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IFUSP/P-737

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Setembro/1988

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ABSTRACT

Three-body corrections to the DWBA inclusive break up cross section are derived and discussed. A new sum rule, similar in structure to the Ichimura, Austern and Vincent sum rule, but contains the full three-body corrections at the DWBA level, is derived and analyzed within a Glauber-type approximation.

* Supported in part by the CNPq

** J. S. Guggenheim Fellow

August 1988

The inclusive nuclear break-up (IB) cross section should describe the process

$$a + A \equiv (b+x) + A \longrightarrow b + \sum_{\substack{all \\ states}} (x+A)$$

where A refers to the target nucleus, and the projectile denoted by a considered to be a two-cluster configuration which breaks up into the ejectile (spectator) b and the participant nucleus x. All states of the strongly interacting $x+A$ system must be taken into account. In all discussion to follow, the spectator hypothesis is invoked, namely the b-particle scatters off A without causing non-elastic transitions.

A rather heated debate has been going on concerning the theory of the IB cross section. On the one hand Austern et.al.[1], and Hussein and McVoy[2] describe the IB cross sections as being proportional to a simple matrix element, $\langle \hat{\Psi}_x^{(+)} | W_{xA} | \hat{\Psi}_x^{(+)} \rangle$, with W_{xA} being the imaginary part of the xA scattering system and $\hat{\Psi}_x^{(+)}$ is, according to 1), an overlap of the exact $b-x-A$ Faddeev wave function with the optical wave function of the spectator $\chi_b^{(-)}$, and according to 2), the overlap $(\chi_b^{(-)} | \chi_a^{(+)} \varphi_{xb} \rangle$[3], where $\chi_a^{(+)}$ is the $a-A$ distorted wave and φ_{xb} is the intrinsic xb wave function.

More specifically, 1) and 2) write

$$\frac{d^2\sigma^{IB}}{d\Omega_b \, dE_b} = \rho(E_b) \frac{2}{\hbar v_a} \langle \hat{\Psi}_x^{(+)} | W_{xA} | \hat{\Psi}_x^{(+)} \rangle \qquad (1)$$

$$|\hat{\Psi}_x^{(+)}\rangle_{Aust.} = (\chi_b^{(-)} | \Psi^{(+)} \rangle \;, \; |\hat{\Psi}_x^{(+)}\rangle_{HM} = (\chi_b^{(-)} | \chi_a^{(+)} \varphi_{xb} \rangle \qquad (2)$$

Udagawa, Tamura and collaborators, have argued on several ocasions that Eq.(1) contains unphysical non-break up a-A components which would render it too large when compared with the IB data. Further, these authors[5] propose that a component of $\frac{d^2\sigma^{IB}}{d\Omega_b\, dE_b}$, which they call elastic break-up fusion (E.B.F.) cross section, is given by an expression similar to Eq.(1), with $|\hat{\Psi}_x^{(+)}\rangle$ given by

$$|\hat{\Psi}_x^{(+)}\rangle_{UT} = G_x^{(+)} (\chi_b^{(-)}|(U_{xA} + U_{bA} - U_{aA})\,|\chi_a^{(+)}\varphi_{xb}\rangle \quad (3)$$

where $G_x^{(+)}$ describes the optical propagation of x in the presence of the target <u>and</u> the spectator b, U_{xA} and U_{bA} are (optical) x and b interactions and U_{aA} the optical potential of the aA system. By the definition of 5), their cross section is less inclusive than Eq.(1).

Another important question which has been discussed extensively in this debate is that, apparently, $\frac{d^2\sigma^{IB}}{d\Omega_b\, dE_b}$, Eq.(1), when calculated with $|\hat{\Psi}_x^{(+)}\rangle$ replaced by its DWBA version should yield a cross section composed of three terms: the HM term, ${}_{HM}\langle \hat{\Psi}_x^{(+)} | W_{xA} | \hat{\Psi}_x^{(+)}\rangle_{HM}$ the UT term, obtained from Eq.(3) and the mixed HM-UT term $2\,\mathrm{Re}\,{}_{HM}\langle \hat{\Psi}_x^{(+)} | W_{xA} | \hat{\Psi}_x^{(+)}\rangle_{UT}$. This result was first derived by Ishimura, Austern and Vincent (IAV)[6]. It should be contrasted with that obtained by merely replacing the exact 3-body wave function of the Austern et.al.[1] relation by its DWBA version which would immediately yield the HM cross-section.

The purpose of this paper is to elucidate these matters through a thorough consideration of the intrinsically three-body character of these reactions. Although Austern et.al.[1] have addressed these questions within a three-body framework, we believe that we have been able to go further and clearly identify what we may call the first-order "two-body" IB cross section calculated with distorted waves, with the HM expression, and associate the UT cross section, with one piece of a heirarchy of three-body corrections. In this respect it is not proper to call the UT cross section, the EBF piece of $\frac{d^2\sigma^{IB}}{d\Omega_b\, dE_b}$, since it is as inclusive as the HM cross section contrary to what has been believed up to now. Clearly, the Austern et.al. cross section[1], Eq.(1) does contain all of the above terms.

We start our discussion with the spectator expression for given by Austern et.al.[1]

$$\frac{d^2\sigma^{IB}}{d\Omega_b\, dE_b} = -\frac{2\rho(E_b)}{\hbar v_a}\int d^3r_x\, W_{xA}(\vec{r}_x)\,|\,(\chi_b^{(-)}(\vec{r}_b)|\Psi^{(+)}(\vec{r}_x,\vec{r}_b)\rangle|^2 \quad (4)$$

$$\equiv -\frac{2\rho(E_b)}{\hbar v_a}\langle \Psi^{(+)}|\chi_b^{(-)})\,W_{xA}\,(\chi_b^{(-)}|\Psi^{(+)}\rangle \quad (5)$$

Where, again, $\chi_b^{(-)}$ is the distorted wave describing the spectator particle and $\Psi^{(+)}$ is the exact b-x-A three-body wave function. W_{xA} is the xA imaginary interaction. It is interesting to remark that we have been able to derive the Austern et.al. cross-section starting with the exact three-body break up amplitude using the coupled channel Faddeev equations. This enabled us to use closure to perform the sum over final break-up channels which are here described by a manifestly complete set of momentum and target intrinsic states. This, jontly with the development of Ref. 1 should put an end to the question of unphysical non-break up component in Eq.(5).

We start now decomposing Eq.(5) into a DWBA cross section plus three-body corrections. We use the approximation that $m_A \gg m_x$, $m_A \gg m_b$. We first write

$$\Psi^{(+)} = G_{x,b} V_{xb} \Psi^{(+)} \tag{6}$$

$$\equiv G_{x,b} V_{xb} \left(\Psi_{xb}^{(+)} + \Psi_{xA}^{(+)} + \Psi_{bA}^{(+)} \right) \tag{7}$$

where

$$G_{x,b}^{(+)} = (E - H_{xA} - H_{bA} + i\varepsilon)^{-1} \tag{8}$$

$$= G_0 + G_{x,b} (U_{xA} + U_{bA}) G_0 \tag{9}$$

describes the x, b exact propagation with V_{xb} turned off the H's are the subsystems Hamiltonians and the $\mathcal{U}$'s the corresponding optical interactions.

With (9) we can rewrite Eq.(6) as

$$\Psi^{(+)} = \Psi_{xb}^{(+)} + G_{x,b} (\mathcal{U}_{xA} + \mathcal{U}_{bA}) \Psi_{xb}^{(+)} \tag{10}$$

where $\Psi_{xb}^{(+)} = G_0 V_{xb} \Psi^{(+)}$. It is a simple matter to show through formal manipulation that the xb bound state projected wave function $\langle \varphi_{xb} | \Psi_{xb}^{(+)} \rangle$ satisfies the following equation

$$\left(H_{0,\alpha} + \mathcal{V}_\alpha^B + \mathcal{V}_{pol.}^B \right) \langle \varphi_{xb} | \Psi_{xb}^{(+)} \rangle \tag{11}$$

$$= (E + |E_{xb}|) \langle \varphi_{xb} | \Psi_{xb}^{(+)} \rangle$$

where

$$\mathcal{V}_a^{B} = \langle \varphi_{xb} | V_{xb} G_{x,b} (\mathcal{U}_{xA} + \mathcal{U}_{bA}) | \varphi_{xb} \rangle \qquad (12)$$

$$\equiv \langle \varphi_{xb} | V_3 | \varphi_{xb} \rangle$$

$$\mathcal{V}_{pol.}^{B} = \langle \varphi_{xb} | V_3 | Q \rangle \left[E - E_Q - H_{0,a} - \langle Q | V_3 | Q \rangle + i\varepsilon \right]^{-1} \qquad (13)$$

$$\cdot \langle Q | V_3 | \varphi_{xb} \rangle$$

In (13), $|Q\rangle$ is defined such that $|\varphi_{xb}\rangle\langle\varphi_{xb}| + |Q\rangle\langle Q| = 1$ and represents a continuum state of the xb system. $\mathcal{V}_a^{B}$ is the aA interaction calculated within the three-body model and $\mathcal{V}_{pol}^{B}$ represents the polarization interaction arising from the coupling between the bound state and the continuum xb state Q. A similar equation to Eq. (11) can be obtained for the break-up component $\langle Q | \Psi_{xb}^{(+)} \rangle$ whose solution follows immediatly

$$\langle Q | \Psi_{xb}^{(+)} \rangle = \left(E^{(+)} - E_Q - H_{0,a} - \langle Q | V_3 | Q \rangle + i\varepsilon \right)^{-1} \cdot \qquad (14)$$

$$\cdot \langle Q | V_3 | \varphi_{xb} \rangle \langle \varphi_{xb} | \Psi_{xb}^{(+)} \rangle$$

We introduce now the optical potential U_{aA} of the aA system and after adding and subtracting it in Eq.(11) we obtain the following practical representation for $\langle \varphi_{xb} | \Psi_{xb}^{(+)} \rangle$

$$\langle \varphi_{xb} | \Psi_{xb}^{(+)} \rangle = \chi_a^{(+)} + G_a^{(+)}(E - E_{xb}) \left[\mathcal{V}_a^{B} + \mathcal{V}_{pol.}^{B} - \mathcal{U}_{aA} \right] \cdot \qquad (15)$$

$$\cdot \langle \varphi_{xb} | \Psi_{xb}^{(+)} \rangle$$

The difference, $\mathcal{V}_a^{B} + \mathcal{V}_{pol.}^{B} - \mathcal{U}_{aA} \equiv \mathcal{V}_a^{res.}$, is a residual three-body interaction which can be rendered very small if $\mathcal{U}_{aA}$ is chosen to fit very well the elastic aA scattering data. Eqs.(10) and (14) are the basic three-body equations which we use below to generate the three-body corrections to the DWBA inclusive break-up cross section . Thus with the reasonable approximation $\langle \varphi_{xb} | \Psi_{xb}^{(+)} \rangle \simeq |\chi_a^{(+)}\rangle$ we obtain the following practical DWBA + lowest order three-body correction representation for

$$|\Psi_{xb}^{(+)}\rangle = |\varphi_{xb}\rangle |\chi_a^{(+)}\rangle + |Q\rangle \left(E - E_Q - H_{0,a} - \langle Q | V_3 | Q \rangle + i\varepsilon \right)^{-1} \cdot$$

$$\cdot \langle Q | V_3 | \varphi_{xb} \rangle \; |\chi_a^{(+)}\rangle \qquad (16)$$

$$= |\varphi_{xb}\rangle |\chi_a^{(+)}\rangle + |Q\rangle G_a^{Q} \; V_a^{QB} \; |\varphi_{xb}\rangle |\chi_a^{(+)}\rangle$$

In what follows we shall use Eq. (16) to derive a new inclusive break-up sum rule.

Using Eq.(10) in the matrix element

$$\langle \Psi^{(+)} | \chi_b^{(-)}) \, W_{xA} \, (\chi_b^{(-)} | \Psi^{(+)} \rangle$$

we obtain the following new exact sum rule.

$$\langle \Psi^{(+)} | \chi_b^{(-)}) \, W_{xA} \, (\chi_b^{(-)} | \Psi^{(+)} \rangle = \langle \Psi_{xb}^{(+)} | \chi_b^{(-)}) \, W_{xA} \, (\chi_b^{(-)} | \Psi_{xb}^{(+)} \rangle +$$

$$+ \langle \Psi_{xb}^{(+)} | (\mathcal{U}_{xA}^{\dagger} + \mathcal{U}_{bA}^{\dagger}) | \chi_b^{(-)}) \, G_x^{(+)\dagger}(E_x) \, W_{xA} \, G_x^{(+)}(E_x) \, (\chi_b^{(-)} | (\mathcal{U}_{xA} + \mathcal{U}_{bA}) | \Psi_{xb}^{(+)} \rangle$$

$$+ 2 \, \mathrm{Re} \, \langle \Psi_{xb}^{(+)} | \chi_b^{(-)}) \, W_x \, G_x^{(+)}(E_x) \, (\chi_b^{(-)} | (\mathcal{U}_{xA} + \mathcal{U}_{bA}) | \Psi_{xb}^{(+)} \rangle \qquad (17)$$

The above is very similar to the post form version of the IAV sum rule[6], except that the wave function $\Psi_{xb}^{(+)}$ is the exact Faddeev wave function and the second term on the RHS, which is commonly referred to as the elastic break-up fusion (BF) piece, does not contain the optical potential U_{aA} in the matrix element.

With the help of Eq.(16), we can now write the desired representation for the inclusive cross section

$$\frac{d^2\sigma^{IB}}{d\Omega_b dE_b} = \frac{-2}{\hbar v_a}\rho(E_b)\langle \chi_a^{(+)}\varphi_{xb}|\chi_b^{(-)})W_{xA}(\chi_b^{(-)}|\chi_a^{(+)}\varphi_{xb}\rangle + \\ + \;TBC \tag{18}$$

where the three body correction term TBC is to lowest order given by

$$\begin{aligned}TBC = \frac{-2}{\hbar v_a}\rho(E_b)\Big[& 2Re\langle \chi_a^{(+)}\varphi_{xb}|\chi_b^{(-)})W_{xA}(\chi_b^{(-)}|G_a^{(+)}v_a^{res.}|\chi_a^{(+)}\varphi_{xb}\rangle \\ & +\langle \chi_a^{(+)}\varphi_{xb}|v_a^{res.\dagger}G_a^{(+)\dagger}|\chi_b^{(-)})W_{xA}(\chi_b^{(-)}|G_a^{(+)}v_a^{res.}|\chi_a^{(+)}\varphi_{xb}\rangle \\ & +2Re\langle \chi_a^{(+)}\varphi_{xb}|\chi_b^{(-)})W_{xA}(\chi_b^{(-)}|Q\rangle G_a^{Q}v_a^{QB}|\chi_a^{(+)}\varphi_{xb}\rangle \\ & +\langle \chi_a^{(+)}\varphi_{xb}|v_a^{BQ\dagger}G_a^{Q\dagger}\langle Q|\chi_b^{(-)}\rangle W_{xA}(\chi_b^{(-)}|Q\rangle G_a^{Q}v_a^{QB}|\chi_a^{(+)}\varphi_{xb}\rangle \\ & +2Re\langle \chi_a^{(+)}\varphi_{xb}|\chi_b^{(-)})W_{xA}G_x^{(+)}(\chi_b^{(-)}|(U_{xA}+U_{bA})|\chi_a^{(+)}\varphi_{xb}\rangle \\ & +\langle \chi_a^{(+)}\varphi_{xb}|(U_{xA}^{\dagger}+U_{bA}^{\dagger})|\chi_b^{(-)})G_x^{(+)\dagger}W_{xA}G_x^{(+)}(\chi_b^{(-)}|(U_{xA}+U_{bA})\cdot \\ & \cdot|\chi_a^{(+)}\varphi_{xb}\rangle\end{aligned} \tag{19}$$

$$\begin{aligned}\approx \frac{-2}{\hbar v_a}\rho(E_b)\;\cdot & \\ \Big[2Re & \langle \chi_a^{(+)}\varphi_{xb}|\chi_b^{(-)})W_{xA}(\chi_b^{(-)}|Q\rangle G_a^{Q}v_a^{QB}|\chi_a^{(+)}\varphi_{xb}\rangle \\ & +2Re\langle \chi_a^{(+)}\varphi_{xb}|\chi_b^{(-)})W_{xA}G_x^{(+)}(\chi_b^{(-)}|(U_{xA}+U_{bA})|\chi_a^{(+)}\varphi_{xb}\rangle \\ & +\langle \chi_a^{(+)}\varphi_{xb}|(U_{xA}^{\dagger}+U_{bA}^{\dagger})|\chi_b^{(-)})G_x^{(+)\dagger}W_{xA}G_x^{(+)}\cdot \\ & \cdot(\chi_b^{(-)}|(U_{xA}+U_{bA})|\chi_a^{(+)}\varphi_{xb}\rangle\end{aligned} \tag{20}$$

where the first two and the fourth, terms in 19 have been dropped compared to the IAV-like last two terms[7].

Eq. (20) for the TBC exhibits two important features. First, the last two terms contain the matrix element $(\chi_b^{(-)}|(U_{xA}+U_{bA})|\chi_a^{(+)}\varphi_{xb}\rangle$ instead of $(\chi_b^{(-)}|(U_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\varphi_{xb}\rangle$ of the IAV sum rule and the UT expression. The second feature's is that the first term is a new correction, which may be as large as the last term (the UT - like term).

The original prior-form IAV sum rule is obtained by dropping the last two Faddeev components, $\Psi_{xA}^{(+)}$ and $\Psi_{bA}^{(+)}$ in Eq.(7) and writing for the wave function $\Psi^{(+)}$ the following approximate form

$$\Psi^{(+)} \simeq G_{x,b}V_{xb}\Psi_{xb}^{(+)} \approx G_{x,b}V_{xb}\chi_a^{(+)}\varphi_{xb} \tag{21}$$

Three-body effects are still present in the exact optical propagator.

With Eq.(21) for $\Psi^{(+)}$ one immediately recovers the original IAV sum rule. In details, we have

$$\langle \Psi^{(+)} | \chi_b^{(-)}) W_{xA} (\chi_b^{(-)} | \Psi^{(+)} \rangle \simeq$$

$$\langle \chi_a^{(+)} \varphi_{xb} | V_{xb}^{\dagger} G_{x,b}^{(+)\dagger} | \chi_b^{(-)}) W_{xA} (\chi_b^{(-)} | G_{x,b}^{(+)} V_{xb} | \chi_a^{(+)} \varphi_{xb} \rangle \qquad (22)$$

since

$$\left(E - H_x^{o} - H_b^{o} - V_{xb} - \mathcal{U}_{aA}\right) \chi_a^{(+)} \varphi_{xb} = 0 \qquad (23)$$

and

$$\left(E_b - H_b^{o} - \mathcal{U}_{bA}^{\dagger}\right) \chi_b^{(-)} = 0 \qquad (24)$$

we have

$$V_{xb} \chi_a^{(+)} \varphi_{xb} = \left(E - H_x^{o} - H_b^{o} - \mathcal{U}_{aA}\right) \chi_a^{(+)} \varphi_{xb} \qquad (25)$$

thus

$$\begin{aligned}
\hat{\Psi}_x^{(+)\,IAV} &\equiv (\chi_b^{(-)} | G_{x,b} V_{xb} | \chi_a^{(+)} \varphi_{xb} \rangle \\
&= G_x^{(+)}(E_x)\, (\chi_b^{(-)} | (E - H_x^{o} - H_b^{o} - \mathcal{U}_{aA}) | \chi_a^{(+)} \varphi_{xb} \rangle \\
&= G_x^{(+)}(E_x)\, (\chi_b^{(-)} | (E_x - H_x^{o} + \mathcal{U}_{bA} - \mathcal{U}_{aA}) | \chi_a^{(+)} \varphi_{xb} \rangle \\
&= G_x^{(+)}(E_x)\, (\chi_b^{(-)} | (E_x - H_x^{o} - \mathcal{U}_{xA}) | \chi_a^{(+)} \varphi_{xb} \rangle \\
&\quad + G_x^{(+)}(E_x)\, (\chi_b^{(-)} | (\mathcal{U}_{xA} + \mathcal{U}_{bA} - \mathcal{U}_{aA}) | \chi_a^{(+)} \varphi_{xb} \rangle
\end{aligned} \qquad (26)$$

thus

$$\begin{aligned}
\hat{\Psi}_x^{(+)\,IAV} &= (\chi_b^{(-)} | \chi_a^{(+)} \varphi_{xb} \rangle + G_x^{(+)}(E_x)\, (\chi_b^{(-)} | (\mathcal{U}_{xA} + \mathcal{U}_{bA} - \mathcal{U}_{aA}) \cdot \\
&\qquad \cdot | \chi_a^{(+)} \varphi_{xb} \rangle \\
&= \hat{\Psi}_x^{(+)\,HM} + \hat{\Psi}_x^{(+)\,UT}
\end{aligned} \qquad (27)$$

which when inserted in the Austern et.al. matrix element yields immediately the IAV sum rule. Since $\hat{\Psi}_x^{HM}$ represents a limit where the three-body wave function is replaced by a product of two-body wave functions, it becomes clear that the UT cross section represents but a usually small three-body correction. Of course our new sum rule, Eq.(18) contains more three-body effects since all three Faddeev components have been taken into account at the DWBA level.

In order to gain physical insight into the full three-body correction, exemplified by Eq.(20), we approximate $\Psi_{xb}^{(+)}$ in Eq.(10) by a product of two-body wave functions in the x and b coordinates, respectively, which resembles very much the Glauber approximation employed by HM[2], namely

$$\Psi^{(+)}_{xb} \simeq \chi^{(+)}_{a}\varphi_{xb} \simeq \chi^{(+)}_{x_i}\chi^{(+)}_{b_i}\varphi_{xb} \tag{28}$$

thus

$$\Psi^{(+)} \simeq \left[\chi^{(+)}_{x_i}\chi^{(+)}_{b_i} + G_{x,b}(\mathcal{U}_{xA}+\mathcal{U}_{bA})\chi^{(+)}_{x_i}\chi^{(+)}_{b_i}\right]\varphi_{xb} \tag{29}$$

The second term on the RHS contains two parts , one associated with the b-particle, and the other with the x-particle. The latter one, namely

$$G_{x,b}\,\mathcal{U}_{xA}\,\chi^{(+)}_{x_i}\chi^{(+)}_{b_i}\varphi_{xb}$$

can be rewritten as

$$G_{x,b}\,\mathcal{U}_{xA}\,\chi^{(+)}_{x_i}\chi^{(+)}_{b_i}\varphi_{xb} = \chi^{(+)}_{b_i}\,G^{(+)}_{x}(E_x)\,\mathcal{U}_{xA}\,\chi^{(+)}_{x_i}\varphi_{xb}\,. \tag{30}$$

This piece of the wave function corresponds to the production of X. Since this particle is strongly absorbed by the target, one can safely ignore this component compared to the dominant b-component which may be written as

$$G_{x,b}\,\mathcal{U}_{bA}\,\chi^{(+)}_{x_i}\chi^{(+)}_{b_i}\varphi_{xb} \simeq \chi^{(+)}_{x_i}\,G^{(+)}_{b}(E_{b_i})\,\mathcal{U}_{bA}\,\chi^{(+)}_{b_i}\varphi_{xb} \tag{31}$$

Inserting now the above form for the three-body piece of $\Psi^{(+)}$ into $\hat{\Psi}^{(+)}_{x}$, we obtain

$$\hat{\Psi}^{(+)}_{x} \simeq \chi^{(+)}_{x_i}\left[\langle\chi^{(-)}_{b}|\chi^{(+)}_{b_i}\rangle \quad + (E_{b_i}-E_b+i\varepsilon)^{-1} \cdot \langle\chi^{(-)}_{b}|\,\mathcal{U}_{bA}\,|\chi^{(+)}_{b_i}\rangle\right]\varphi_{xb} \tag{32}$$

The presence of φ_{xb} guarantees that the result above is not singular, since it represents an average over the Fermi-motion of b.

Thus we may safely replace the above by

$$\hat{\Psi}_x^{(+)} \simeq \chi_{x_i}^{(+)} \Big[\langle \chi_b^{(-)} | \chi_{b_i}^{(+)} \rangle \varphi_{xb} + \\ + \left(E_{b_i} - E_b + i\Gamma_F/2\right)^{-1} \overline{\langle \chi_b^{(-)} | \mathcal{U}_{bA} | \chi_{b_i}^{(+)} \rangle} \Big] \quad (33)$$

where the bar denotes Fermi average. It is clear from the above that the second term is a genuine three-body correction since the matrix element is off shell. This three-body correction, which , as the UT correction, is usually small, and peaks at the incident b energy, as does the UT correction. The width of this peaking, Γ_F is entirely related to the Fermi-motion of b.

The far more dominant HM contribution, exemplified by the first term in Eq.(16), peaks at an energy lower than the beam energy and has a width much larger than Γ_F , as the recent calculation by Hussein and Mastroléo[8] clearly demonstrated. To finalize , we write below the full form of the cross section in this "Glauber" limit[9]

$$\frac{d^2\sigma^{IB}}{d\Omega_b dE_b} \simeq \rho(E_b) \sum_{\ell_x} P(\ell_x, q)\, \sigma_R^{xA}(\ell_x)$$

$$P(\ell_x, q) = P_{HM}(\ell_x, q) + P_{3b}(\ell_x, q) + P_{C.T.}(\ell_x, q)$$

$$P_{HM}(\ell_x, q) = \frac{1}{2\pi}\int_0^{2\pi} d\phi \left| \int d^3r_b\, e^{i\vec{q}\cdot\vec{r}_b} S_{bA}(b_b)\, \varphi_{xb}(\vec{r}_b - \vec{r}_x) \right|^2$$

$$P_{3b}(\ell_x, q) = \frac{1}{(E_{b_i} - E_b)^2 + \frac{\Gamma_F^2}{4}} \cdot \frac{1}{2\pi}\int_0^{2\pi} d\phi \left| \int d^3r_b\, e^{i\vec{q}\cdot\vec{r}_b} \cdot \right. \\ \left. \cdot \overline{\langle \chi_b^{(-)} | \mathcal{U}_{bA} | \chi_{b_i}^{(+)} \rangle}\, \varphi_{xb}(\vec{r}_b - \vec{r}_x) \right|^2 \quad (34)$$

and $P_{C.T.}(\ell_x, q)$ is the mixed break-up probability which also peaks at $E_b = E_{bi}$.

In conclusion, we have in this paper derived and discussed the lowest-order three-body corrections to the inclusive inelastic break-up cross section in the DWBA limit. We have identified one piece of these 3-body terms with what Udagawa and Tamura call elastic break-up-fusion cross section. A fuller account of these corrections leads to a new sum rule which resembles very much the Ichimura, Austern and Vincent cross section. The numerical evaluation of this sum rule should be easily performed with the existing UT break up code. A "Glauber"-type analysis was performed on the sum rule which exhibited clearly its general features and clarified the physical content of the 3-body corrections .

ACKNOWLEDGEMENTS

We are grateful to Professor N. Austern for a very careful reading of the manuscript.

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7) It is easy to show, within the Glauber estimate to be presented later, that the leading correction containing $\mathcal{V}_a^{res.}$ (with $G_a^{(+)}(E+|E_{xb}|)$ replaced by its off-shell part).

$$-i\pi \langle \varphi_{xb} | \left[\frac{|S_b|^2}{S_a} \langle \chi_x^{(+)} | W_{xA} | \chi_x^{(+)} \rangle \langle \chi_b^{(-)} \chi_x^{(-)} | \mathcal{V}_a^{res.} | \chi_b^{(+)} \chi_b^{(+)} \rangle \right] | \varphi_{xb} \rangle$$

8) M. S. Hussein and R. C. Mastroléo, Nucl. Phys. A, in press.

9) The first three-body correction term of Eq. 20 will not be considered here for simplicity.