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ABSTRACT

We quantize, via Dirac procedure, gauge covariant chiral bosons using different Lagrange densities. We discuss ambiguities of the quantization procedure, and propose a "correct" procedure. A generalization of the Siegel symmetry is obtained. Finally, using a certain modified Lagrangian, we obtain a theory equivalent to chiral QED in two dimensions.

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1. INTRODUCTION

Recently we have discussed the quantization of genuine chiral bosons coupled to an Abelian gauge field[1] where by "genuine" we mean that the chirality condition is the usual one, $\partial_- \varphi = 0$. However, it is more natural to impose a gauge covariant condition, namely $D_- \varphi = 0$. Although this seems to be a trivial generalization of ref.[1], it is a good laboratory to understand the quantization of systems with an externally imposed constraint. The general procedure will be discussed elsewhere[2]. The covariant constraint will be imposed using a Lagrange multiplier. However, if it is implemented by a linear constraint, the Lagrange multiplier plays the role of a constraint force, describing a new field. This method, in the quantum case, is equivalent to a delta function in the partition function.

A second procedure is the implementation of the constraint by means of a quadratic term. The classical system is first class, and a Siegel symmetry[3] takes place. We argue in favor of the second procedure. As mentioned in the abstract there exists two inequivalent Lagrangians, and only one of them leads to a theory equivalent to chiral QED_2 .

In section 2 we discuss the first model, briefly with linear constraint, then thoroughly with quadratic constraint; in section 3 we discuss the second model and its equivalence to chiral QED_2. In section 4 we draw conclusions.

2. THE MINIMAL COUPLING

We consider the minimal coupling of real bosons to gauge fields, realized by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu} D_\mu \varphi D_\nu \varphi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (\eta^{\mu\nu} - \epsilon^{\mu\nu}) A_\mu D_\nu \varphi + \frac{(a+1)}{2} A_\mu A^\mu \quad , \tag{2.1}$$

with the constraint

$$D_- \varphi = 0 \quad , \tag{2.2}$$

and $D_\mu \varphi = \partial_\mu \varphi + A_\mu$.

The constraint (2.2) may be implemented adding to (2.1) a term

$$\mathcal{L}_{\lambda_+} = \sqrt{2}\lambda_+ D_- \varphi \quad . \tag{2.3}$$

The resulting theory has two primary cosntraints

$$\Omega_1 = \Pi_o \approx 0 \qquad (2.4.a)$$

$$\Omega_2 = \Pi_{\lambda_+} \approx 0 \quad , \qquad (2.4.b)$$

and two secondary constraints

$$\Omega_3 = \Pi_1' - \varphi' - aA_o - \lambda_+ \approx 0 \qquad (2.5.a)$$

$$\Omega_4 = \Pi - \varphi' + A_o - \lambda_+ \approx 0 \quad , \qquad (2.5.b)$$

which determine the Lagrange multiplier λ_+ in terms of the remaining fields. The Dirac brackets of the matter fields are the canonical ones (see also ref. [4]). The lack of interest in this theory, however arises from the fact that the field λ_+ is non trivial, describing the "external force" necessary to enforce the constraint. If we are interested in the solutions of the equations of motion arising from (2.1), which obey (2.2), we add to the Lagrangian

$$\mathcal{L}_{\lambda_{++}} = \lambda_{++}(D_-\varphi)^2 \quad . \qquad (2.6)$$

The equations of motion for λ_{++}, A_+, A_- and φ imply

$$(D_-\varphi)^2 = 0 \quad (\Rightarrow D_-\varphi = 0) \qquad (2.7.a)$$

$$\partial_- E = aA_- - \partial_-\varphi = (a+1)A_- \qquad (2.7.b)$$

$$\partial_+ E = -aA_+ - \partial_+\varphi \qquad (2.7.c)$$

$$\Delta\varphi = E \qquad (2.7.d)$$

$$(\Delta + (a+1))E = 0 \quad , \qquad (2.7.e)$$

where $E = -F_{+-}$, and the gauge field has mass $m^2 = (a+1)$.

Canonical quantization of the model may be performed using the Dirac method[4]. The primary constraints read now

$$\tilde\Omega_1 = \Pi_o \approx 0 \qquad (2.8.a)$$

$$\tilde\Omega_2 = \Pi_{\lambda_{++}} \approx 0 \qquad (2.8.b)$$

The canonical Hamiltonian may be readily computed and we find

$$\mathcal{H}_c = \frac{1}{2}\Pi_1^2 + \frac{1}{2}\varphi'^2 + \frac{a}{2}A_1^2 + \frac{1}{2}(\Pi + A_1)^2 - \frac{a}{2}A_o^2 + A_o(\Pi_1 - \varphi)' + $$

$$- \frac{\lambda_{++}}{2(1+\lambda_{++})}(\Pi - \varphi' + A_o)^2 \quad , \qquad (2.9)$$

The secondary constraints are the Gauss' law, and the equivalent, in phase space, to (2.7.a), namely

$$\tilde\Omega_3 = \Pi_1' - \varphi' - aA_o \approx 0 \qquad (2.10.a)$$

$$\tilde\Omega_4 = \frac{1}{2(1+\lambda_{++})^2}(\Pi - \varphi' + A_o)^2 \approx 0 \qquad (2.10.b)$$

Equation (2.10.b) may be linearized to the equivalent constraint

$$\tilde\Omega_5 = \Pi - \varphi' + A_o \approx 0 \qquad (2.11)$$

Note also that the Lagrangian (2.1) and (2.6) has a local symmetry, since there is a subset of constraints, which is first class. The local symmetry may be obtained by the algorithm of Anderson and Bergmann[2],[6], and reads

$$\delta\varphi = \varepsilon_+ D_-\varphi = \varepsilon_+(\partial_-\varphi + A_-) \qquad (2.12.a)$$

$$\delta\lambda_{++} = -\partial_+\varepsilon_+ + \varepsilon_+\partial_-\lambda_{++} - \lambda_{++}\partial_-\varepsilon_+ + \varepsilon_+\frac{\partial_\mu A^\mu}{D_-\varphi} \quad . \qquad (2.12.b)$$

The commutators are obtained by the Dirac method, fixing the above symmetry, if we take, e.g. $\lambda_{++} = 0$. The non vanishing equal-time commutators are

$$[A_1(t,x), A_1(t,y)] = \frac{i\hbar}{2a(a+1)}\delta'(x-y) \qquad (2.13.a)$$

$$[A_1(t,x), \Pi_1(t,y)] = -i\hbar\delta(x-y) \qquad (2.13.b)$$

$$[A_1(t,x), \Pi(t,y)] = \frac{-i\hbar}{2a}\delta'(x-y) \qquad (2.13.c)$$

$$[\Pi(t,x), \Pi(t,y)] = \frac{i\hbar(a+1)}{2a}\delta'(x-y) \qquad (2.13.d)$$

$$[\varphi(t,x),\varphi(t,y)] = \frac{-i\hbar a}{4(a+1)}\epsilon(x-y) \qquad (2.13.e)$$

$$[\varphi(t,x),\Pi(t,y)] = \frac{i\hbar}{2}\delta(x-y) \qquad (2.13.f)$$

$$[\varphi(t,x),A_1(t,y)] = \frac{i\hbar}{2(a+1)}\delta(x-y) \quad . \qquad (2.13.g)$$

Notice that the commutators are essencially non canonical as oposed to the former case (2.1) and (2.3). The latter case corresponds to the usual quantization of chiral bosons; the Lagrange multiplier is a gauge field and plays no role, while it is defined by eq. (2.5) in the former case, being interpreted there as an external force and therefore presuposes a modification of the problem. Notice also that in general the symmetry (2.12), which is a generalization of Siegel symmetry[3], is anomalous in the quantum theory. Quantization requires a Wess-Zumino term[8]. In the case of chiral bosons, an analisis shows[9] that inclusion of the anomaly does not invalidate the result, because the quantum symmetry replaces (2.12). We postpone further discussions to the next section, where a more interesting model is considered.

The special cases $a = 0, -1$ give rise to new symmetries, as one readily sees from the fact that either $\tilde{\Omega}_3$ commutes with $\tilde{\Omega}_1(a=0)$ or $\tilde{\Omega}_3 - \tilde{\Omega}_5$ commutes with $\tilde{\Omega}_1(a=-1)$.

3. A NEW INTERACTION, AND EQUIVALENCE TO FERMIONIC CHIRAL QED$_2$

We can contemplate studying the effect of adding a term $\eta^{\mu\nu}A_\mu\partial_\nu\varphi$ to the Lagrangian (2.1), obtaining

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}D_\mu\varphi D_\nu\varphi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \epsilon^{\mu\nu}A_\mu D_\nu\varphi + \frac{(a-1)}{2}A_\mu A^\mu \quad . \qquad (3.1)$$

The equations of motion are

$$\Delta\varphi + \partial_\mu A^\mu = E \qquad (3.2.a)$$

$$\partial_- E = aA_- \qquad (3.2.b)$$

$$\partial_+ E = -aA_+ - 2\partial_+\varphi \qquad (3.2.c)$$

$$E = (1-a)\partial_\mu A^\mu \qquad (3.2.d)$$

$$(\Delta + \frac{a^2}{(a-1)})E = 0 \quad . \qquad (3.2.e)$$

Therefore, the gauge field $E = -F_{+-}$ has a mass $m^2 = a^2/(a-1)$, which is the value of the chiral QED$_2$ photon mass[10]. Introducing a linear chiral constraint, the structure of the equations of motion get modified, and we shall not dwell on this case. We eventually go to the (gauge fixed) case $\lambda_{++} = 0$, adding the constraint to our set of equations (the constraint appears as the secondary constraint arising from the conservation of $\Pi_{\lambda_{++}} \approx 0$). The whole set of constraints reads

$$\hat{\Omega}_1 = \Pi_o \approx 0 \qquad (3.3.a)$$

$$\hat{\Omega}_2 = \Pi + \varphi' + A_1 - \Pi_1' + (a-1)A_o \approx 0 \qquad (3.3.b)$$

$$\hat{\Omega}_3 = \Pi_{\lambda_{++}} \approx 0 \qquad (3.3.c)$$

$$\hat{\Omega}_4 = \Pi - \varphi' \approx 0 \qquad (3.3.d)$$

$$\hat{\Omega}_5 = \lambda_{++} - f \approx 0 \quad . \qquad (3.3.d)$$

Note the simplicity of the chiral constraint, since in this formulation

$$\sqrt{2}D_-\varphi = \dot{\varphi} - \varphi' + A_o - A_1 = \Pi - \varphi' \quad , \qquad (3.4)$$

due to the definition of the momentum

$$\Pi = \dot{\varphi} + A_o - A_1 \quad . \qquad (3.5)$$

The constraint $\hat{\Omega}_2$ corresponds to the conservation of $\hat{\Omega}_1$, using the canonical Hamiltonian

$$\mathcal{H}_c = \frac{1}{2}(\Pi + \varphi' - A_o + A_1)^2 + \frac{1}{2}\Pi_1^2 - \Pi\varphi' - A_o'\Pi_1 - \frac{a}{2}(A_o^2 - A_1^2) + \\ - \frac{\lambda_{++}}{2(1+\lambda_{++})}(\Pi - \varphi')^2 \quad . \qquad (3.6)$$

The constraint $\Pi_{\lambda_{++}} \approx 0$, and the corresponding consistency condition, imply, using the algorithm of Anderson and Bergmann[6], the gauge symmetry

$$\delta\varphi = \varepsilon_+ D_-\varphi \qquad (3.7.a)$$

$$\delta\lambda_{++} = -\partial_+\varepsilon_+ + \varepsilon_+\partial_-\lambda_{++} - \lambda_{++}\partial_-\varepsilon_+ \quad . \qquad (3.7.b)$$

We recovered once again a generalization of the Siegel symmetry. The "chiral" Dirac-brackets are

$$\{\varphi(t,x),\varphi(t,y)\} = -\frac{1}{4}\epsilon(x-y) \qquad (3.8.a)$$

$$\{\varphi(t,x),\Pi(t,y)\} = \frac{1}{2}\delta(x-y) \qquad (3.8.b)$$

$$\{\Pi(t,x),\Pi(t,y)\} = \frac{1}{2}\delta'(x-y) \quad . \qquad (3.8.c)$$

After using the constraints strongly we obtain

$$\mathcal{H} = \frac{1}{2}\Pi_1^2 + \frac{(a-1)}{2}A_1^2 + \frac{1}{(a-1)}(2\varphi' + A_1 - \Pi_1')^2 + (\varphi' + A_1)^2 \qquad (3.9)$$

The special case $a = 1$ has two additional constraints

$$\hat{\Omega}_6 = \Pi_1 \approx 0 \qquad (3.10.a)$$

$$\hat{\Omega}_7 = A_o - A_1 \approx 0 \quad , \qquad (3.10.b)$$

and we get the usual chiral boson model[7]

$$\mathcal{H} = (\varphi')^2 \qquad (3.11)$$

$$\{\varphi(t,x),\varphi(t,y)\} = -\frac{1}{4}\epsilon(x-y) \quad . \qquad (3.12)$$

The only loophole in the argument is the fact that the gauge symmetry (3.7) is anomalous, and one should thus add the Wess-Zumino $\mathcal{L}_{WZ}$ term to the Lagrangian[8]

$$\mathcal{L}_{WZ} = \alpha\lambda_{++}\partial_- D_-\varphi \quad , \qquad (3.13)$$

and the system acquires a quantum gauge symmetry[9]

$$\delta\varphi = \varepsilon_+ D_-\varphi - \frac{\alpha}{2}\partial_-\varepsilon_+ \qquad (3.14.a)$$

$$\delta\lambda_{++} = -\partial_+\varepsilon_+ + \varepsilon_+\partial_-\lambda_{++} - \lambda_{++}\partial_-\varepsilon_+ \quad . \qquad (3.14.b)$$

The Hamiltonian obtained with the new term (3.13) is

$$\mathcal{H} = \frac{1}{2}\Pi_1^2 + \frac{1-a}{2}A_o^2 + \frac{1+a}{2}A_1^2 + (\Pi - A_1)\varphi' - A_1\Pi + A_o(\Pi + \varphi' - A_1 - \Pi_1') +$$

$$-\frac{2}{\alpha}\Pi_{\lambda_{++}}(\Pi - \varphi') - \lambda_{++}\Pi'_{\lambda_{++}} - \frac{2}{\alpha^2}(1 + \lambda_{++})\Pi^2_{\lambda_{++}} \quad . \qquad (3.15)$$

It is clear that the system has no longer a constraint for $\Pi_{\lambda_{++}}$. However, we follow ref.[9] and fix the quantum gauge symmetry (3.14) with the condition

$$\tilde{\Omega}_1 = \lambda_{++} + 1 \approx 0 \quad . \qquad (3.16.a)$$

Then we obtain new constraints, from successive time conservation

$$\tilde{\Omega}_2 = \Pi - \varphi' \approx 0 \qquad (3.16.b)$$

$$\tilde{\Omega}_3 = \Pi'_{\lambda_{++}} \approx 0 \quad , \qquad (3.16.c)$$

and the same conclusions as before hold.

There is another Lagrangian formulation which does not make use of the auxiliary field λ_{++}, avoiding the anomaly problem at the price of not having manifest Lorentz invariance :

$$\mathcal{L} = \frac{1}{4}\int dy\chi(x)\epsilon(x-y)\dot{\chi}(y) - \frac{1}{2}\chi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a}{2}A_\mu A^\mu +$$

$$+2\chi A_- - (A_-)^2 \quad , \qquad (3.17)$$

which equations of motion imply

$$\chi(x) = \frac{1}{2}\int dy\epsilon(x-y)\dot{\chi}(y) + 2A_- \qquad (3.18.a)$$

$$\partial_- E = aA_- \quad (3.18.b)$$

$$\partial_+ E = -aA_+ - 2(\chi - A_-) \quad (3.18.c)$$

$$E = (1-a)\partial_\mu A^\mu \quad (3.18.d)$$

$$(\Delta + \frac{a^2}{(a-1)})E = 0 \quad . \quad (3.18.d)$$

The scalar field $\chi(x)$ is related to $\varphi(x)$ through

$$\chi = \sqrt{2}\varphi' \quad ; \quad \varphi(x) = \frac{\sqrt{2}}{4}\int dy \epsilon(x-y)\chi(y) \quad (3.19.a)$$

$$D_\mu \chi \equiv \partial_\mu \chi + \sqrt{2}A'_\mu \quad . \quad (3.19.b)$$

Dirac-bracket quantization of (3.17) leads to:

$$\mathcal{H} = \frac{1}{2}(\chi + \sqrt{2}A_1)^2 + \frac{1}{2}\Pi_1^2 + \frac{(a-1)}{2}A_1^2 + \frac{1}{2(a-1)}(\sqrt{2}\chi + A_1 - \Pi_1')^2 \quad (3.20.a)$$

$$[\chi(t,x), \chi(t,y)] = i\hbar\delta'(x-y) \quad (3.20.b)$$

$$[A_1(t,x), \Pi_1(t,y)] = -i\hbar\delta(x-y) \quad (3.20.c)$$

$$\Pi = \frac{1}{2}\int dy \epsilon(x-y)\chi(y) \quad (3.20.d)$$

$$\Pi_o = 0 \quad (3.20.e)$$

$$A_o = \frac{1}{(a-1)}(\sqrt{2}\chi + A_1 - \Pi_1') \quad (3.20.f)$$

for $a \neq 1$, and

$$\mathcal{H} = \frac{1}{2}\chi^2 \quad (3.21.a)$$

$$[\chi(t,x), \chi(t,y)] = i\hbar\delta'(x-y) \quad (3.21.b)$$

$$\Pi = \frac{1}{2}\int dy \epsilon(x-y)\chi(y) \quad (3.21.c)$$

$$\Pi_o = 0 \quad (3.21.d)$$

$$\Pi_1 = 0 \quad (3.21.e)$$

$$A_o = A_1 = -\sqrt{2}\chi \quad (3.21.f)$$

for $a = 1$.

The model above agrees with the chiral boson model proposed in ref.7 in the absence of the gauge field A_μ.

The Lagrangian (3.1) may be written as

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\eta^{\mu\nu} - \epsilon^{\mu\nu})A_\mu\partial_\nu\varphi + \frac{a}{2}A_\mu A^\mu \quad , \quad (3.22)$$

which is the bosonized form of chiral QED_2

$$\mathcal{L}_{CQED_2} = \bar{\psi} i \not{\partial}\psi + eA_\mu\bar{\psi}\gamma^\mu\frac{(1-\gamma_5)}{2}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a}{2}A_\mu A^\mu \quad . \quad (3.23)$$

First notice that the solution of eqs. (3.2) is given by:

$$A_- = -\partial_-\varphi \quad (3.24.a)$$

$$A_+ = \frac{(a-2)}{a}\partial_+\varphi - \partial_+\phi(x^+) \quad (3.24.b)$$

$$(\Delta + \frac{a^2}{(a-1)})\varphi = 0 \quad , \quad (3.24.c)$$

while the solution of (3.23) is given by

$$A_- = -\frac{1}{e}\partial_-\sigma \quad (3.25.a)$$

$$A_+ = \frac{(a-2)}{ea}\partial_+\sigma - \partial_+ h \quad . \quad (3.25.b)$$

The massless field h is trivial, in the sense that it does not contribute to the anomaly. The mass of the σ field can also be read from (3.24.c). However the φ field has a "chiral" commutation relation (see (3.8)) while σ has a canonical one.

4. CONCLUSIONS

We have discussed different versions of chiral bosons interactions with gauge fields. Our first main result concerns the way we implement the external constraint, with Lagrange multipliers: as it turns out, the correct results are obtained using first class constraints; in turn, they become first class if we square them (see eqs. (2.6), (2.9)). Using the modified Lagrangian (3.1), we obtained a bosonic model related to chiral QED_2. However, the fields have the so called "chiral" commutation relations (3.8); nevertheless the solution of the gauge fields in terms of the bosonic fields is exactly the same as in the chiral QED_2 case (compare (3.24) with (3.25)).

The first class symmetry obtained is a generalization of the Siegel symmetry. However, it is anomalous and the solution of the problem requires the addition of a WZ term to the Lagrangian in order to cancel the anomaly (eq.(3.13)). In this case the symmetry is recovered at the quantum level (3.14), and a short discussion shows that the naive results obtained in the absence of the WZ term still hold, after fixing the Siegel symmetry (3.16).

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