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COMMENTS ON "BRST QUANTIZATION OF THE EXTENDED SUPERSYMMETRIC SPINNING PARTICLE"

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COMMENTS ON "BRST QUANTIZATION OF THE EXTENDED SUPERSYMMETRIC SPINNING PARTICLE"

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Abstract

We show that when properly regularized the divergences appearing in the Feynman kernel of the extended supersymmetric spinning particle do not impose any restriction on the number of supersymmetries and the space-time dimension (for even space-time dimensions).

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In a recent paper Qiong-gui Lin and Guang-jiong Ni [1] calculated the Feynman kernel for the relativistic spinning-particle with N-extended supersymmetries using the BRST path-integral formalism of Batalin-Fradkin-Vilkovisky [2].The kernel obtained by the authors diverges in even-dimensional space-times for every N except for $N = 2$. Hence they concluded that in particular for $d = 4$ a massless particle can only have spin 1 in contradiction with former results for spin 0 and $\frac{1}{2}$ [3] [4] and also in contradiction with the results obtained by the Dirac quantization of the theory [5].

The purpose of this comment is to point out that when properly treated the path integral quantization brings no constraints on N (for even dimensions) and therefore agrees with the former results. A complete exposition of the BRST-BFV quantization of the spinning particle with N-extended supersymmetries can be found in [6].

It is well known that several functional integrals diverge and what we have to do is to use a technique for obtaining their finite values. We therefore have to adopt some regularization prescription. Sometimes this procedure breaks the classical symmetries of the action giving rise to anomalies. We will demonstrate that the divergence of the kernel obtained in ref.[1] can be eliminated in this way and does not give rise to any anomaly. Then the kernel makes sense for every value of N contrary to what is claimed by the authors of ref.[1].

The kernel obtained by Qiong-gui Lin and Guang-jiong Ni [1] is:

$$Z = A\frac{i}{p_1^2 + i\epsilon}\delta(p_2 - p_1)\Big\langle\bar{\eta}_2\Big|\prod_i p_{1\mu}\lambda_i^\mu \prod_{i<j}\delta(M_{ij})\Big|\eta_1\Big\rangle \tag{1}$$

where λ_i^μ are Grassman variables satisfying $\{\lambda_i^\mu, \lambda_j^\nu\} = -i\delta_{ij}\eta^{\mu\nu}$, $M_{ij} = i\lambda_i^\mu\lambda_{j\mu}$ and

$$A = \prod_{n=1}^{\infty} n^{f(N,d)} \tag{2}$$

$$f(N,d) = \frac{1}{2}N^2 + \frac{(d-2)}{2}N - (d-1)$$

The coefficient A clearly diverges for $f(N,d) > 0$ because $n^{f(N,d)}$ increases without bound. In order to remove this divergence we evaluate the infinity product in (2) using the zeta function regularization.

$$\prod_{n=1}^{\infty} n^{f(N,d)} = \prod_{n=1}^{\infty} \exp\log(n^{f(N,d)}) = \exp\sum_{n=1}^{\infty} f(N,d)\log n =$$
$$= \exp[f(N,d)\sum_{n=1}^{\infty}\log n] = exp[-f(N,d)\lim_{s\to 0}\frac{d}{ds}\sum_{m=i}^{\infty} m^{-s}] \tag{3}$$

In the last term in (3) the expression multiplying the function $f(N,d)$ is the derivative of the zeta function $\zeta(s)$ at $s=0$. Since $\zeta'(0) = -\frac{1}{2}\ln 2\pi$ we finally have:

$$\prod_{n=1}^{\infty} n^{f(N,d)} = (2\pi)^{\frac{f(N,d)}{2}} \tag{4}$$

which is just a finite numerical factor for any value of N and d and can be absorved in the normalization factor of the kernel.

We can give another argument to show that the factor (2) really can be absorved by an overall normalization. In our work [6] we obtained some indetermined determinants like $\det \partial_t$ (without enough boundary conditions to determine their eigenvalues). We verified that their appearance depends on the order of integration of the several variables of the theory. Also they do not depend on the variables of the theory. This points out that they have no meaning and justify their absortion in the overall normalization of the kernel. Now, making a comparison with our work we observe that the factor (2) is nothing but our indetermined determinants.

The next point of interest that we would like to comment is on the gauge-fixing function used by the authors. As we have shown in our paper a gauge choice compatible with the local $O(N)$ symmetry is $f_{ij} = 0$ or $f_{ij} = constant$ but not $\dot{f}_{ij} = 0$ which is implicitly adopted in ref.[1]. The reason for this is that the local $O(N)$ symmetry is an internal symmetry and the action is invariant without any boundary conditions on the parameters of this symmetry. With our gauge choices we reproduce the results obtained

by the Dirac quantization. Finally we can give a physical interpretation for the Feynman kernel, what Qiong-gui Lin and Guang-jiong Ni [1] cannot achieve using the gauge $\dot{f}_{ij} = 0$.

Our final result is that for even dimensions the transition amplitude is given by

$$Z = \int dp\ e^{ip\Delta x} \prod_{i=1}^{N} \frac{(p\cdot\gamma_i)}{p^2 + i\epsilon}$$

The γ_i can be represented in terms of the Dirac gamma matrices γ_μ and we can identify the path-integral with the propagator of the field strength which describes fields with spin $N/2$. For more details see ref.[6] where we extensively discuss this approach to the problem.

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