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PERIODIC BEHAVIOUR IN A CLASS OF PERIODICALLY FORCED DISSIPATIVE INTEGRABLE SYSTEMS

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PERIODIC BEHAVIOUR IN A CLASS OF PERIODICALLY FORCED DISSIPATIVE INTEGRABLE SYSTEMS+

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ABSTRACTS: We consider bidimensional nonautonomous dissipative systems subjected to delta peaks periodic forcing, such that the unperturbed systems are integrable. We study numerically the related maps and analyse the existence and stability of their fixed points.

In the study of non–linear oscillators there are some systems which show equilibrium solutions, even when perturbed by periodic forces. As an example[1], the harmonic damped oscillator settles down in a fixed point (in phase space) after some time. If a periodically applied force acts upon this system, a new equilibrium solution arises, namely a limit cycle. This kind of periodic behaviour can evolve to a chaotic regime in a variety of ways, as control parameters are changed.

Limit–cycle systems are not very common, and its study can help us to understand many physical systems of interest. In a recent paper[2], two of us proposed the following bidimensional model with delta–peaked forcing:

$$\frac{d\underset{\sim}{Y}}{dt} = \underset{\sim}{M}[\underset{\sim}{Y}]\,\underset{\sim}{Y} + \sum_{m=1}^{\infty} c\,\delta(t-mT)\,\underset{\sim}{N}[\underset{\sim}{Y}]\,\underset{\sim}{Y} \qquad (1)$$

where $\underset{\sim}{Y}$ stands for $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and $\underset{\sim}{M}$, $\underset{\sim}{N}$ are nonlinear matrices, the former being chosen so that the unperturbed system is integrable between the pulses.

In order to investigate the dynamical behaviour of this model we use an analytical Poincaré (stroboscopic) map. This is feasible when the forcing is modelled by simple functions, like in eq. (1), where the pulses are represented by a sequence of delta functions with period T.

We construct this map by using:

$$\underset{\sim}{Y}_m = \lim_{\epsilon\to 0} \{ \underset{\sim}{Y}(t=mt-\epsilon) + \frac{c}{2}\underset{\sim}{N}[\underset{\sim}{Y}(t=mt-\epsilon)]\,\underset{\sim}{Y}(t=mt-\epsilon) + $$
$$+ \frac{c}{2}\underset{\sim}{N}[\underset{\sim}{Y}(t=mT+\epsilon)]\,\underset{\sim}{Y}(t=mT+\epsilon)\} \qquad (2)$$

In our case, namely when there are distributions in the coefficients of the related differential equation, the solutions themselves depend at a certain extent on the nature of the jumps involved (Kurzweil's theorem)[3]. This fact allows us to redefine the map assuming different conditions for the approach to the solution. We have pursued this task and results will be published elsewhere.

There is another method to know the behaviour of the systems such as given by eq.(1) that does not make use of a recurrent map but instead of the so called "Isochrone portrait"[4] that has been used in some nonlinear oscillators driven by a sequence of impulsive forces.

Using polar coordinates (such that $y_1 = r\cos\theta$ and $y_2 = r\sin\theta$) to uncouple equations we study the following case:

$$[\underset{\sim}{M}] = \begin{bmatrix} f(r)/r & 1 \\ -1 & f(r)/r \end{bmatrix} \qquad [\underset{\sim}{N}] = \begin{bmatrix} g(r)/r & 0 \\ 0 & g(r)/r \end{bmatrix} \tag{3}$$

A first choice to be considered is:

$$f(r) = a^2 - r^2 \qquad \text{and} \qquad g(r) = \alpha r + \beta \tag{4}$$

In this case, eq. (1) without forcing has a stable limit cycle at r=a. When c≠0 one obtains from eq. (2) the map:

$$r_{n+1} = \xi^{-1}\,\frac{r_n + a\rho}{r_n\rho + a} + \eta \tag{5}$$

$$\theta_{n+1} = \theta_n - T \ (\mathrm{mod.}\ 2\pi)$$

where

$$\rho = \tanh(aT) \tag{6}$$

$$\xi = \frac{1}{a}\left[\frac{2 - c\alpha}{2 + c\alpha}\right] \tag{7}$$

$$\eta = \frac{2\,c\,\beta}{2 - c\alpha} \tag{8}$$

We restrict ourselves to the range $0 \le c\alpha < 4$ for the perturbation strength such that the map (5) is well–defined[5]. Fig. 1 shows a phase portrait of this map for a given initial condition. In this case the points of the limit cycle form a circle of radius:

$$r^* = \frac{\eta}{2} + \frac{1 - \xi a}{2\xi\rho} + \frac{\Lambda^{1/2}}{2\xi\,(1 - e^{-2aT})} \tag{9}$$

where

$$\Lambda = (1-\xi a)^2\,(1 + e^{-2aT})^2 + (\xi^2\,\eta^2 + 4\xi a)\,(1 - e^{-2aT})^2 + [2\xi\eta\,(1-\xi a) + 4a\xi^2\eta]\,(1 - e^{-4aT}) \tag{10}$$

This stable fixed point depends monotonically on the strength as well as the period of forcing (see fig. 2). As $c\alpha$ approaches 4.0, the radius grows to $+\infty$. This is also the case when the period between the pulses is very short. The large period limit is a constant value.

Other interesting class of limit–cycle system occurs when

$$f(r) = (a - r)(b - r) \qquad (a > b > 0) \text{ and}$$

$$g(r) = \alpha r + \beta \tag{11}$$

The unperturbed case has two limit cycles: a stable and an unstable one, located at r=b and r=a, respectively. Considering the kicks we need to analyse the map for radial coordinate:

$$r_{n+1} = \xi^{-1} \frac{r_n \left[\frac{b - a\lambda}{a(1-\lambda)}\right] - b}{r_n - b\left[\frac{a - b\lambda}{b(1-\lambda)}\right]} + \eta \tag{12}$$

where

$\lambda = \exp[T(b-a)]$ and ξ, η are defined before.

Within the intervals $T \geq 0.02$ and $-0.26 < c < 0$, eq. (12) is well–defined and furnishes two limit cycles (one of them at least is stable) whose radii are given by:

$$r^* = \frac{-a\lambda[\xi(b+\eta)+1] + \xi a(a+\eta) + b \pm \Omega^{1/2}}{2a\,\xi\,(1-\lambda)} \tag{13}$$

with:

$$\Omega = \{a\lambda[\xi(b+\eta)+1] - \xi a(a+\eta) - b\}^2 - 4a^2\xi(1-\lambda)[\xi a\eta + b - b\lambda(\xi\eta+1)] \tag{14}$$

and showing a similar (and still monotonic) variation with both c and T (see fig. 3).

As far as our investigation was concerned we did not find any kind of chaotic behaviour. This is partially explained when one considers the maps (5) and (12). They show no quadratic or higher power maxima in order to generate period m–tupling cascades[6] and neither they are likely to generate intermittent behaviour.

Nevertheless we have proved that, for the systems considered in this note, limit–cycle solutions can survive even under a periodic forcing (except, of course, when there are no longer attractors, for $c \to \infty$ and $T \to 0$).

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FIGURE CAPTIONS

Fig. 1: Phase portrait of map (5) with $a = \beta = 1$, $\alpha = 0.5$, $c = 0.01$, $T = 0.001$ and $(r_0, \theta_0) = (0.1;0)$.

Fig. 2: Radius of limit cycles for map (5) with $a = \beta = 1$, $\alpha = 0.5$.
a) As a function of c with $T = 0.5$; b) As a function of T with $c=1$.

Fig. 3: Radii of limit cycles for map (12) with $a = \beta = 1$, $\alpha = b = 0.5$. The solid and dashed lines represent stable and unstable attractors, respectively. a) As a function of c with $T = 1$; b) As a function of T with $c = -0.01$.

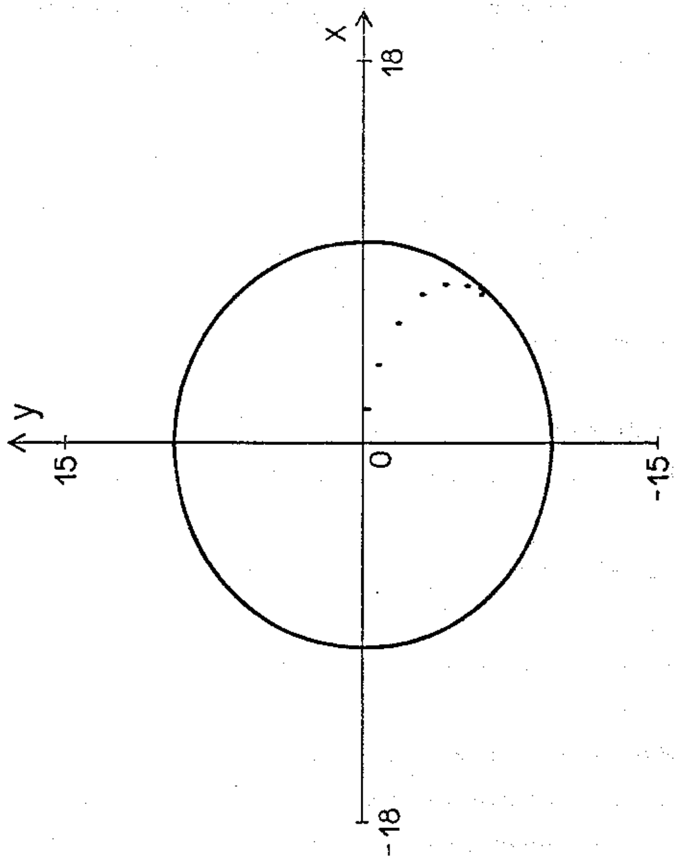

FIG. 1

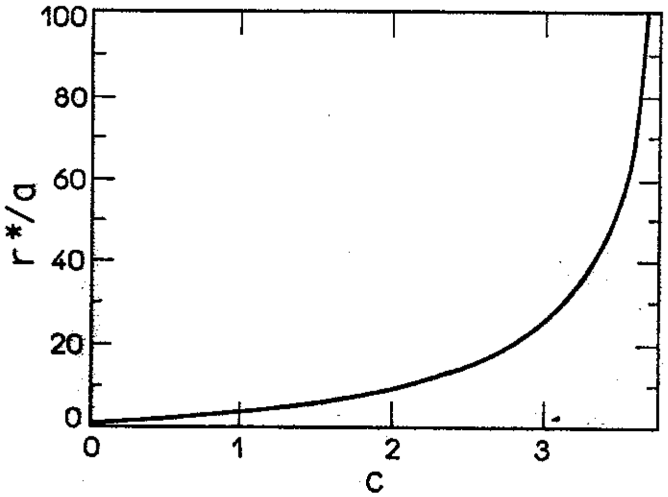

FIG. 2a

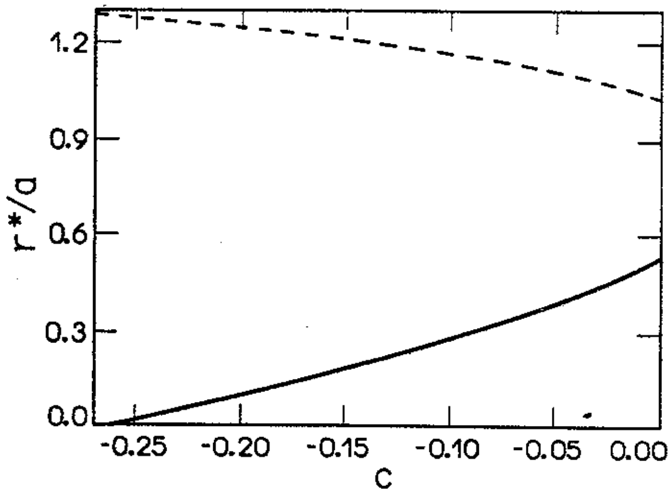

FIG. 3a

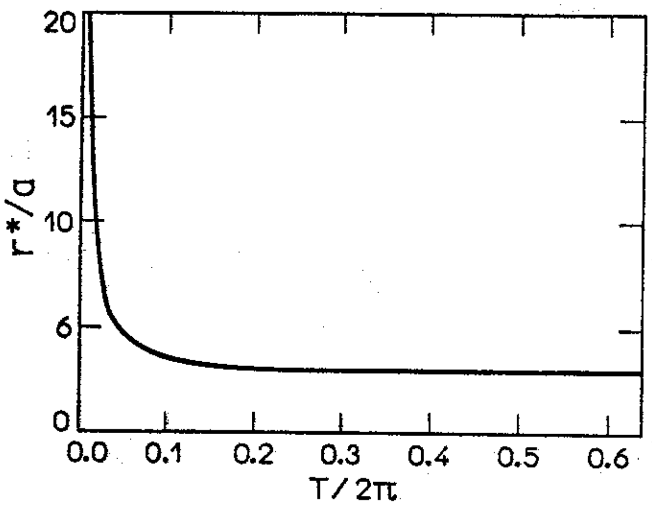

FIG. 2b

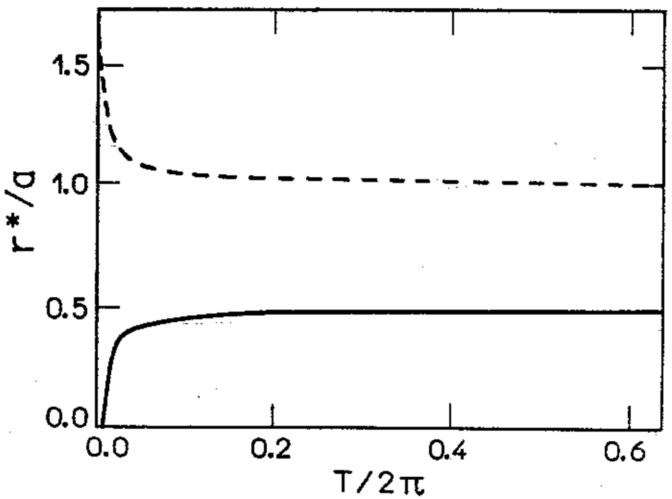

FIG. 3b