(b_0) = \left[\hat{a}_{\mu i}(b_0) \right]^{+} \quad , \tag{1.2–b}$$

where b_0 is the harmonic oscillator size parameter. The expression of the sp(3.R) operators, eqs. (1.1), in terms of the creation and annihilation operators, eqs. (1.2), gives the new basis:

$$\hat{A}_{\mu\nu}(b_0) = \sum_{i=1}^{A-1} \hat{a}^{+}_{\mu i}\ b_0)\ \hat{a}^{+}_{\nu i}(b_0) \qquad \text{raising operators} \tag{1.3–a}$$

$$\hat{B}_{\mu\nu}(b_0) = \sum_{i=1}^{A-1} \hat{a}_{\mu i}(b_0)\ \hat{a}_{\nu i}(b_0) \qquad \text{lowering operators} \tag{1.3–b}$$

$$C_{\mu\nu}(b_0) = \frac{1}{2} \sum_{i=1}^{A-1} \left[\hat{a}^{+}_{\mu i}(b_0)\ \hat{a}_{\nu i}(b_0) + \hat{a}_{\nu i}(b_0)\ \hat{a}^{+}_{\mu i}(b_0) \right] \qquad \text{U(3) operators} \tag{1.3–c}$$

The U(3) operators are further subdivided into raising, lowering and weight operators

$$\hat{C}_{\mu\nu} \qquad \mu > \nu \qquad \text{raising operators} \tag{1.4–a}$$

$$\hat{C}_{\mu\nu} \qquad \mu < \nu \qquad \text{lowerig operators} \tag{1.4–b}$$

$$\hat{C}_{\mu\mu} \qquad \text{weight operators} \tag{1.4–c}$$

A basis of states in the IR $\{f\} = \{f_1,f_2,f_3\}$ of Sp(3,R) is given by the repeated action of the operators $\hat{A}_{\mu\nu}(b_0)$ on a basis states $|\{f\};\alpha>$ of the U(3) lowest–weight IR {f}, which satisfy

$$\hat{B}_{\mu\nu}|\{f\};\alpha> \ = \ 0 \quad , \tag{1.5}$$

where α indexes the basis of states.

In turn, a basis of states in the U(3) IR {f} is given by the repeated action of the U(3) raising operators, eq. (1.4–a), on the U(3) lowest–weight state, which obey the equations

$$\hat{C}_{\mu\mu}|\{f\},LW> = \left[f_\mu + \frac{A-1}{2} \right] |\{f\},LW> \qquad (1.6\text{–a})$$

$$\hat{C}_{\mu\nu}|\{f\},LW> = 0 \qquad \mu < \nu \qquad (1.6\text{–b})$$

where f_μ is the number of oscillator quanta in the direction μ, of the state $|\{f\},LW>$. This state, which satisfies eq. (1.5), is an Sp(3,R) lowest–weight state.

Thus we see that the SCM is a generalization of the SU(3) model of Elliott (Elliott 1958). In the Elliott model the basis states, $|\{f\},\alpha>$, have the same harmonic oscillator energy. In the SCM, besides these states, the basis states carry states of excitation energy equal to $2n\hbar w$, $n \neq 0$ with respect to the states of the Elliott model. The configuration of these states are given by the successive addition of two oscillator quanta, by the action of the operators $\hat{A}_{\mu\nu}(b_0)$, to the states of the Elliott model.

Due to the complexity of the SCM, microscopic calculations have been restricted to submodels of the complete model. In these submodels we supress some of the degrees of freedom of the SCM. For prolate (oblate) nuclei one successful submodel is the Sp(1,R) model which, in the intrinsic frame, considers only oscillations parallel (perpendicular) to the direction of axial symmetry, (Arickx et al. 1979, Arickx et al. 1982).

An analysis of the results of the application of the Sp(1,R) model in the description of the properties of the states in the ground state band in ^{20}Ne, demonstrates the importance of the excited states, $n \neq 0$, (Okhrimenko and Steshenko 1981). For example, in the model ground state, basis states with $n \lesssim 3$ are equally probable, basis states with $1 \leq n \leq 4$ give the dominant contribution to the BE2 between the first 2^+ and 0^+ states, and to achieve numerical convergence, one has to use up to $n \simeq 20$ basis states. All these facts are a consequence of the strong correlations due to the deformation which is described in the spherical harmonic oscillator basis only by the inclusion of these

excited basis states. This fact canceals the physical interpretation of the effects due to the deformation and raises difficulties to the extension of these calculations to include mixture of Sp(3,R) IR spaces.

Given the basis of operators 1.3 of the sp(3,R) algebra and the basis states in the IR space, we can find equivalent basis of operators and of states by symplectic unitary transformations. One of these symplectic unitary transformations is a change of scale in the direction μ, whose infinitesimal generator is the operator $\hat{D}_{\mu\mu}$, (Rowe 1985)

$$e^{-i\alpha_\mu \hat{D}_{\mu\mu}}\, \hat{a}_{\mu i}(b_0)\, e^{+i\alpha_\mu \hat{D}_{\mu\mu}} = \hat{a}_{\mu i}(b_0\, e^{\alpha_\mu}) \ .$$

The action of this unitary transformation in the basis states replaces the spherical harmonic oscillator wave functions by deformed harmonic oscillator wave functions of size parameters $b_0\, e^{\alpha_\mu}$, $\mu = 1,2,3$. As is well known, the basis constructed in terms of deformed harmonic oscillator wave functions is, from a variational point of view, the optimal one.

In our paper we present a qualitative analysis of the results of an Sp(1,R) calculation in ^{8}Be, in an optimized deformed harmonic oscillator basis. Our purpose is not only to show that the physical interpretation of the effects due to the deformation is more transparent in this basis, but also to demonstrate that a numerical calculation in this basis is feasible. The first steps in this direction has already been taken by Arickx et al. 1982, from a slightly different point of view. Our paper elaborate further and advance this program.

Our paper is organized as follows: In section II we discuss briefly the results of an Sp(1,R) calculation for ^{8}Be, in the spherical harmonic oscillator basis. In section III we show how to construct the deformed harmonic oscillator basis by the method of variation after projection and we present the results of an Sp(1,R) calculation for ^{8}Be in this basis. Our conclusions and final remarks are the subject of section four.

II. THE $Sp_{\parallel}(1,R)$ CALCULATION IN ^{8}Be

A basis of states of the $Sp_{\parallel}(1,R)$ model for prolate nuclei is given by the angular momentum projection of states defined in an $Sp(1,R)$ IR space.

The $sp_{\parallel}(1,R)$ algebra is spanned by the operators (Arickx et al. 1979)

$$\hat{A}_{\parallel}(b_0) = \frac{1}{2}\sum_{i=1}^{A-1} \hat{a}^{\dagger}_{1i}(b_0)\,\hat{a}^{\dagger}_{1i}(b_0) \qquad (2.1\text{–a})$$

$$\hat{B}_{\parallel}(b_0) = \frac{1}{2}\sum_{i=1}^{A-1} \hat{a}_{1i}(b_0)\,\hat{a}_{1i}(b_0) \qquad (2.1\text{–b})$$

$$\hat{C}_{\parallel}(b_0) = \frac{1}{4}\sum_{i=1}^{A-1} \left[\hat{a}^{\dagger}_{1i}(b_0)\,\hat{a}_{1i}(b_0) + \hat{a}_{1i}(b_0)\,\hat{a}^{\dagger}_{1i}(b_0)\right] \quad , \qquad (2.1\text{–c})$$

which corresponds to longitudinal vibrations (in the direction of axial symmetry).

A lowest weight state of $Sp_{\parallel}(1,R)$ is a state annihilated by $\hat{B}_{\parallel}(b_0)$ and an eigenstate of $\hat{C}_{\parallel}(b_0)$,

$$\hat{B}_{\parallel}(b_0)|0;b_0\rangle = 0 \qquad \hat{C}_{\parallel}(b_0)|0;b_0\rangle = K_{\parallel}|0;b_0\rangle \quad ,$$

where $K_{\parallel}$ labels the $Sp_{\parallel}(1,R)$ IR.

From the above definition, we see that a lowest weight state of the $Sp(3,R)$ IR $\{f_0\}$ is a lowest weight state of the IR $K_{\parallel} = (f_{0_1} + \frac{A-1}{2})/2$ of $sp_{\parallel}(1,R)$.

Given the lowest weight state, we can construct a basis in an irreducible representation space by the repeated action of $\hat{A}_{\parallel}(b_0)$ on $|0;b_0\rangle$,

$$|n;b_0\rangle = \hat{A}_{\parallel}(b_0)^{n}|0;b_0\rangle \sqrt{\frac{\Gamma(2K_{\parallel})}{\Gamma(n+1)\;\Gamma(2K_{\parallel}+n)}} \qquad (2.2)$$

where $|n;b_0\rangle$ are called stretched states (Arickx et al. 1979).

Since an $Sp(3,R)$ lowest weight state is also an $SU(3)$ lowest weight state, it follows that $|0;b_0\rangle$ is a lowest weight state of the $SU(3)$ IR (λ_0,μ_0),

$$\lambda_0 = f_{01} - f_{02} \qquad \mu_0 = f_{02} - f_{03} = 0 \ .$$

As a consequence, the stretched states $|n;b_0\rangle$ are lowest weight states of the $SU(3)$ IR $(\lambda_0 + 2n,0)$.

A basis in the $Sp_{\parallel}(1,R)$ model is given by the angular momentum projection of the stretched states

$$|nLM;b_0\rangle = P^{L}_{M0}|n;b_0\rangle \quad , \qquad (2.3)$$

where P^{L}_{M0} is the angular momentum projection operator and, as the stretched states are axially symmetric, we can project only $K = 0$ states.

The basis states (2.3) are orthogonal, since they are eigenstates of the spherical harmonic oscillator hamiltonian in the center of mass frame with eigenvalue $(N_0+2n)\hbar\omega_0$, $N_0 = f_{01} + f_{02} + f_{03} + \frac{3(A-1)}{2}$, and they span the $(\lambda_0 + 2n,0)$ $SU(3)$ IR spaces.

The stretched states have a very simple interpretation in terms of the $sp_{\parallel}(1,R)$ Holstein–Primakoff bosons, $\hat{S}^{\dagger}_{\parallel}(b_0)$ and $\hat{S}_{\parallel}(b_0)$, defined only in the IR space

$K_{\shortparallel}$, of $Sp_{\shortparallel}(1,R)$ (Arickx et al. 1982, Broeckhove et al 1984)

$$\hat{A}_{\shortparallel}(b_0) = \hat{S}^{+}_{\shortparallel}(b_0)(2K_{\shortparallel} + \hat{S}^{+}_{\shortparallel}(b_0)\,\hat{S}_{\shortparallel}(b_0))^{1/2} \tag{2.4--a}$$

$$\hat{B}_{\shortparallel}(b_0) = (2K_{\shortparallel} + \hat{S}^{+}_{\shortparallel}(b_0)\,\hat{S}_{\shortparallel}(b_0))^{1/2}\,\hat{S}_{\shortparallel}(b_0) \tag{2.4--b}$$

$$\hat{C}_{\shortparallel}(b_0) = K_{\shortparallel} + \hat{S}^{+}_{\shortparallel}(b_0)\,\hat{S}_{\shortparallel}(b_0) \quad . \tag{2.4--c}$$

From eqs. (2.4) one deduces that $\hat{S}^{+}_{\shortparallel}(b_0)$ and $\hat{S}_{\shortparallel}(b_0)$ act on the basis states in canonical fashion

$$\hat{S}^{+}_{\shortparallel}(b_0)\,|n;b_0> = (n+1)^{1/2}\,|n+1;b_0>$$

$$\hat{S}_{\shortparallel}(b_0)\,|n;b_0> = n^{1/2}\,|n-1;b_0>$$

$$[\hat{S}_{\shortparallel}(b_0)\,,\,\hat{S}^{+}_{\shortparallel}(b_0)]\,|n;b_0> = |n;b_0> \quad .$$

As a consequence the stretched states can be written as

$$|n;b_0> = \frac{\hat{S}^{+}_{\shortparallel}(b_0)}{\sqrt{n!}}\,|0;b_0> \qquad \hat{S}_{\shortparallel}(b_0)\,|0;b_0> = 0$$

where $|0;b_0>$ is the longitudinal boson vacuum and $|n;b_0>$ the n–longitudinal boson state. In the case of ^{8}Be, the state $|0;b_0>$ is the intrinsic component of the Slater determinant

$$|\psi_0,b_0> = \frac{1}{\sqrt{8!}}\det\,(000)^4\,(100)^4 \quad , \tag{2.5}$$

where $(n_z n_x n_y)$ is the harmonic oscillator wave function in the cartesian basis. This state is non–spurious, has permutation symmetry [44] and $T = S = 0$.

It can be easily shown that this state is, respectively, a lowest weight state of the (4,0,0) Sp(3,R) IR and of the $\frac{15}{4}$ $Sp_{\shortparallel}(1,R)$ IR.

The model states are determined by diagonalization of a microscopic hamiltonian in the basis of eq. (2.3). In our calculation we use the Brink–Boeker hamiltonian (Brink 1967). Matrix elements of the hamiltonian and of the collective operators are calculated by the generating function method (Arickx et al. 1979) and b_0 is fixed at the value which minimizes the model ground state energy. To achieve convergence we must use basis states up to $n_{max} \simeq 30$.

Our results agree with Arickx et al. 1979 and have the same qualitative behaviour of the ^{20}Ne calculations of Okhrimenko et al. 1981, Vassanji et al. 1983. Figs. 1 and 2 demonstrates the importance of the excited basis states. These figures are a graph of the partial contribution of the excited basis states to the $BE2(2^+_0 \to 0^+_0)$ and to the quadrupole moment of the first 2^+ state (Okhrimenko et al. 1981). The values of these quantities, considering states up to n, is given by the sum of the previous blocks plus the block which starts at the point n. From these figures, it is evident the importance of the excited basis states. The explanation of this behaviour is well known. The spherical harmonic oscillator basis is not optimized with respect to the deformation. This fact suggests a qualitative study of an $Sp_{\shortparallel}(1,R)$ calculation in an equivalent basis optimized by variational methods.

III. $Sp_\parallel(1,R)$ CALCULATIONS IN A DEFORMED HARMONIC OSCILLATOR BASIS

Given the stretched states 2.2, constructed in terms of spherical harmonic oscillator wave functions, we can find equivalent basis by $Sp_\parallel(1,R)$ unitary transformations. One of those is a scale transformation in the longitudinal direction, whose generator is $\hat{D}_{11}$,

$$\hat{D}_{11} = i(\hat{A}_\parallel(b_0) - \hat{B}_\parallel(b_0)) \quad . \tag{3.1}$$

The longitudinal scale transformation of the stretched states,

$$|n;b_\parallel b_0\rangle = e^{-i \ln \frac{b_\parallel}{b_0} \hat{D}_{11}} |n;b_0\rangle \tag{3.2.a}$$

and of the operators 2.3,

$$\begin{array}{ll} \hat{A}_\parallel(b_\parallel) & \hat{A}_\parallel(b_0) \\ \hat{B}_\parallel(b_\parallel) = e^{-i \ln \frac{b_\parallel}{b_0} \hat{D}_{11}} & \hat{B}_\parallel(b_0)\ e^{+i \ln \frac{b_\parallel}{b_0} \hat{D}_{11}} \\ \hat{C}_\parallel(b_\parallel) & \hat{C}_0(b_0) \end{array} \tag{3.2–b}$$

leads to:

$$|n;b_\parallel b_0\rangle = \hat{A}_\parallel(b_\parallel)^n |0;b_\parallel b_0\rangle \left[\frac{\Gamma(2K_\parallel)}{\Gamma(n+1)\ \Gamma(2K_\parallel+n)} \right]^{1/2}$$

$$\hat{A}_\parallel(b_\parallel) = \frac{1}{2} \sum \hat{a}^\dagger_{1i}(b_\parallel)\, \hat{a}^\dagger_{1i}(b_\parallel) \quad ,$$

$$\hat{B}_\parallel(b_\parallel) = \frac{1}{2} \sum_i \hat{a}_{1i}(b_\parallel)\, \hat{a}_{1i}(b_\parallel)$$

$$\hat{C}_\parallel(b_\parallel) = \frac{1}{4} \sum \left[\hat{a}^\dagger_{1i}(b_\parallel)\, \hat{a}_{1i}(b_\parallel) + \hat{a}_{1i}(b_\parallel)\, \hat{a}^\dagger_{1i}(b_\parallel) \right] \quad .$$

The operators $\hat{A}_\parallel(b_\parallel)$, $\hat{B}_\parallel(b_\parallel)$, $\hat{C}_\parallel(b_\parallel)$ are an equivalent basis for the $sp_\parallel(1,R)$ algebra, (2.1). Equally well, the states $|n;b_\parallel b_0\rangle$ are an equivalent basis of states in the $Sp_\parallel(1,R)$ IR space labelled by $K_\parallel$. The effect of the scale transformation of $|n,b_0\rangle$ is to replace the spherical harmonic oscillator wave functions with $b_x = b_y = b_z = b_0$ by deformed harmonic oscillator wave functions with $b_x = b_y = b_0$ and $b_z = b_\parallel$.

The states $|n;b_\parallel b_0\rangle$ have the same properties and physical interpretation as the states $|n;b_0\rangle$, only now in terms of an equivalent basis of operators, given by the scaling transformation of the operators (1.3). Analogous to the spherical case, the states $|n;b_\parallel b_0\rangle$ can be interpreted as n–deformed longitudinal phonon states, where the deformed longitudinal boson, $\hat{S}^+_\parallel(b_\parallel)$, is given by the scaling transformation, eq. (3.2), of the spherical boson $\hat{S}^+_\parallel(b_0)$.

The coefficients of the expansion of $|n;b_\parallel b_0\rangle$ in the spherical boson states $|n;b_0\rangle$,

$$|n;b_\parallel b_0\rangle = \sum_{n'} |n';b_0\rangle\, U_{n'n} \quad , \tag{3.3}$$

can be calculated explicitly:

$$U_{n'n} = \langle n';b_0| \, e^{-i \ln \frac{b_\shortparallel}{b_0} \hat{D}_{11}} \, |n;b_0\rangle = \sqrt{\frac{\Gamma(n+1)\Gamma(n'+1)}{\Gamma(n+2K_\shortparallel)\Gamma(n'+2K_\shortparallel)}} \left[(1-\tau_0^2)\right]^{K_\shortparallel}$$

$$\times \sum_{r=0}^{\{n,n'\}} \frac{\Gamma(2K_\shortparallel+n'+n-r)}{\Gamma(n-r+1)\ \Gamma(n'-r+1)\ \Gamma(r+1)} (-1)^{n-r} \tau_0^{n+n'-2r} ,$$

where τ_0 is the deformation parameter

$$\tau_0 = \frac{1 - \left[\frac{b_0}{b_\shortparallel}\right]^2}{1 + \left[\frac{b_0}{b_\shortparallel}\right]^2} .$$

Projecting states of good angular momentum we can find the relation between the equivalent basis of the $Sp_\shortparallel(1,R)$ model:

$$|nLM;b_\shortparallel b_0\rangle = \sum_{n'} |n'LM;b_0\rangle \, U_{n'n} . \qquad (3.4)$$

The states $|nLM;b_\shortparallel b_0\rangle$ are not orthogonal and for $L > \lambda_0$ they are linearly dependent, the number of states linearly dependent being equal to $n_L = \frac{L-\lambda_0}{2}$.

The equivalent basis of states depends on a parameter, $b_\shortparallel$, which is the oscillator size parameter in the direction of axial symmetry. We fixed it by the method of variation after projection, imposing that the expectation value of the hamiltonian in the vacuum of the deformed phonon, projected into angular momentum L, be a minimum:

$$\partial_{b_\shortparallel} \frac{\langle OLM ; b_\shortparallel b_0 | \hat{H} | OLM; b_\shortparallel b_0\rangle}{\langle OLM; b_\shortparallel b_0 | OLM ; b_\shortparallel b_0\rangle} = 0 .$$

This condition is equivalent to impose that the matrix element between the L projected vacuum and one deformed phonon state, orthogonalized by the method of Gram–Schmidt, vanishes:

$${}_{GS}\langle 0LM;b_\shortparallel b_0|H|1LM;b_\shortparallel b_0\rangle_{GS} = 0 .$$

where

$$|0LM;b_\shortparallel b_0\rangle_{GS} = |0LM;b_\shortparallel b_0\rangle$$

$$|1LM;b_\shortparallel b_0\rangle = |1LM;b_\shortparallel b_0\rangle - |0LM;b_\shortparallel b_0\rangle \frac{\langle 0LM;b_\shortparallel b_0|1LM;b_\shortparallel b_0\rangle}{\langle 0LM;b_\shortparallel b_0|0LM;b_\shortparallel b_0\rangle} .$$

Given the states $|nLM;b_\shortparallel b_0\rangle$, an orthonormal basis $|nLM;b_\shortparallel b_0\rangle_{OGS}$, can be constructed by the method of Gram–Schmidt.

In our paper, we calculated the matrix elements of the operators in the deformed harmonic oscillator basis in terms of the ones in the spherical harmonic oscillator basis, truncating the expansion (3.4) at $n'_{max} = 30$, and orthonormalizing the basis states by the method of Gram–Schmidt. We have verified that this truncation does not affect the matrix elements of the observables of interest, for the states up to four phonons. This procedure is an extension to $n \neq 0$ of a method proposed by Asherova et al. 1981. Table I shows the values of $b_\shortparallel(L)$ which minimizes the expectation value of the hamiltonian for L = 0,2,4,6 . The probability of finding in the first two states of a given L, the L projected vacuum, one and two deformed phonon state, orthonormalized by the method of Gram–Schmidt, is shown in Table II.

For the states in the ground state band we show in figs. 3 and 4 the contribution of the basis states $|nLM;b_\shortparallel b_0\rangle_{OGS}$ to the $BE2(2_0^+ \to 0_0^+)$ and to the electric quadrupole moment of the first 2^+ state. To investigate the behaviour of the states in an excited band we present in fig. 5 an identical graph for the monopole transition between the first two 0^+ states. Finally the excitation energy and the geometrical characteristics of the states in the ground band, calculated by diagonalizing the hamiltonian in a basis truncated at $n_{max} = 0$–4 are given in Table III.

IV. DISCUSSION OF THE RESULTS AND CONCLUDING REMARKS

Table II shows that the first two states of a given L are dominated, respectively, by the L projected vacuum and one deformed phonon state. This dominance is reflected in the matrix elements of the collective observables as shown in figs. 3 and 4. The contribution of the projected vacuum for the $BE2(2_0^+ \to 0_0^+)$ and for $Q_{e\ell_{2_0^+}}$ is greater than 99%. This dominance is not so overwhelming when we consider states in the first excited band as shown in fig. 5. In this case the contribution of the vacuum and one deformed phonon state is greater than 90%. In all cases, the contribution coming from states with more than four phonons is negligeable.

This is further demonstrated in Table III. We truncate the basis states (3.3) at $n_{max} = 0$–4. At each n_{max} we diagonalize the hamiltonian and we calculate the excitation energy and the geometrical characteristics of the states in the ground band. We see that the truncation at the two deformed phonon state gives results practically identical to the exact ones, the BE2 being the most sensible observable. For the lower L, this is true, even when we only consider the projection of the vacuum.

To conclude, we see that the states in the ground band is well described by the angular momentum projection of a single L dependent state, the vacuum of the

deformed phonon, $\hat{S}_{\shortparallel}(b_{\shortparallel}(L))$. These deformed phonons are associated to the giant monopole and quadrupole resonances. The addition of a few more states, the projection of one and two deformed phonon states, leads to results practically identical to the exact ones. Thus, the Sp(1,R) model can be view as an improvement of the variation after projection deformed harmonic oscillator model (Abgrall et al. 1969, Bouten et al. 1981).

The fast convergence and the feasibility of numerical calculations in the deformed basis, opens up the possibility of performing microscopic Sp(3,R) calculations with mixture of Sp(3,R) IR (Carvalho et al. 1987), in a scheme that is a generalization of the work of Bouten et al. 1981. This improves the predictions of the symplectic collective model as suggested below. The symplectic shell model decomposes the Hilbert space of the harmonic oscillator shell model into symplectic shells (Rowe 1985). Each symplectic shell carries an Sp(3,R) IR and the symplectic collective model is restricted to a single Sp(3,R) IR. By construction, there is no quadrupole and monopole transitions between states belonging to different symplectic shells. However experimental quadrupole and monopole transitions between low–lying states having dominant components in different symplectic shells, (examples are the first two 0^+ states in ^{12}C and ^{16}O), indicates the importance of mixture of symplectic shells. This mixture is also important to solve the problem of excessive collectivity of a single Sp(3,R) IR calculation (Vassanji et al. 1983).

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TABLE CAPTIONS

Table I – Values of $b_{''}(L)$ which minimizes the expectation value of the hamiltonian in the vacuum of the deformed phonon, projected to angular momentum L. $b_0 = 1.39$ fm.

Table II – Probability of finding the states $|nLM;b_{''}(L)b_0>_{OGS}$, n = 0,1,2 , in the exact model states, for the first two states of L = 0–6 , $|\bar{C}_n^{\nu}(L)|^2 =$ $= |<\nu LM|nLM,b_{''}(L)b_0>_{OGS}|^2$.

Table III – Excitation energy and the geometrical characteristics of the states in the ground band calculated by diagonalizing the hamiltonian in a basis truncated at $n_{max} = 0$–4 . The energies are given in Mev and $<r^2>^{1/2}$ in fermis. The BE2 and the electric quadrupole moment are given, respectively, in $e^2\,fm^4$ and $e\,fm^2$.

FIGURE CAPTIONS

Fig. 1 – Partial contribution to the $BE2(2_0^+ \rightarrow 0_0^+)$, from a systematic inclusion, in the expansion $|\nu LM\rangle = \sum_{n=0}^{n_{max}} C_n^{\nu}(L)\,|nLM;b_0\rangle$, of the basis states $|nLM;b_0\rangle$. The value of the BE2 at point n is found by considering, in the above expansion, the contribution of all terms up to n, inclusive.

Fig. 2 – As in fig. 1, for the electric quadrupole moment of the first 2^+ state.

Fig. 3 – As in fig. 1. The only difference is that the expansion is in the basis states $|nLM;b_{\shortparallel}(L)\,b_0\rangle_{OGS}$, $|\nu LM\rangle = \sum_n \bar{C}_n^{\nu}(L)\,|nLM;b_{\shortparallel}(L)b_0\rangle_{OGS}$. Notice the change of scale.

Fig. 4 – As in fig. 3, for the electric quadrupole moment of the first 2^+ state.

Fig. 5 – As in fig. 3, for the monopole transition $M(0_1^+ \rightarrow 0_0^+)$.

TABLE I

	L = 0	L = 2	L = 4	L = 6
$b_{\shortparallel}$ (fm)	2.27	2.23	2.17	2.50

TABLE II

	$\lvert\bar{C}_n^{\nu}(0)\rvert^2$		$\lvert\bar{C}_n^{\nu}(2)\rvert^2$		$\lvert\bar{C}_n^{\nu}(4)\rvert^2$		$\lvert\bar{C}_n^{\nu}(6)\rvert^2$	
n \ ν	0	1	0	1	0	1	0	1
0	0.999	–	0.998	–	0.988	0.001	0.992	0.001
1	–	0.954	–	0.941	–	0.921	–	0.920
2	–	0.037	0.001	0.044	0.011	0.031	0.007	0.043

TABLE III

	EXACT				VACUUM			
L	EXC.E	$\langle r^2\rangle^{1/2}$	BE2	$Q_{e\ell}$	EXC.E	$\langle r^2\rangle^{1/2}$	BE2	$Q_{e\ell}$
0	–	2.48	–	–	–	2.48	–	–
2	3.15	2.48	20.12	– 9.09	3.14	2.48	20.07	– 9.04
4	11.15	2.50	29.05	–11.88	11.39	2.49	28.81	–11.70
6	25.62	2.87	41.50	–19.41	25.72	2.86	37.13	–19.18

	2 – PHONON				3 – PHONON			
L	EXC.E	$\langle r^2\rangle^{1/2}$	BE2	$Q_{e\ell}$	EXC.E	$\langle r^2\rangle^{1/2}$	BE2	$Q_{e\ell}$
0	–	2.48	–	–	–	2.49	–	–
2	3.11	2.48	20.16	– 9.09	3.13	2.48	20.28	– 9.14
4	11.09	2.50	29.12	–11.92	11.12	2.50	29.13	–11.87
6	25.55	2.87	41.62	–19.44	25.58	2.87	41.13	–19.40

	4 – PHONON			
L	EXC.E	$\langle r^2\rangle^{1/2}$	BE2	$Q_{e\ell}$
0	–	2.48	–	–
2	3.14	2.48	20.20	– 9.10
4	11.12	2.50	29.08	–11.89
6	25.59	2.87	41.52	–19.41

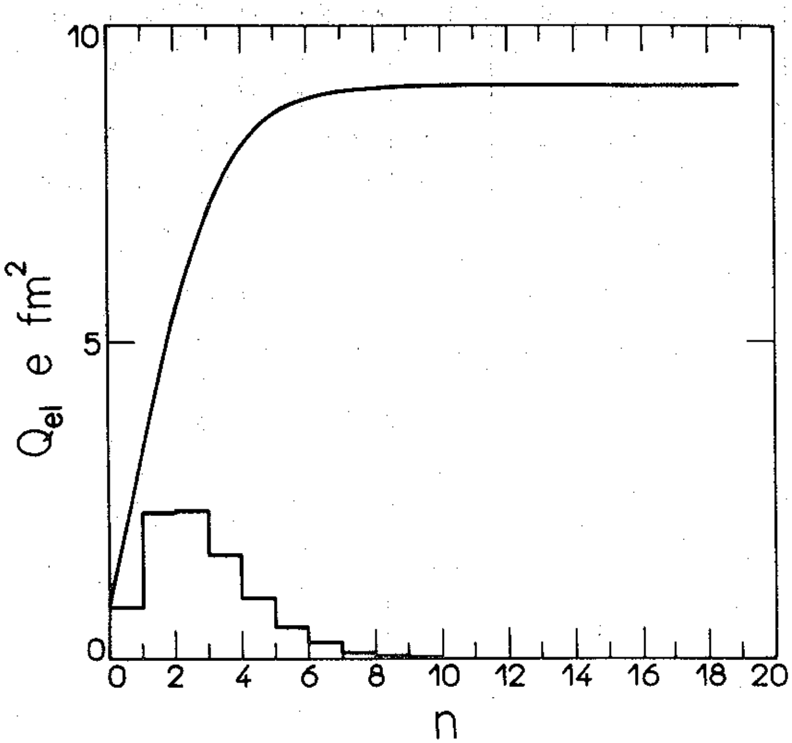

FIG. 1

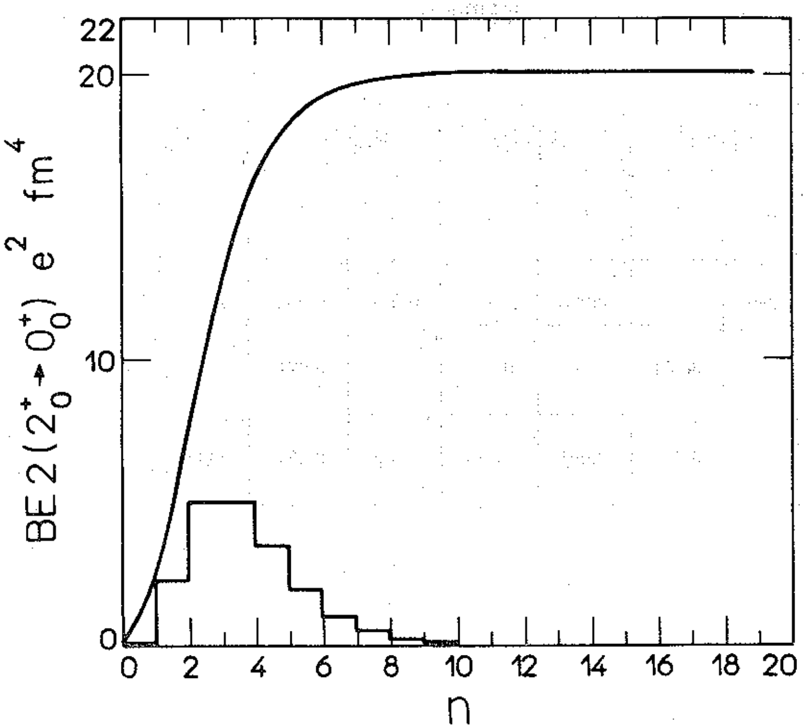

FIG. 2

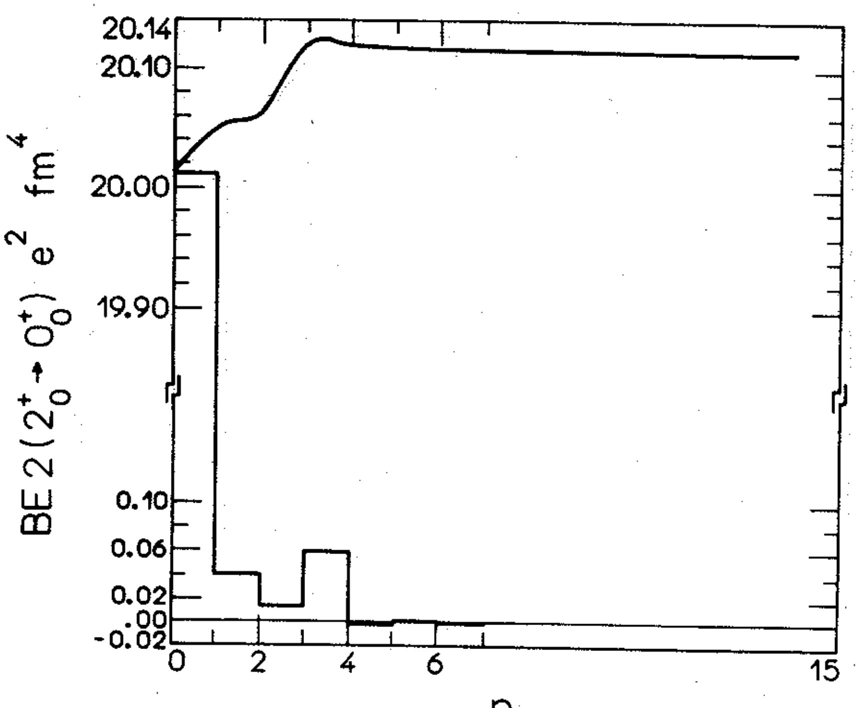
20.14
20.10
20.00
19.90
0.10
0.06
0.02
0.00
-0.02
0
2
4
6
15
n
BE 2 (2$_0^+$ → 0$_0^+$) e^2 fm^4

FIG. 3

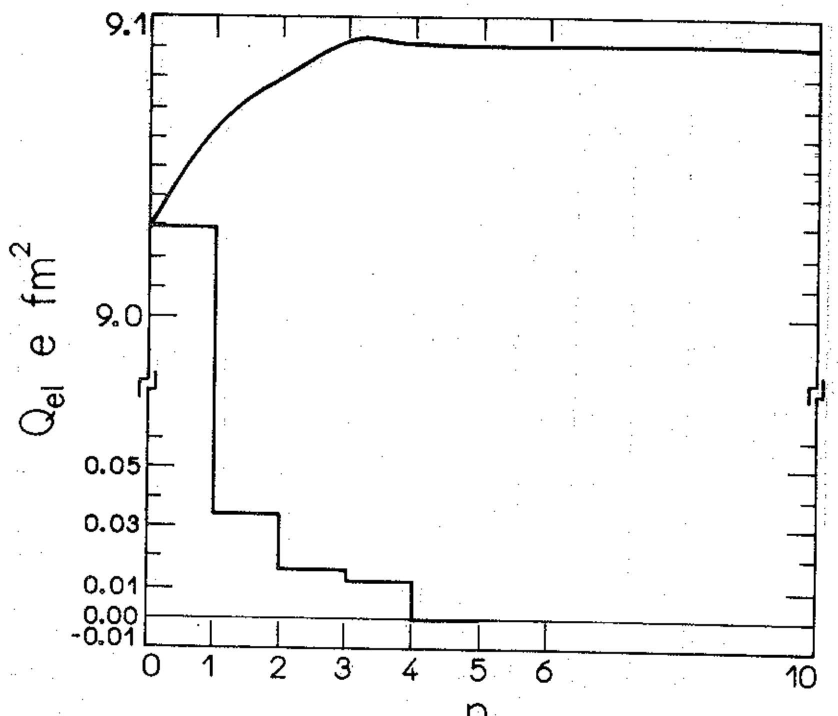
9.1
9.0
0.05
0.03
0.01
0.00
-0.01
0
1
2
3
4
5
6
10
n
Q_{el} e fm^2

FIG. 4

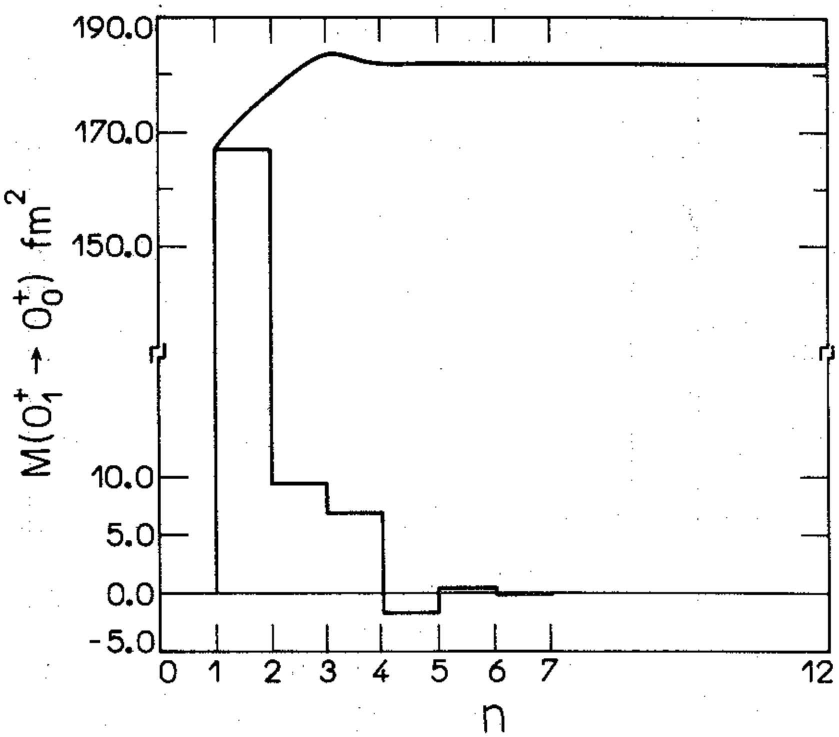
190.0
170.0
150.0
10.0
5.0
0.0
-5.0
$M(0_1^+ \rightarrow 0_0^+)$ fm^2
0
1
2
3
4
5
6
7
12
n

FIG. 5