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SYMMETRIES OF CHERN-SIMONS THEORY IN LANDAU GAUGE

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Abstract

In $d = 3$ dimensions the $IOSp(d|2)$ algebra admits a non-trivial modification. We show that 3-dimensional Chern-Simons theory in Landau gauge has a global symmetry based on this large Lie superalgebra.

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Interest in the quantization of a gauge theory based on a pure Chern-Simons action in $d = 3$ dimensions arises for several reasons. This type of theory is one example of a quantum field theory which is ***topological*** in the sense of being independent of the space-time metric on the three-manifold on which it is defined. As such it certainly deserves to be studied in its own right. Another approach to Chern-Simons theory comes from its connection to conformal field theories in one dimension lower[1,2]. This latter connection, and its relation to knot theory, has recently been explored from several points of view, and by now various groups have also performed explicit one and two loop perturbative computations in Chern-Simons theory, some of them motivated by the relation of this gauge theory to 2-dimensional conformal field theories[3-7].

One remarkable property of Chern-Simons theory is the existence of a so far unexplained new "supersymmetry" when this theory is considered in Landau gauge[3,8,9]. It is only a symmetry once the gauge has been fixed (it couples non-trivially between the Chern-Simons term and the ghost sector), and it appears difficult to extend it to other gauges. This is a quite puzzling situation which ought to be better understood.

In this note we shall demonstrate that this new symmetry in fact is only one part of a larger invariance group, based on a slightly modified $IOSp(3|2)$ algebra. Before doing this, let us recapitulate a few facts about the ordinary $IOSp(d|2)$ algebra, and show why the case $d = 3$ is rather special. Many details about the $IOSp(d|2)$ algebra, and in particular its importance in string theory, can be found in refs.[10,11].

The usual inhomogeneous orthosymplectic $IOSp(d|2)$ algebra has bosonic generators $J_{\mu\nu} = -J_{\nu\mu}, \mu = 0, 1, \cdots, d-1$; $J_{AB}, A, B = 1, 2$ (or $+$, $-$) and P_μ. The fermionic generators are $J_{\mu A}$ and P_A. It is a superspace generalization of the usual Poincaré algebra, with $J_{\mu\nu}$ generating ordinary Lorentz transformations, and P^μ generating the usual space-time translations. The objects J_{AB} and P^A are the corresponding generators in the anticommuting coordinates. The full algebra is defined by the following non-trivial (anti)commutation relations, with $\eta_{\mu\nu}$ denoting the space-time metric in the d-dimensional space, and C_{AB} being the symplectic metric in the two extra dimensions:

$$\begin{aligned}
[J_{\mu\nu}, J_{\rho\sigma}] &= \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} + \eta_{\nu\sigma}J_{\mu\rho} - \eta_{\nu\rho}J_{\mu\sigma} &(1)\\
[J_{AB}, J_{CD}] &= C_{AC}J_{BD} + C_{AD}J_{BC} + C_{BC}J_{AD} + C_{BD}J_{AC} &(2)\\
\{J_{\mu A}, J_{\nu B}\} &= -\eta_{\mu\nu}J_{AB} + C_{AB}J_{\mu\nu} &(3)\\
[J_{\mu\nu}, J_{\rho A}] &= \eta_{\mu\rho}J_{\nu A} - \eta_{\nu\rho}J_{\mu A} &(4)\\
[J_{AB}, J_{\mu C}] &= C_{AC}J_{\mu B} + C_{BC}J_{\mu A} &(5)\\
[P_\mu, P_\nu] &= \{P_A, P_B\} = [P_\mu, P_A] = 0 &(6)\\
[J_{\mu\nu}, P_\lambda] &= \eta_{\mu\lambda}P_\nu - \eta_{\nu\lambda}P_\mu &(7)\\
[J_{\mu\nu}, P_A] &= [J_{AB}, P_\mu] = 0 &(8)\\
[J_{AB}, P_C] &= C_{AC}P_B + C_{BC}P_A &(9)\\
[J_{\mu A}, P_\nu] &= \eta_{\mu\nu}P_A &(10)\\
\{J_{\mu A}, P_B\} &= -C_{AB}P_\mu &(11)
\end{aligned}$$

with all other (anti)commutation relations vanishing. The special situation arising in $d = 3$ dimensions is due to the possibility of also using the $\epsilon_{\mu\nu\rho}$-symbol in the defining equations of the algebra. One point in the above sequence of equations where this is possible is the case of $\{J_{\mu A}, J_{\nu B}\}$; consider the replacement of eq.(3) by

$$\{J_{\mu A}, J_{\nu B}\} = \epsilon_{\mu\nu\rho} C_{AB} P^\rho \tag{12}$$

Making the double replacement of $\mu \leftrightarrow \nu$ and $A \leftrightarrow B$, we see that this could be a consistent *ansatz*. But of course we also need to check if all Jacobi identities can be satisfied as well. This is found to be the case provided we simultaneously change eq.(10) to

$$[J_{\mu A}, P_\nu] = 0 \tag{13}$$

So the system of equations (1)-(11), with the modifications (12) and (13), does indeed define a non-trivial modification of the $IOSp(d|2)$ algebra, peculiar to $d = 3$ dimensions. Notice that this is an extension of the usual space-time supersymmetry algebra, where the anticommutators of the supersymmetry generators yields a space-time translation. Here we have an extended supersymmetry generator which carries an additional space-time index.

To specify the modified algebra more precisely, we introduce an ordinary Lie algebra G_0 defined by the semisimple product between $SO(3)\oplus Sp(2)$ and space-time translations. This is just $ISO(3)\oplus Sp(2)$. Our graded Lie algebra G is then given by $G = G_0 \oplus G_1$, where the G_0-module G_1 is defined by $G_1 \equiv \{P_A, J_{\mu A}\}$. It is easy to see that with a product $*$ defined by the commutation relations (1)-(11) (and the modifications (12) and (13)), we have $G_i * G_j \subseteq G_{i+j}$ for $i, j \in Z(2)$. With this $Z(2)$-grading, and all Jacobi identities satisfied, G does indeed form a Lie superalgebra. There is no change of basis (redefinition of the generators) which can relate this super Lie algebra to that of $IOSp(3|2)$, although the two algebras are so similar in form. (This would in any case be rather surprising, since our algebra is very specifically related to $d = 3$ dimensions, while that of $IOSp(d|2)$ depends on dimensionality in a completely trivial way).

Curiously, our super Lie algebra can also be obtained by an Inönü-Wigner contraction of the simple exceptional superalgebra $D(2|1;\alpha)$ (in Kac's classification scheme[12]). $D(2|1;\alpha)$ has bosonic generators $Q^i_\mu(\mu = 1,2,3;\ i = 1,2,3)$ and fermionic generators $R_{ABC}(A, B, C, = +, -)$. The commutation relations can be written

$$\begin{aligned}
[Q^i_\mu, Q^j_\nu] &= \delta^{ij}\epsilon_{\mu\nu\rho} Q^{i\rho} \\
[Q^1_\mu, R_{ABC}] &= -\frac{i}{2}(\sigma^\mu)_{A'A} R_{A'BC} \\
[Q^2_\mu, R_{ABC}] &= -\frac{i}{2}(\sigma^\mu)_{B'B} R_{AB'C} \\
[Q^3_\mu, R_{ABC}] &= -\frac{i}{2}(\sigma^\mu)_{C'C} R_{ABC'} \\
\{R_{ABC}, R_{A'B'C'}\} &= i\alpha_1 (C\sigma^\mu)_{AA'} C_{BB'} C_{CC'} Q^1_\mu + i\alpha_2 C_{AA'} (C\sigma^\mu)_{BB'} C_{CC'} Q^2_\mu \\
&\quad + i\alpha_3 C_{AA'} C_{BB'} (C\sigma^\mu)_{CC'} Q^3_\mu
\end{aligned} \tag{14}$$

where σ^μ are the Pauli matrices and $C \equiv i\sigma^2$. The real numbers α_i are arbitrary, except for the constraint $\alpha_1 + \alpha_2 + \alpha_3 = 0$.

We can realize this superalgebra if we modify our algebra as follows (let $J_{\mu\nu} = \epsilon_{\mu\nu\rho} J^\rho$ in d = 3 dimensions):

$$\begin{aligned}
[J_\mu, J_\nu] &= \epsilon_{\mu\nu\rho} J^\rho \\
[J_\mu, P_\nu] &= \epsilon_{\mu\nu\rho} P^\rho \\
[P_\mu, P_\nu] &= m^2 \epsilon_{\mu\nu\rho} J^\rho \\
[J_\mu, J_{\nu A}] &= \epsilon_{\mu\nu\rho} J^\rho_A \\
[J_\mu, P_A] &= 0 \\
[P_\mu, J_{\nu A}] &= -m\frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2}\eta_{\mu\nu} P_A \\
[P_\mu, P_A] &= m\frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} J_{\mu A} \\
[J_{\mu A}, J_{\nu B}] &= \epsilon_{\mu\nu\rho} C_{AB}\left(P^\rho + m\frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} J^\rho\right) - m\eta_{\mu\nu}\frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} J_{AB} \\
[J_{\mu A}, P_B] &= -C_{AB}\left(P_\mu + m\frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} J_\mu\right) \\
[P_A, P_B] &= -m(\alpha_1 - \alpha_2) J_{AB} \\
[J_{AB}, J_{CD}] &= C_{AC} J_{BD} + C_{AD} J_{BC} + C_{BC} J_{AD} + C_{BD} J_{AC} \\
[J_{\mu\nu}, J_{AB}] &= 0 \\
[J_{AB}, J_{\mu C}] &= C_{AC} J_{\mu B} + C_{BC} J_{\mu A} \\
[J_{AB}, P_\mu] &= 0 \\
[J_{AB}, P_C] &= C_{AC} P_B + C_{BC} P_A
\end{aligned} \tag{15}$$

When $m \to 0$ we get back our superalgebra. The transformation between our generators and those of $D(2|1;\alpha)$ listed above is

$$\begin{aligned}
Q^1_\mu &= \frac{1}{2}(J_\mu + \frac{P_\mu}{m}) \\
Q^2_\mu &= \frac{1}{2}(J_\mu - \frac{P_\mu}{m}) \\
Q^3_\mu &= \frac{i}{4}(\sigma_\mu C)_{AB} J_{AB} \\
R_{ABC} &= \pm\frac{1}{2}\sqrt{\frac{\alpha_1 - \alpha_2}{m}}\, i(C\sigma^\mu)_{AB} J_{\mu C} \mp \frac{\alpha_1 + \alpha_2}{2\sqrt{m(\alpha_1 - \alpha_2)}} C_{AB} P_C
\end{aligned} \tag{16}$$

This contraction transformation is indeed singular when $m \to 0$. However, as we have seen, the *algebra* itself is well defined in this limit, where it reduces to ours.

What is the relation between this large graded Lie algebra and 3-dimensional Chern-Simons theory? Let us first fix notation. We define

$$S_{CS}[A_\mu] = \frac{k}{4\pi}\int d^3x Tr\{\epsilon_{\mu\nu\rho}(A^\mu\partial^\nu A^\rho + \frac{2}{3}A^\mu A^\nu A^\rho)\} \tag{17}$$

where k is restricted to be an integer (since otherwise the exponential of the action is not invariant under large gauge transformations). As in the perturbative treatment[6], it is convenient to introduce a more conventional coupling constant $g^2 \equiv 4\pi k^{-1}$ and then absorb it partially by a rescaling of the gauge potential, $A_\mu \to gA_\mu$. After gauge fixing into Landau gauge ($\partial_\mu A^\mu = 0$) the resulting action is of the form $S = S_{CS}[A_\mu] + S_{GF}[A_\mu, \bar{c}, c, b]$:

$$S = \int d^3x Tr\{\epsilon_{\mu\nu\rho}(A^\mu\partial^\nu A^\rho + \frac{2}{3}gA^\mu A^\nu A^\rho) + \bar{c}\partial_\mu D^\mu c - b\partial_\mu A^\mu\} \tag{18}$$

where the trace is taken over the fundamental representation of the group ($SU(N)$ for definiteness). With t^a being the generators of $SU(N)$, the trace is normalized by $Tr(t^a t^b) = \frac{1}{2}\delta^{ab}$. The covariant derivative is, on account of the shift $A_\mu \to gA_\mu$, defined by $D_\mu \equiv \partial_\mu + g[A_\mu, \;]$. To avoid unnecessary notational inconveniences, we have restricted ourselves to a manifold with flat metric $g_{\mu\nu}$. Otherwise, although the theory is still generally covariant, explicit $detg$ factors appear in the gauge fixing terms.

The full action (18) is of course invariant under the ordinary BRST symmetry

$$\begin{aligned} \delta A_\mu &= \epsilon D_\mu c \\ \delta c &= -\epsilon\frac{1}{2}[c,c] \\ \delta\bar{c} &= \epsilon b \\ \delta b &= 0 \end{aligned} \qquad (19)$$

as well as the anti-BRST symmetry

$$\begin{aligned} \bar{\delta} A_\mu &= \bar{\epsilon} D_\mu \bar{c} \\ \bar{\delta}\bar{c} &= -\bar{\epsilon}\frac{1}{2}[\bar{c},\bar{c}]r \\ \bar{\delta} c &= -\bar{\epsilon} b - \bar{\epsilon}[\bar{c},c] \\ \bar{\delta} b &= [b,\bar{c}] \end{aligned} \qquad (20)$$

The surprising observation of ref.[3] is that in addition the action (13) is found to be invariant under the following transformations:

$$\begin{aligned} \delta' A_\mu &= -\epsilon_{\mu\nu\rho}\epsilon^\nu \partial^\rho c \\ \delta' c &= 0 \\ \delta'\bar{c} &= \epsilon^\mu A_\mu \\ \delta' b &= -\epsilon^\mu D_\mu c \end{aligned} \qquad (21)$$

and a similar set of transformations corresponding to the anti-BRST invariance[9]:

$$\begin{aligned} \bar{\delta}' A_\mu &= \epsilon_{\mu\nu\rho}\bar{\epsilon}^\nu \partial^\rho \bar{c} \\ \bar{\delta}' c &= \bar{\epsilon}^\mu A_\mu \\ \bar{\delta}'\bar{c} &= 0 \\ \bar{\delta}' b &= \bar{\epsilon}^\mu D_\mu \bar{c} \end{aligned} \qquad (22)$$

These transformations involve anticommuting vector parameters ϵ_μ and $\bar{\epsilon}_\mu$, and one is naturally led to the idea that this new invariance could be related to the $IOSp(3|2)$ algebra, - either in its usual form, or in the modified form discussed earlier. If true, the only possibility would be to identify the generator of (21) with $J_{\mu+}$ and that of (22) with $J_{\mu-}$ (the choice of + or - being a matter of convention). To see if this first step in identifying the algebra is a consistent assumption, we first compute the anticommutator $\{J_{\mu+}, J_{\nu-}\}$ using the above identification, and indeed we find that $\{\bar{\delta}', \delta'\}[A_\mu, \bar{c}, c, b] = \epsilon_\mu\bar{\epsilon}_\nu\epsilon^{\mu\nu\rho}\partial_\rho[A_\mu, \bar{c}, c, b]$, which precisely corresponds to eq.(12). This holds only when one uses the equations of motion, so the symmetry can at most be realized on-shell.

Next, we identify P_A with the BRST and anti-BRST generators. In this respect we differ from related work on $IOSp$ symmetries by Siegel and Zwiebach[10], who instead identify the BRST generators with a light cone component of $J_{\mu A}$. Now all other (anti)commutation relations of eqs.(1)-(13) involving $J_{\mu A}, J_{\mu\nu}, P^\mu$ and P^A are easily seen to be satisfied as well. In a few instances the equations of motion must be used.

It now only remains to identify the generators J_{AB}. Since they involve rotations in the anticommuting space of coordinates and refer only to the theory after gauge fixing, we expect them to act trivially on the A_μ field. Ghost number counting gives us yet another hint of what kind of symmetry it should be, since the ghost number of $c, \bar{c}$ and b have already been fixed (at -1,1 and 0, respectively). Combining this with the constraints imposed by the scaling dimensions of these fields, we find after a little experimentation:

$$\begin{aligned} \Delta A_\mu &= 0 \\ \Delta c &= \epsilon\bar{c} \\ \Delta\bar{c} &= 0 \\ \Delta b &= \epsilon\frac{1}{2}g[\bar{c},\bar{c}] \end{aligned} \qquad (23)$$

as well as

$$\begin{aligned} \bar{\Delta} A_\mu &= 0 \\ \bar{\Delta} c &= 0 \\ \bar{\Delta}\bar{c} &= \bar{\epsilon} c \\ \bar{\Delta} b &= \bar{\epsilon}\frac{1}{2}g[c,c] \end{aligned} \qquad (24)$$

The commutator of these transformations yields yet another symmetry, which turns out to be generated by the ghost number charge Q_c:

$$\begin{aligned} \tilde{\Delta} A_\mu &= 0 \\ \tilde{\Delta} c &= -\tilde{\epsilon} c \\ \tilde{\Delta}\bar{c} &= \tilde{\epsilon}\bar{c} \\ \tilde{\Delta} b &= 0 \end{aligned} \qquad (25)$$

We identify the generator of (23) with J_{++}, the generator of (24) with J_{--}, and finally the generator of (25) with J_{+-}. In fact, one could have guessed this from the beginning, since these generators simply satisfy the $Sp(2)$ algebra

$$[\sigma^+, \sigma^-] = \sigma^0 \;;\;\; [\sigma^0, \sigma^\pm] = \pm 2\sigma^\pm \qquad (26)$$

of general gauge theories in Landau gauge[13].

Actually, Landau gauge seems to be rather special also at this point. As one can easily check, there is a different invariance:

$$\begin{aligned} \Delta A_\mu &= 0 \\ \Delta c &= 0 \\ \Delta\bar{c} &= \epsilon\partial^\mu A_\mu \\ \Delta b &= \epsilon\partial^\mu D_\mu c \end{aligned} \qquad (27)$$

and the related anti-BRST type symmetry obtained basically by replacing the ghost by its antighost.

Note that this "equation of motion symmetry" (it reduces to the identity on-shell) holds for *any Yang-Mills theory gauge fixed to Landau gauge, in any number of dimensions.* There is clearly an infinite set of such symmetries, all reducing to the identity on shell. Very similar kinds of equation of motion symmetries have been found for particle, superparticle and string actions[14]. However, the generators of this type of symmetry can *not* be identified with J_{AB}.

It is now straightforward to confirm that including the former identifications of J_{AB} (eqs.(23)-(25)), we indeed have a full set of generators of the supe r Lie algebra defined by eqs.(1)-(11), with the modifications (12) and (13). To summarize, the identifications are the following:

$$\begin{aligned} P_+ &\longleftrightarrow \delta \\ P_- &\longleftrightarrow \bar{\delta} \\ J_{\mu+} &\longleftrightarrow \delta' \\ J_{\mu-} &\longleftrightarrow \bar{\delta}' \\ J_{++} &\longleftrightarrow \Delta \\ J_{--} &\longleftrightarrow \bar{\Delta} \\ J_{+-} &\longleftrightarrow \tilde{\Delta} \end{aligned}$$

A number of questions obviously still remain to be answered. What is the deeper reason for the appearance of this large symmetry group for this particular theory, and this particular gauge? Can it be extended to other theories or to other gauges? Can it be made to close off-shell by additional auxiliary fields (without changing the theory)? It would also be interesting to understand the consequences of this extra symmetry. Is it perhaps responsible for the perturbative infrared-finiteness[4] of Chern-Simons theory in Landau gauge?

In fact very similar symmetries do appear in other gauge theories as well. As a trivial example, consider pure (free) QED given by the action

$$S = \int d^dx[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{c}\partial^2 c + b\partial_\mu A^\mu + \frac{1}{2}\alpha b^2] \tag{28}$$

Apart from the usual BRST and anti-BRST invariance this theory in $\alpha = -1$ gauge is also invariant under

$$\begin{aligned} \delta A_\mu &= -\epsilon_\mu c \\ \delta c &= 0 \\ \delta \bar{c} &= \epsilon_\mu A^\mu \\ \delta b &= -\epsilon_\mu \partial^\mu c \end{aligned} \tag{29}$$

and a similar symmetry with the ghost replaced by the antighost. A systematic analysis of such accidental symmetries of Yang-Mills theories in certain gauges, and their consequences in terms of further constraints on the usual Ward Identities, will be published elsewhere.

Finally, one obvious question is whether the extra symmetry of Chern-Simons theory, which has been demonstrated at the classical level only, can be anomalous. There have in fact been indications that this might be the case[6], but a recent analysis[15] has shown that at least the subalgebra (18)-(19) is non-anomalous.[1] It thus appears very unlikely that the full global symmetry group discussed here will be broken by quantum fluctuations.

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[1]See also the recent study of scale invariance in this theory[16].

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