UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
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M.C. Nemes, M. Nielsen and **M.M. de Oliveira**
Instituto de Física, Universidade de São Paulo

J. da Providência
Centro de Física Teórica (INIC), Universidade de Coimbra
P-3000 Coimbra, Portugal

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M. C. NEMES, M. NIELSEN and M. M. de OLIVEIRA

Instituto de Física, Universidade de São Paulo Caixa Postal 20516 - 01498 São Paulo, Brazil

J. da PROVIDÊNCIA[1]

Centro de Física Teórica (INIC), Universidade de Coimbra, P–3000 Coimbra, Portugal

Abstract: A semiclassical version of the linear σ model is studied in a variational approach for the generators of the mesonic excitations. The method offers a convenient framework to study the mesonic spectroscopy for both bound states and resonances. The strength distribution of the pionic mode is obtained.

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The linear σ model is a field theoretical model originally introduced by Gell - Mann and Lévy [1] as an example of a phenomenological model which realizes one important characteristic feature of quantum cromodynamics: chiral symmetry and partial conservation of the axial current. It is generally used to describe low lying hadronic properties of the barinic sector [2,3], whereas the mesonic sector is generally described by models such as the Nambu Jona-Lasinio (NJL) model [4,5], where mesons are seen as collective quark-antiquark ($q\bar{q}$) modes built on the non-perturbative vacuum.

A natural question which arises at this point concerns to the possibility of describing both sectors with the same model. In ref.[6] it is shown that the chiral soliton model can be considered as an approximation to the Hartree solution (zero-boson and one-fermion loop) of the NJL model. In such case, the σ model would also be appropriate to describe the mesonic sector.

The aim of the present paper is twofold: First the study of the mesonic sector of the chiral linear σ model. In particular in this model the physical pion must be understood as composed of two very distinct ingredients contained in the model: a) the structureless pion field term which gives rise to the Goldstone boson; b) a collective $q\bar{q}$ component built on the non-perturbative vacuum. The $q\bar{q}$ part of the pion is the part which couples to the axial current which is bilinear in the quark fields. Second, we consider excitations of $q\bar{q}$ pairs whose energy lie in the continuum. The mesons in the continuum show up as $q\bar{q}$ resonances with a certain spreading width. The positions of the center of gravity of those resonances can be compared with the experimental meson masses.

We will restrict ourselves to the time dependent Hartree-Fock approach in the small amplitude limit of the mean-field description. Therefore we will obtain equations which describe the dynamics of the collective modes in the framework of the random phase approximation (RPA). Up to now mainly the discrete low-lying modes (bound states), i.e., excitations whose energies satisfy the condition $\omega \leq 2M$ where $2M$ is the threshold of the $q\bar{q}$continuum, of the RPA like equations have been considered. However the collectivity of the RPA modes is a characteristic of the bound states as well as the mesonic excitations in the $q\bar{q}$ continuum. The mesons in the continuum appear naturally as $q\bar{q}$ resonances.

Previous analysis of the continuum have been made in the NJL model [7] and in a semiclassical approach in the scalar plasma model [8] and have shown good agreement with the experimental spectrum. Ref.[8] deals with a continuum of particle-hole states in hadronic matter (both the particle and the hole refer to positive energy states, above and below the Fermi level, respectively) while in ref.[7] $q\bar{q}$ states are produced by creating a particle in a + energy state and anihilating another particle in a − energy state. In the present paper we study the pion excitationin the continuum in the context of the linear σ model. We determine the strength distribution of that mode and the respective sum rule.

In a semiclassical realization of the linear σ model, the scalar field σ and the pseudoscalar-isovector field $\vec{\Psi}$ are classical fields. The effective hamiltonian describing a system of N fermions occupying either positive or negative energy states, interacting with these classical fields is

$$H = \sum_{j=1}^{N}[\vec{p}_j.\vec{\alpha}_j + g\beta_j(\sigma(x_j) + i\gamma_5(j)\vec{\tau}_j.\vec{\Psi}(x_j))]$$

$$+\frac{1}{2}\int d^3x\,(\Pi_\sigma^2 + \vec{\nabla}\sigma\cdot\vec{\nabla}\sigma + \Pi_\Psi^2 + \sum_{i=1}^{3}\vec{\nabla}\Psi_i\cdot\vec{\nabla}\Psi_i) + \int d^3x\left(\frac{g^2}{2}(\sigma^2+\Psi^2-\sigma_0^2)^2 - c\sigma\right)$$

$$+2\xi\int\frac{d^3pd^3x}{(2\pi)^3}\sqrt{p^2+g^2(\sigma^2+\Psi^2)}\,\Theta(\Lambda^2-p^2) + \frac{\lambda-1}{2}\int d^3x\,(\Pi_\sigma^2+\Pi_\Psi^2)\,, \qquad (1)$$

where $\vec{\alpha}$, β and γ_5 are the usual Dirac matrices, $\vec{\tau}$ correspond to the matrices of the fundamental flavor representation SU(2),Π_σ and $\vec{\Pi}_\Psi$ are the conjugate momenta associated with the classical fields σ and $\vec{\Psi}$ respectively .The constants c and σ_0 are given by

$$c = m_\pi^2 M_0/g \qquad (2.a)$$

$$g^2\sigma_0^2 = M_0^2 - \frac{m_\pi^2}{2} \qquad (2.b)$$

so that the Goldberger-Treiman relation on the quark level (g_A(quark)=1) : $f_\pi = M_0/g$ is fulfilled and the σ field expectation value in the vacuum gives the constituent quark mass : $\sigma = M_0/g$. In equation (2) m_π is the phenomenological pion mass. The last two terms in eq. (1) are renormalization terms, which depend on a cutoff parameter Λ. These terms insure that all physical quantities are independent of Λ in the limit $\Lambda \to \infty$. The second renormalization term which contains the parameter λ allows for the definition of the scalar meson mass in the vacuum. The factor ξ stands for the degeneracy of the system and will be taken equal to six (we shall be considering three colours).

The ground state of the model is described by the density matrix ρ_0, which is determined variationally. In quark homogeneous matter we can write

$$\rho_0 = \frac{1}{2}\left(I + \frac{\vec{p}.\vec{\alpha}+\beta M}{\sqrt{p^2+M^2}}\right)\Theta(P_F^2-p^2) + \frac{1}{2}\left(I - \frac{\vec{p}.\vec{\alpha}+\beta M}{\sqrt{p^2+M^2}}\right)\theta(\Lambda^2-p^2)\,. \qquad (3)$$

Here M is a variational parameter, representing the quark mass at the momentum P_F, to be fixed by the usual energy minimization procedure.It gives:

$$M = g\sigma\,, \qquad (4.a)$$

$$\Psi_i = 0\,, \qquad (4.b)$$

and the self consistency equation is

$$2(M^2 - M_0^2 + \frac{m_\pi^2}{2})M - M_0 m_\pi^2 = -\frac{2\xi g^2 M}{(2\pi)^3}\int d^3p\frac{\theta(P_F^2-p^2)}{\sqrt{p^2+M^2}}\,. \qquad (5)$$

The time evolution of a density matrix $\rho(t)$ corresponding to a Slater determinant slightly displaced from equilibrium can be written as

$$\rho(t) = e^{iS(t)}\rho_0 e^{-iS(t)} \qquad (6)$$

where $S(t)$ is a hermitian, one body, time dependent operator. For small amplitude fluctuations, it is sufficient to consider the effect of $S(t)$ up to second order. Due to the coupling terms in the hamiltonian (eq. (1)), there will also be fluctuations in the scalar and pseudoscalar classical fields: $\delta\sigma$ and $\delta\Psi_i$, which are non vanishing and time dependent. Consistently we shall treat these terms only up to second order.

The Lagrangian describing small amplitude oscillations around the equilibrium state is [9]

$$L = \frac{i}{2}tr(\rho_0[S,\dot{S}]) - \frac{1}{2}tr(\rho_0[S,[h_0,S]]) - itr(\rho_0[S,\delta h]) + \frac{\Omega}{2}\left(\frac{(\delta\dot{\sigma})^2}{\lambda} + \frac{(\delta\dot{\Psi}_i)^2}{\lambda}\right)$$

$$-\Omega((3M^2 - M_0^2 - \frac{m_\pi^2}{2})(\delta\sigma)^2 + (M^2 - M_0^2 - \frac{m_\pi^2}{2})(\delta\Psi_i)^2) - \xi g^2\int\frac{d^3pd^3x}{(2\pi)^3}\left(\frac{p^2}{\epsilon}(\delta\sigma)^2 - \frac{1}{\epsilon}(\delta\Psi_i)^2\right) \quad (7)$$

where

$$h_0 = \vec{p}.\vec{\alpha} + \beta M\,, \qquad (8.a)$$

$$\delta h = \beta g\delta\sigma + ig\beta\gamma_5\vec{\tau}.\delta\vec{\Psi}\,, \qquad (8.b)$$

$$\epsilon = \sqrt{p^2+M^2}\,, \qquad (8.c)$$

and Ω is the renormalization volume.

The generators of the scalar and pseudoscalar homogeneous excitations for zero momentum transfer are respectively given by

$$S_\sigma = \vec{p}.\vec{\alpha}\Phi_1(p^2,t) + i\beta\vec{p}.\vec{\alpha}\Phi_2(p^2,t)\,, \qquad (9)$$

$$S_\Psi = i\beta\gamma_5\vec{\tau}.\vec{S}_1(p^2,t) + \gamma_5\vec{\tau}.\vec{S}_2(p^2,t)\,. \qquad (10)$$

Inserting eq.(10) in eq.(7) and using the dimensionless quantities :

$$\vec{Q} = \delta\vec{\Psi}/M_0 \qquad (11.a)$$

$$\vec{P} = \vec{\Pi}_\psi / M_0^2 = \delta\dot{\vec{\Psi}}/\lambda M_0^2 \tag{11.b}$$

$$x = \epsilon^2 / M_0^2 \tag{11.c}$$

$$m = M/M_0 \tag{11.d}$$

$$f(x) = \frac{\xi}{2\pi^2}\sqrt{\frac{x - m^2}{x}} \tag{11.e}$$

we get

$$\frac{L_\psi}{M_0^4\Omega} = \frac{1}{2}(\vec{P^*}.\dot{\vec{Q}} - \dot{\vec{P}}.\vec{Q^*}) - m\int_{x_F}^{x_\Lambda} dx f(x)(\vec{S}_2^*.\dot{\vec{S}}_1 - \vec{S}_1^*.\dot{\vec{S}}_2)$$

$$-2\int_{x_F}^{x_\Lambda} dx f(x)(m^2 \mid \vec{S}_2 \mid^2 + \mid \vec{S}_1 \mid^2) + gm\vec{Q^*}.\int_{x_F}^{x_\Lambda} dx f(x)\vec{S}_2$$

$$+ gm\vec{Q}.\int_{x_F}^{x_\Lambda} dx f(x)\vec{S}_2^* - \frac{1}{2}(\lambda \mid \vec{P} \mid^2 + 4\alpha^2 \mid \vec{Q} \mid^2) \tag{12}$$

where all time derivatives are relates to $\tau = M_0 t$ and

$$4\alpha^2 = 2(m^2 - 1 + \frac{m_\pi^2}{2M_0^2}) + g^2\int_{m^2}^{x_\Lambda} dx f(x) . \tag{13}$$

The Euler - Lagrange equations are

$$\dot{\vec{Q}} - \lambda\vec{P} = 0 , \tag{14.a}$$

$$\dot{\vec{P}} + 4\alpha^2\vec{Q} - 2mg\int_{x_F}^{x_\Lambda} dx f(x)\vec{S}_2(x) = 0 , \tag{14.b}$$

$$m\dot{\vec{S}}_2(x) - 2x\vec{S}_1(x) = 0 , \tag{14.c}$$

$$\dot{\vec{S}}_1(x) - g\vec{Q} + 2m\vec{S}_2(x) = 0 . \tag{14.d}$$

The ansatz

$$\begin{bmatrix} \vec{Q}(\tau) \\ \vec{P}(\tau) \\ \vec{S}_1(x,\tau) \\ \vec{S}_2(x,\tau) \end{bmatrix} = \begin{bmatrix} \vec{Q}_\omega \\ \vec{P}_\omega \\ \vec{S}_{1\omega}(x) \\ \vec{S}_{2\omega}(x) \end{bmatrix} e^{i\omega\tau} \tag{15}$$

leads to the equations for the normal modes

$$i\omega\vec{Q}_\omega = \lambda\vec{P}_\omega , \tag{16.a}$$

$$i\omega\vec{P}_\omega = -4\alpha^2\vec{Q}_\omega + 2mg\int_{x_F}^{x_\Lambda} dx f(x)\vec{S}_{2\omega}(x) , \tag{16.b}$$

$$i\omega\vec{S}_{1\omega}(x) = g\vec{Q}_\omega - 2m\vec{S}_{2\omega}(x) , \tag{16.c}$$

$$i\omega\vec{S}_{2\omega}(x) = \frac{2x}{m}\vec{S}_{1\omega}(x) . \tag{16.d}$$

There are, as usual [8,10,11], two types of solutions of eqs.(16). If $\omega = \pm\omega_z$, $\omega_z^2 < 4x_F$, there are two collective modes described by

$$\vec{Q}_\pm = \vec{n} , \tag{17.a}$$

$$\vec{P}_\pm = \pm\frac{i\omega_z}{\lambda}\vec{n} , \tag{17.b}$$

$$\vec{S}_{1\pm}(x) = \pm\frac{i\omega_z g}{4}\frac{\vec{n}}{x - \omega_z^2/4} , \tag{17.c}$$

$$\vec{S}_{2\pm}(x) = \frac{gx}{2m}\frac{\vec{n}}{x - \omega_z^2/4} , \tag{17.d}$$

where ω_z is given by the dispersion relation for "zero sound"

$$\frac{\omega_z^2}{4\lambda} - \alpha^2 + \frac{g^2}{4}\int_{x_F}^{x_\Lambda} dx f(x)\frac{x}{x - \omega_z^2/4} = 0 \tag{18}$$

and $\vec{n}$ is an arbitrary unit isovector.

If, on the other hand, $4x_F < \omega^2 < 4x_\Lambda$, there is a continuum of solutions:

$$\vec{Q}_\omega = -\frac{2m}{g}a(\omega^2/4)\vec{n} , \tag{19.a}$$

$$\vec{P}_\omega = -\frac{i\omega}{\lambda}\frac{2m}{g}a(\omega^2/4)\vec{n} , \tag{19.b}$$

$$\vec{S}_{1\omega}(x) = i\omega\frac{m}{2x}\vec{S}_{2\omega}(x) , \tag{19.c}$$

$$\vec{S}_{2\omega}(x) = \left(\delta(\omega^2/4 - x) + \frac{x a(\omega^2/4)}{\omega^2/4 - x}\right)\vec{n} , \tag{19.d}$$

where $a(\omega^2/4)$ satisfies the equation

$$a(\omega^2/4) = \frac{g^2 f(\omega^2/4)}{\omega^2/\lambda - 4\alpha^2 + g^2\int_{x_F}^{x_\Lambda} dx f(x)\frac{x}{x-\omega^2/4}} . \tag{20}$$

In eq.(20) and in what follows, integrals involving the factor $1/(x - \omega^2/4)$ have to be interpreted as principal value integrals.

From eqs.(16) we can show that solutions defined in eqs.(17) and (19) satisfy the following orthogonality relations

$$i\left(\vec{P}_{\omega'}^*.\vec{Q}_\omega - \vec{Q}_{\omega'}^*.\vec{P}_\omega - 2m\int_{x_F}^{x_\Lambda} dx f(x)(\vec{S^*}_{2\omega'}.\vec{S}_{1\omega} - \vec{S^*}_{1\omega'}.\vec{S}_{2\omega})\right) = m^2\frac{f(\omega^2/4)}{\omega^2/4}\delta(\omega/2 - \omega'/2) , \tag{21.a}$$

$$i\left(\vec{Q}_\pm.\vec{P^*}_\omega - \vec{P}_\pm.\vec{Q^*}_\omega - 2m\int_{x_F}^{x_\Lambda} dx f(x)(\vec{S}_{1\pm}.\vec{S^*}_{2\omega} - \vec{S}_{2\pm}.\vec{S^*}_{1\omega})\right) = 0 , \quad (21.b)$$

$$i\left(\vec{Q}_\pm.\vec{P^*}_\pm - \vec{P}_\pm.\vec{Q^*}_\pm - 2m\int_{x_f}^{x_\Lambda} dx f(x)(\vec{S}_{1\pm}.\vec{S^*}_{2\pm} - \vec{S}_{2\pm}.\vec{S^*}_{1\pm})\right) = \pm\eta , \quad (21.c)$$

where

$$\eta = 2\omega_z\left(\frac{1}{\lambda} + \frac{g^2}{4}\int_{x_F}^{x_\Lambda} dx f(x)\frac{x}{(x-\omega_z^2/4)^2}\right) . \quad (22)$$

It has been shown by van Kampem [12], for the electron plasma, and in ref.[10] for the nuclear case, that this set of solutions is complete. This means that given an arbitrary initial state

$$\Psi_0 = \begin{bmatrix} \vec{Q}(0) \\ \vec{P}(0) \\ \vec{S}_1(x,0) \\ \vec{S}_2(x,0) \end{bmatrix} = \begin{bmatrix} \vec{Q}_0 \\ \vec{P}_0 \\ \vec{H}_1(x) \\ \vec{H}_2(x) \end{bmatrix} \quad (23)$$

there is a function $c(\omega)$ and numbers C_+, C_- such that

$$\begin{bmatrix} \vec{Q}_0 \\ \vec{P}_0 \\ \vec{H}_1(x) \\ \vec{H}_2(x) \end{bmatrix} = \int_{2\sqrt{x_F}}^{2\sqrt{x_\Lambda}} c(\omega)\begin{bmatrix} \vec{Q}_\omega \\ \vec{P}_\omega \\ \vec{S}_{1\omega}(x) \\ \vec{S}_{2\omega}(x) \end{bmatrix} d\omega + C_+\begin{bmatrix} \vec{Q}_+ \\ \vec{P}_+ \\ \vec{S}_{1+}(x) \\ \vec{S}_{2+}(x) \end{bmatrix} + C_-\begin{bmatrix} \vec{Q}_- \\ \vec{P}_- \\ \vec{S}_{1-}(x) \\ \vec{S}_{2-}(x) \end{bmatrix} . \quad (24)$$

Following van Kampen [12] we get

$$C_\pm = \pm\frac{i}{\eta}\left(\vec{P}^*_\pm.\vec{Q}_0 - \vec{Q}^*_\pm.\vec{P}_0 - 2m\int_{x_F}^{x_\Lambda} dx f(x)(\vec{H}_1(x).\vec{S}^*_{2\pm}(x) - \vec{H}_2(x).\vec{S}^*_{1\pm}(x))\right) , \quad (25)$$

$$c(\omega) = \frac{\tilde{c}(\omega)}{1+\pi^2 a^2(\omega^2/4)\omega^2/4} , \quad (26)$$

$$\tilde{c}(\omega) = \frac{i\omega^2/4}{4m^2 f(\omega^2/4)}\left(\vec{Q}_0.\vec{P}^*_\omega - \vec{P}_0.\vec{Q}^*_\omega - 2m\int_{x_F}^{x_\Lambda} dx f(x)(\vec{H}_1(x).\vec{S}^*_{2\omega}(x) - \vec{H}_2(x).\vec{S}^*_{1\omega}(x))\right) . \quad (27)$$

The unexpected factor in the denominator of eq.(26) appears because of the singularities in $\vec{S}_{1\omega}(x)$ and $\vec{S}_{2\omega}(x)$.

The solution of the initial value problem satisfyng both eq.(14) and the initial condition eq.(23) is therefore

$$\begin{bmatrix} \vec{Q}(\tau) \\ \vec{P}(\tau) \\ \vec{S}_1(x,\tau) \\ \vec{S}_2(x,\tau) \end{bmatrix} = \int_{2\sqrt{x_F}}^{2\sqrt{x_\Lambda}} c(\omega)\begin{bmatrix} \vec{Q}_\omega \\ \vec{P}_\omega \\ \vec{S}_{1\omega}(x) \\ \vec{S}_{2\omega}(x) \end{bmatrix} e^{i\omega\tau}d\omega + C_+\begin{bmatrix} \vec{Q}_+ \\ \vec{P}_+ \\ \vec{S}_{1+}(x) \\ \vec{S}_{2+}(x) \end{bmatrix} e^{i\omega_z\tau} + C_-\begin{bmatrix} \vec{Q}_- \\ \vec{P}_- \\ \vec{S}_{1-}(x) \\ \vec{S}_{2-}(x) \end{bmatrix} e^{-i\omega_z\tau} . \quad (28)$$

The use of sum rules is a complementary approach to the use of equations of motion when one is looking at the collective states microscopically . Following ref.[11], we find that the amplitudes $\tilde{c}(\omega), C_+$ and C_- satisfy the following energy wheighted sum rule (EWSR):

$$4m^2\int_{2\sqrt{x_F}}^{2\sqrt{x_\Lambda}} d\omega\frac{|\tilde{c}(\omega)|^2 f(\omega^2/4)}{\omega/2(1+\pi^2a^2(\omega^2/4)\omega^2/4)} + \frac{\omega_z}{2}\eta\left(|C_+|^2+|C_-|^2\right) = \frac{1}{2}\left(\lambda|\vec{P}_0|^2 + 4\alpha^2|\vec{Q}_0|^2\right)$$

$$+2\int_{x_F}^{x_\Lambda} dx f(x)(m^2|\vec{H}_2|^2 + x|\vec{H}_1|^2) - gm\vec{Q}_0.\int_{x_F}^{x_\Lambda} dx f(x)\vec{H}^*_2(x) - gm\vec{Q}^*_0.\int_{x_F}^{x_\Lambda} dx f(x)\vec{H}_2(x) . \quad (29)$$

The strength function representing the pionic mode in the $q\bar{q}$ continuum is

$$s_\pi(\omega) = \frac{4m|\tilde{c}(\omega)|^2 f(\omega^2/4)}{\omega/2(1+\pi^2a^2(\omega^2/4)\omega^2/4)} . \quad (30)$$

The simplest example of initial condition which favours the mode in the continuum is [8]

$$\Psi_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vec{n} \end{bmatrix}$$

for this condition, the strength function in the vacuum ($P_F = 0, m = 1$) becomes

$$s_\pi(\omega) = \frac{\omega/2 f(\omega^2/4)(m_\pi/2M_0)^4}{\left(\frac{g^2}{4}\frac{\omega^2}{4}\int_1^{x_\Lambda} dx\frac{f(x)}{\omega^2/4-x} + \left(\frac{m_\pi}{2M_0}\right)^2 - \frac{\omega^2}{4\lambda}\right)^2 + \pi^2\frac{g^4}{16}\frac{\omega^2}{4}f(\omega^2/4)} \quad (31)$$

and, from eq.(29) the EWSR is

$$m_1 = 2\int_1^{x_\Lambda} dx f(x) . \quad (32)$$

The fraction of the EWSR exhausted by the discrete frequency $\pm\omega_z$, in this case, is

$$\%\omega_z = \frac{4\omega_z^3}{\eta g^2}\left(\left(\frac{m_\pi}{M_0\omega_z}\right)^2 - \frac{1}{\lambda}\right)^2\frac{1}{m_1} , \quad (33)$$

and ω_z is given by eq. (18)that we can rewrite as

$$\omega_z^2\left(\frac{1}{\lambda} + \frac{g^2}{4}\int_1^{x_\Lambda} dx\frac{f(x)}{x-\omega_z^2/4}\right) = \left(\frac{m_\pi}{M_0}\right)^2 . \quad (34)$$

At his point we may ask what is the value of m_π. We must remember that m_π represents the mass of the phenomenological structureless pion related to the pseudoscalar isovector field $\vec{\Psi}$. The mass of the physical pion is ω_z and we will choose m_π such that it gives $\omega_z = 138MeV/M_0$.

As said before, the parameter λ allows for the definition of the scalar meson mass in the vacuum, and if we choose $m_\sigma = 2M_0$ (a known result of NJL model [9]) we get [13]

$$\frac{1}{\lambda} = 1 - \frac{g^2}{4}\int_1^{x_\Lambda} dx\frac{f(x)}{x} . \quad (35)$$

We are now in a position to derive numerical results. In order to reproduce the experimental pion decay constant, for instance $f_\pi = 93 MeV$, for a constituent quark mass $M_{u,d} = 320 MeV$ we get $g = 3.44$ for the coupling constant. In spite of both m_π and the maximum in $s(\omega)$ being independent of Λ in the limit $\Lambda \to \infty$, we have to work with a finite value for Λ because the result of eq.(32) is divergent. The strength function eq.(31) shown in Fig.1 was calculated with $\Lambda/M_0 = 15$. It exhibits a pronounced maximum around 2200 MeV which is of the same order of the experimental mass of the (not well established) resonance $\pi(1300)$[14], which is indicated by an arrow.

For this initial condition, 98% of the total EWSR lies in the continuum.

In conclusion, we have analyzed pionic collective modes in the σ linear model corresponding to small amplitude oscillations around a stationary state, including the continuum. We have obtained a set of stationary linear modes of excitation satisfying orthogonality and completeness relations, and the corresponding EWSR. Our calculations show that the strength distribution function of the pionic excitation in the $q\bar{q}$ continuum exhausts almoust all the EWSR for our choice of the initial state, and has a pronounced maximum of the same order of the pion resonance.

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FIGURE CAPTIONS

Fig.1- Strength function representing the pion resonance, as a function of ω. The arrow idicates the experimental mass.