IFUSP/P-796

UNIVERSIDADE DE SÃO PAULO

PUBLICAÇÕES

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

IFUSP/P-796

DYNAMICAL PARITY VIOLATION AND THE CHERN SIMONS TERM

M. Gomes, V. O. Rivelles and A. J. da Silva
Instituto de Física, Universidade de São Paulo

Julho/1989

DYNAMICAL PARITY VIOLATION AND THE CHERN SIMONS TERM

M. Gomes, V. O. Rivelles and A. J. da Silva

Instituto de Física da USP - Departamento de Física Matemática

C. P. 20516 - São Paulo, S.P., Brazil

Abstract

We consider a three dimensional model of two component spinors with a quadrilinear self interaction. In the 1/N expansion the model turns out to be renormalizable and a mass term is generated, violating parity. This allows for the generation of a Chern Simons term if the spinor is coupled to an external gauge field. The parity violation and the associated induction of the topological term ceases at a computable critical temperature.

This work was partially supported by CNPq.

Quantum field theory in three space time dimensions has recently attracted considerable interest, particularly due to its possible relevance to the quantized Hall effect[1] and to high T_c superconductivity[2]. Indeed, there is a class of three dimensional theories exihibiting interesting and impressive features such as exotics statistics, fractional spin[3,4] and massive gauge fields[5,6]. It has been pointed out that these peculiarities are of a topological nature and that they can be produced via the addition of a Chern Simons term to the Lagrangian describing the system under consideration[7,8]. It is therefore important to understand the mechanism by which a topological term of the Chern-Simons type can be generated. Being a pseudo scalar density this term is odd under parity. Parity violation is so a prerequisite to generate the Chern Simons term. This breakdown of parity can be accomplished by coupling the gauge field to massive, two component spinors. In fact, it is well known that a fermionic mass term is odd under parity. Here we would like to indicate another route to parity violation, namely the parity symmetry breakdown through radiatiave corrections. This dynamical violation of parity will be explicitly verified in a context of a model with a four fermion interaction. Although perturbatively non renormalizable the model has a well defined 1/N expansion. Formally, it is described by the Lagrangian density

$$\mathcal{L} = i\overline{\psi}\partial\!\!\!/\psi - \frac{g}{2N}(\overline{\psi}\psi)^2 \tag{1}$$

The dimension of ψ is one so that the quadrilinear term has dimension four, signaling a (perturbatively) nonrenormalizable theory.

There are two inequivalent two dimensional representations of the Dirac algebra. One of these has

$$\gamma^o = \sigma^3, \quad \gamma^1 = i\sigma^1 \quad \text{and} \quad \gamma^2 = i\sigma^2 \tag{2}$$

as a typical explicit realization; the other inequivalent representation has a typical representative which differs from the above just by the sign of one of the gamma matrices. For definiteness, we use the representation (2) whenever convenient.

The parity transformation, corresponding to the inversion of one of the axis, x_1, let us say, leaves the action corresponding to (1) invariant if

$$\psi(x^o, x^1, x^2) \quad \rightarrow \quad \gamma^1\psi(x^o, -x^1, x^2) \tag{3}$$

Note however that a mass term $\overline{\psi}\psi$ would change sign under such transformation[6].

The most efficient way to derive the $1/N$ expansion for this model is to use the equivalent Lagrangian

$$\mathcal{L} = i\overline{\psi}\partial\!\!\!/\psi - \sigma(\overline{\psi}\psi) + \frac{N}{2g}\sigma^2 \tag{4}$$

where σ plays the role of an auxiliary field(classically, $\sigma = \frac{g}{N}\overline{\psi}\psi$). As in two dimensions[9], there is the possibility for σ to acquire a nonzero vacuum expectation value, $\langle 0|\sigma|0\rangle = \sigma_o$, at the quantum level. Making the replacement $\sigma \to \sigma' = \sigma + \sigma_o$ the Lagrangian density in eq. (4) becomes

$$\mathcal{L} = i\overline{\psi}\partial\!\!\!/\psi - \sigma_o(\overline{\psi}\psi) - \sigma(\overline{\psi}\psi) + \frac{N}{2g}\sigma_o^2 + \frac{N}{2g}\sigma^2 + \frac{N}{g}\sigma_o\sigma \tag{5}$$

The requirement that the shifted field σ now has zero vacuum expectation value implies that

$$\frac{1}{g}\sigma_o = -\int \frac{d^3k}{(2\pi)^3}\mathrm{Tr}\frac{i}{k\!\!\!/ - \sigma_o} = -2i\sigma_o \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^2 - \sigma_o^2} \tag{6}$$

where in the (logarithimically divergent) integral an appropriate ultraviolet cutoff is understood. This relation fixes g as a function of the generated mass. In particular, if a Pauli Villars regulator is employed, we get

$$\frac{1}{g(\Lambda)} + \frac{1}{2\pi}(\Lambda - \sigma_o) = 0 \tag{7}$$

so that, introducing a renormalized coupling constant, g_R, through

$$\frac{1}{g_R} = \frac{1}{g(\Lambda)} + \frac{1}{2\pi}(\Lambda - \mu) \tag{8}$$

we obtain

$$\frac{1}{g_R} = \frac{\sigma_o - \mu}{2\pi} \tag{9}$$

where the massive parameter μ plays the role of the renormalization spot. Different renormalization prescriptions will introduce the mass parameter in different ways, but they can be related through a renormalization group transformation[10].

Let us look now to the propagator for the σ field. In the dominant order of the $1/N$ expansion it is given by the inverse of

$$\begin{aligned}\Gamma_\sigma &= i\frac{N}{g} + N\mathrm{Tr}\int \frac{d^3k}{(2\pi)^3}\frac{i}{k\!\!\!/ - \sigma_o}\frac{i}{(k\!\!\!/ + p\!\!\!/) - \sigma_o} \\ &= iN(\frac{1}{g} + 2i\int \frac{d^3k}{(2\pi)^3}\frac{k\cdot(k+p) + \sigma_o^2}{(k^2 - \sigma_o^2)((k+p)^2 - \sigma_o^2)})\end{aligned} \tag{10}$$

Now, if one replaces (6) into the above equation, we get a finite result

$$\Gamma_\sigma = N(p^2 - 4\sigma_o^2)\int \frac{d^3k}{(2\pi)^3}\frac{1}{(k^2 - \sigma_o^2)((k+p)^2 - \sigma_o^2)} \tag{11}$$

which exhibits a bound state pole for the propagator $\Delta_\sigma = (\Gamma_\sigma)^{-1}$ at $p^2 = 4\sigma_o^2$. For large p^2, the sigma's propagator behaves like

$$\Delta_\sigma(p) \overset{p\to\infty}{\rightarrow} \frac{1}{\sqrt{p^2}} \tag{12}$$

This expression illustrates the fact that, by summing an infinite chain of one loop diagrams as the $\frac{1}{N}$ expansion does, one may improve the ultraviolet behaviour of field theories. Here each fermionic loop at high momenta has gained additional decaying factors corresponding to the sigma lines in it.

Higher orders contributions to the $\frac{1}{N}$ expansion can now be systematically constructed. They must use

Fermion propagator: $\frac{i}{\not{p}-\sigma_o}$

Sigma propagator: Δ_σ, given above

Trilinear vertex: the vertex associated to the term $-\sigma(\overline{\psi}\psi)$ (13)

Graphs containing as subgraphs either the one loop contribution to the σ propagator or the one loop contribution to the tadpole should be omitted since they have been explictly taken into consideration. Similarly to the four component spinor case[11], with these rules we obtain that the degree of superficial divergence associated to a proper graph γ is given by

$$d(\gamma) = 3 - N_F - N_\sigma \tag{14}$$

where N_F and N_σ are the number of external fermion and sigma lines, respectively. From this we see that the $\frac{1}{N}$ expansion defines a renormalizable theory. Graphs having $N_\sigma = 2$ and $N_F = 0$ are linearly divergent but, due to Lorentz covariance, only a counterterm proportional to σ^2 is actually needed to absorb this divergence; the counterterm corresponds to a coupling constant renormalization. Differently, in four dimensions the same type of diagrams

is quadratically divergent and need a counterterm of the type $(\partial_\mu\sigma)^2$ making the $1/N$ expansion unrenormalizable.

A Chern Simons term can be generated by coupling a gauge field A_μ to ψ through the interaction

$$\mathcal{L}_{int} = \frac{e}{\sqrt{N}}\overline{\psi}\gamma_\mu\psi A^\mu \tag{15}$$

so that, in the dominant order of $\frac{1}{N}$ the polarization tensor $\pi_{\mu\nu}$ is given by

$$\pi^{\mu\nu}(p) = e^2\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}[\gamma^\mu\frac{i}{\not{k}-\sigma_o}\gamma^\nu\frac{i}{\not{p}+\not{k}-\sigma_o}] \tag{16}$$

Taking into account that $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho] = -2i\epsilon^{\mu\nu\rho}$, we obtain

$$\pi^{\mu\nu}(p) = 2ie^2\sigma_o\epsilon^{\mu\rho\nu}p_\rho F(p^2) + \text{ other terms} \tag{17}$$

where $F(p^2)$ is the same integral found in the calculation of the sigma propagator, i. e.,

$$F(p^2) = \int\frac{d^3k}{(2\pi)^3}\frac{1}{(k^2-\sigma_o^2)((k+p)^2-\sigma_o^2)} \tag{18}$$

For very low momenta this expression reduces itself to $\frac{e^2}{4\pi}(\text{sign }\sigma_o)\epsilon^{\mu\rho\nu}p_\rho$. It corresponds to an induced Chern Simons term

$$\frac{e^2}{8\pi}(\text{sign }\sigma_o)\epsilon^{\mu\rho\nu}A_\mu\partial_\rho A_\nu \tag{19}$$

in the effective Lagrangian density. We arrive thus to the conclusion that the Chern Simons term can be generated via a dynamical violation of the parity symmetry.

The Chern Simons produces a rotational anomaly as follows. Firstly notice that the equation of motion for the A_o component of the gauge field is

$$\frac{e}{\sqrt{N}}\overline{\psi}\gamma^o\psi + \frac{e^2}{4\pi}(\text{ sign }\sigma_o)\epsilon_{ij}\partial_i A_j = 0 \tag{20}$$

implying that the "magnetic" field, $\epsilon_{ij}\partial^i A^j$, creates a charge. In Dirac's Hamiltonian formalism for constrained systems this is a secondary constraint induced by the primary constraint $\pi_o = 0$ (π_o is the canonical momentum conjugate to A_o). Choosing the gauge $\vec{\nabla}\cdot\vec{A} = 0$, $A_o = 0$ permits one to integrate eq. (20) to

$$A_i(x) = \frac{2(\text{ sign } \sigma_o)}{e^2}\epsilon_{ij}\int d^2y \frac{x_j - y_j}{|\vec{x} - \vec{y}|^2} j_o(y) \tag{21}$$

where $j_o = \frac{e}{\sqrt{N}}\overline{\psi}\gamma_o\psi$ is the charge density. Now, the symmetric gauge invariant energy-momentum tensor is given by

$$T^{\mu\nu} = \frac{i}{4}\Big(\overline{\psi}\gamma^\mu D^\nu\psi + \overline{\psi}\gamma^\nu D^\mu\psi - D^\nu\overline{\psi}\gamma^\mu\psi - D^\mu\overline{\psi}\gamma^\nu\psi\Big) - g^{\mu\nu}\mathcal{L} \tag{22}$$

where $D^\mu\psi = (\partial^\mu - i\frac{e}{\sqrt{N}}A^\mu)\psi$ is the covariant derivative. In particular, the component T^{oi} has the form

$$T^{oi} = \frac{i}{4}\Big(3\overline{\psi}\gamma^o\partial^i\psi - \partial^i\overline{\psi}\gamma^o\psi - \partial^k(\overline{\psi}\gamma^k\gamma^o\gamma^i\psi)\Big) + \frac{e}{\sqrt{N}}\overline{\psi}\gamma^o\psi A^i \tag{23}$$

so that the generator of rotations

$$L = \int d^2x(x_j\epsilon_{ji}T_{oi}) \tag{24}$$

turns out to be equal to

$$L = \int d^2x i\Big\{\epsilon_{ji}x_j\overline{\psi}\gamma_o\partial_i\psi - \frac{\epsilon_{ik}}{4}\overline{\psi}\gamma^o\gamma^k\gamma^i\psi + \frac{e}{\sqrt{N}}\epsilon_{ij}x_i\overline{\psi}\gamma_o\psi A_j\Big\} \tag{25}$$

The last term in this expression which can be rewritten as $\frac{(\text{ sign } \sigma_o)}{e^2}Q^2$, where Q is the charge operator, is responsible for the fractional spin.

As we shall see now, these facts are temperature dependent and we shall determine a critical temperature beyond which no parity violation occurs and, consequently, no Chern Simons term is induced.

At finite temperature, eq. (6) changes to

$$\frac{1}{g} - \frac{2}{\beta}\sum_n\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2 - \sigma_T^2} = 0 \tag{26}$$

where the sum is over k_o equal to odd integer multiples of πiT; β is the inverse of the temperature. The sum can be transformed into an integral, giving

$$\frac{1}{g} + \int\frac{d^2k}{(2\pi)^2}\frac{1}{w_k}(1 - \frac{2}{e^{\beta w_k} + 1}) = 0 \tag{27}$$

where $w_k = \sqrt{\vec{k} + \sigma_T^2}$. This equation shows that σ_T decreases monotonically from σ_o at $T = 0$ till zero at the critical temperature $T_c = \frac{1}{4\ln 2}\sigma_o \approx 0,3607\sigma_o$. We should remark that a similar result holds in two dimensions where there is also a critical temperature at the leading $\frac{1}{N}$ order, associated to a chiral symmetry violation[12].

At $T = T_c$ parity ceases to be violated and there is no induced Chern Simons term. In fact, one rapidly finds that the finite temperature anologue of the first term in eq (17) contains a factor σ_T which takes the would be Chern Simons term to zero for all values of $p \neq 0$. The isolated singularity at $p = 0$ is integrable so that the Chern Simons term really tends to zero in a distributional sense.

We could have considered a more general Lagrangian with four fermion interactions. However, because of the identities

$$(1 + \frac{2}{N})(\overline{\psi}\psi)^2 = -(\overline{\psi}\gamma^\mu\psi)(\overline{\psi}\gamma_\mu\psi) - (\overline{\psi}\lambda^a\psi)(\overline{\psi}\lambda^a\psi), \tag{28}$$

$$\begin{aligned}2(2 + \frac{1}{N})(\overline{\psi}\psi)^2 + \frac{2}{N}(\overline{\psi}\gamma^\mu\psi)(\overline{\psi}\gamma_\mu\psi) + (\overline{\psi}\lambda^a\psi)(\overline{\psi}\lambda^a\psi)\\ + (\overline{\psi}\gamma^\mu\lambda^a\psi)(\overline{\psi}\gamma_\mu\lambda^a\psi) = 0,\end{aligned} \tag{29}$$

where λ^a, $a = 1, \ldots, N$, indicate the SU(N) generators, there are only two linearly independent, U(N) invariant, quadrilinear interactions. These could be $(\overline{\psi}\psi)^2$ and $(\overline{\psi}\gamma^\mu\psi)(\overline{\psi}\gamma_\mu\psi)$. If we incorporate this last interaction the treatment is very similar to what we have already described. Besides the σ field, one introduces an auxiliary vector field, W^μ, through the combination $N\,W^2/\overline{g} - W^\mu\overline{\psi}\gamma_\mu\psi$, where $\overline{g}$ is another coupling constant, associated to the new interaction. Due to $\mathrm{Tr}\gamma^\mu = 0$, the tadpole equation, eq. (6), remains the same. The W^μ propagator turns out to have a transversal part which decays for large momentum as $\frac{1}{\sqrt{p^2}}$. The longitudinal piece, nonetheless, behaves like a constant. This bad behaviour will still be under control if the regularization procedure keeps the current $\overline{\psi}\gamma^\mu\psi$ conserved.

On the other hand, if the only interaction is $(\overline{\psi}\gamma^\mu\psi)(\overline{\psi}\gamma_\mu\psi)$, the induced mass will possibly be generated at higher orders of $\frac{1}{N}$. This is presently under investigation.

References

[1] S. M. Girvin and A. H. MacDonald, Phys. Rev Lett. 58, 1252, 1987.

[2] B. Wiegman, Phys. Rev. Lett. 60, 821, 1988.
I. Dzyaloshinskii, A. M. Polyakov and P. W. Wiegmann, Phys. Lett. A127, 112, 1988.

[3] J. M. Leinaas and J. Myrheim, Il Nuovo Cimento 37B, 1, 1977.

[4] F. Wilczek and A. Zee, Phys. Rev. Lett. 51, 2250, 1983.
T. H. Hansson, M. Rocek, I. Zahed and S. C. Zhang, Phys. Lett 214B, 475, 1988.
L. Huerta and M. Ruiz-Altaba, Phys. Lett. 216B, 371, 1989.
R. Mackenzie and F. Wilczek, Int. J. of Mod. Phys. 3, 2827, 1988.

[5] J. F. Schonfeld, Nucl Phys. B185, 157, 1981.

[6] S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. 48, 975, 1982; Ann. Phys. 140, 372, 1982.

[7] A. N. Redlich, Phys. Rev. Lett. 52, 18, 1984; Phys. Rev. D29, 2366, 1984.
S. Deser and A. N. Redlich, Phys. Rev. Lett. 61, 1541, 1988.

[8] A. Niemi and G. W. Semenoff, Phys. Rev. Lett. 23, 2077, 1983.
G. W. Semenoff, Phys. Rev. Lett 61, 517, 1988.

[9] D. J. Gross and A. Neveu, Phys. Rev. D10, 3235, 1974.

[10] For a discussion of this question in two dimensions see H. Fleming, K. Furuya and J. L. de Lyra, Il Nuovo Cimento 53A, 405, 1979.

[11] B. Rosenstein, B. J. Warr and S. H. Park, Phys. Rev. Lett. 62, 1433, 1989.

[12] R. F. Dashen, S-k Ma and R. Rajaraman, Phys. Rev. D11, 1499, 1975.