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LOCAL BOSONIC SYMMETRIES IN THE STRING'S ACTION

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ABSTRACT

The string action enjoys Sierra's bosonic local symmetry on which Siegel's κ–symmetry closes. This bosonic symmetry is analyzed in Dirac's Hamiltonian formalism and its generators are identified.

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In an interesting paper, Sierra observed[1] that the actions for a massless relativistic particle and for the superparticle[2] enjoy invariance under local transformations, which include the usual reparametrization invariance. The local Siegel symmetry closes on these bosonic symmetries[3]. In this letter, following arguments similar to Sierra's[1], and using Dirac's formalism for constrained Hamiltonian systems[4, 5, 6], we show that the bosonic string enjoys a similar local symmetry and find its generators.

The bosonic string is described by the constraints

$$\begin{aligned} \mathcal{H}_{\perp} &= \frac{1}{2}\left(P^2 + (\partial_\sigma X)^2\right) \approx 0 \\ \mathcal{H}_1 &= P \cdot \partial_\sigma X \approx 0 \end{aligned} \tag{1}$$

where $X^\mu(\tau,\sigma)$ is the position of the string in space–time, and $P^\mu(\tau,\sigma)$ is its conjugate momentum, with equal time Poisson bracket

$$[P^\mu(\sigma), X^\nu(\sigma\prime)] = \eta^{\mu\nu}\delta(\sigma - \sigma\prime). \tag{2}$$

Instead of (1), we find more convenient the linear combinations

$$\mathcal{H}_{\pm} = \frac{1}{2}(\mathcal{H}_{\perp} \pm \mathcal{H}_1) = \frac{1}{4}Q_{\pm}^2, \tag{3}$$

with

$$Q_{\pm}^{\mu} = P^\mu \pm \partial_\sigma X^\mu. \tag{4}$$

The string action

$$S = \int d\tau d\sigma \left[P^\mu \dot{X}^\mu - \lambda_+\mathcal{H}_- - \lambda_-\mathcal{H}_-\right] \tag{5}$$

enjoys the following local gauge invariance:

$$\begin{aligned} \delta X^\mu &= \frac{1}{2}\left(\epsilon_+ Q_+^\mu + \epsilon_- Q_-^\mu\right) \\ \delta P^\mu &= \frac{1}{2}\partial_\sigma\left(\epsilon_+ Q_+^\mu - \epsilon_- Q_-^\mu\right) \\ \delta\lambda_+ &= \dot{\epsilon}_+ + (\partial_\sigma\lambda_+)\epsilon_+ - \lambda_+\partial_\sigma\epsilon_+ \\ \delta\lambda_- &= \dot{\epsilon}_- - (\partial_\sigma\lambda_-)\epsilon_- - \lambda_-\partial_\sigma\epsilon_- \end{aligned} \tag{6}$$

This is the natural generalization of Sierra's gauge invariance for the particle. The variation of the action (5) under (6) is

$$\begin{aligned} \delta S &= \frac{1}{4}\int_{\sigma_1}^{\sigma_2} d\sigma(\epsilon_+ + \epsilon_-)(P^2 - \partial_\sigma X^2)\Big|_{\tau_1}^{\tau_2} \\ &+ \frac{1}{2}\int_{\tau_1}^{\tau_2} d\tau \int_{\sigma_1}^{\sigma_2} d\sigma\partial_\sigma \left\{\epsilon_+(Q_+^\mu \dot{X}_\mu - 2\lambda_+\mathcal{H}_+) - \epsilon_-(Q_-^\mu \dot{X}_\mu - 2\lambda_-\mathcal{H}_-)\right\}, \end{aligned} \tag{7}$$

which vanishes only if $\epsilon_\pm = 0$ at the end points. Hence, the symmetry (6) does not admit the interpretation of $P^2 - (\partial_\sigma X)^2$ as a symmetry generator (as in Ref. 1 for the particle). This was to be expected, because the string is a general–covariant system and the gauge symmetry is not internal[7, 8, 9]. The interpretation of Ref. 1 is possible, nevertheless, if we require the action to be invariant under the transformations

$$\begin{aligned} \delta X^\mu &= \frac{1}{2}\left(\beta_+^\mu Q_+^2 + \beta_-^\mu Q_-^2\right) \\ \delta P^\mu &= \frac{1}{2}\,\partial_\sigma\left(\beta_+^\mu Q_+^2 - \beta_-^\mu Q_-^2\right), \end{aligned} \tag{8}$$

with $\delta\lambda_\pm$ to be determined. One finds that δS vanishes if and only if

$$\begin{aligned} \delta\lambda_+ &= 2\beta_+^\mu \partial_\sigma(\lambda_+ Q_{+\mu}) - \beta_+^\mu \dot{Q}_{+\mu} \\ \delta\lambda_- &= -2\beta_-^\mu \partial_\sigma(\lambda_- Q_{-\mu}) - \beta_-^\mu \dot{Q}_{-\mu} \end{aligned} \tag{9}$$

with periodic boundary conditions for $\lambda_\pm$. Identifying the total derivative term as

$$\delta S = \int d\tau d\sigma \partial_\tau \partial_\sigma \{\beta \cdot (\text{generator or constraint})\},$$

the generators of the β–symmetry for the string are

$$F_\pm^\mu = P^\mu Q_\pm^2 \approx 0. \tag{10}$$

Could these considerations be generalized to the case of higher–dimensional extended objects, such as membranes? The answer is not clear, since in general one does not know how to write the constraints as a perfect square Q^2. Work along this lines is in progress[10].

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