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ON THE FLUXES OF THE ELECTRON

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The paper is concerned with the explicit quantization of the Coulomb field of the electron. This problem had been considered by Dirac[1], who introduced the physical field of the electron, ψ^*, multiplying the original Fermi field by a unitary operator $\exp ie V_{\vec{x}}$, that accounts for the Coulomb field mode. $V_{\vec{x}}$ is a functional of the vector potential longitudinal components.

Here I treat the problem, beginning with the quantization of fluxes[2]. I also propose a manner of introducing the magnetic field produced by the electron, and consider a mechanism of local transference of flux.

The description of the electromagnetic waves can be done with different representations for the fields. An alternative for the conventional representation in terms of the field $\vec{A}(\vec{x})$, is given by the $\vec{T}$–field, such that $\vec{E} = \vec{\nabla} \times \vec{T}$ and $\vec{B} = \dot{\vec{T}}$. As far as the description of the electromagnetic waves is concerned, both methods are equally good.

The corresponding local commutation relations in each one of these representations are

$$\left[E_i(\vec{x}) , A_j(\vec{y})\right] = i\, \delta_{ij}\, \delta(\vec{x}-\vec{y}) \quad ; \quad \text{and} \quad \left[B_i(\vec{x}) , T_j(\vec{y})\right] = -i\, \delta_{ij}\, \delta(\vec{x}-\vec{y}) \quad . \tag{1}$$

I take the electromagnetic field to be complete in the sense that this couple of commutators should hold simultaneously.

Let us consider a closed line Γ_1, and an open surface S_2 with an intersection, as it is shown in Figure 1. S_1 is a surface encircled by Γ_1, and Γ_2 is the perimetral line of S_2.

Let also $\hat{\phi}_E$ be the electric flux through surface S_2, and $\hat{\phi}_B$ the magnetic flux through surface S_1.

The flux commutation law

$$\left[\hat{\phi}_E , \hat{\phi}_B\right] = i \quad ; \quad \text{or} \quad \left[\oint_{\Gamma_2} \vec{T}\cdot d\vec{\ell} , \oint_{\Gamma_1} \vec{A}\cdot d\vec{\ell}\right] = i \quad , \tag{2}$$

belongs to both representations.

Figure 2 shows a closed surface S, and a line Γ, with an end at the point $\vec{x}$. One defines the electric flux through S, and a magnetic–like flux along Γ,

$$\phi_{E,S} = \oint_S \vec{E}\cdot d\vec{s} \quad ; \quad \text{and} \quad \theta_\Gamma(\vec{x}) = \int_\Gamma^{\vec{x}} \vec{A}\cdot d\vec{\ell} \quad , \tag{3}$$

and again, by using the local commutators (2), one can verify that

$$\left[\phi_{E,S} , \theta_\Gamma(\vec{x})\right] = -i \tag{4}$$

if $\vec{x}$ is inside S. But the commutator vanishes if $\vec{x}$ is outside the surface.

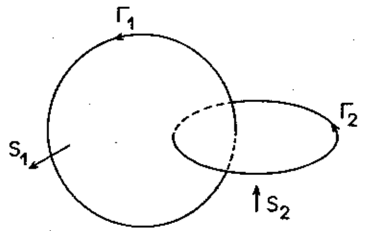

Figure 1

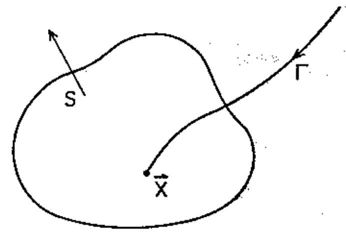

Figure 2

The Coulomb field is a single mode of the electric field, which can be created centred in a particle. Taking the commutator (4), one can verify that, for every closed surface S that contains the point $\vec{x}$ inside, the field functional

$$C^\dagger(\vec{x}) = \exp ie \int_\Gamma^{\vec{x}} \vec{A}\cdot d\vec{\ell} \tag{5}$$

is an eigenstate of the electric flux operator $\phi_{E,S}$, with eigenvalue e. However, if $\vec{x}$ is outside S, then the electric flux eigenvalue vanishes.

Now, I take a formal step that shows another aspect of the Coulomb mode $C^\dagger(\vec{x})$. The exponent $ie\,\theta_\Gamma(\vec{x})$ can be written as

$$ie\,\theta_\Gamma(\vec{x}) = i\int d^3\vec{y}\; e\,\delta(\vec{x}-\vec{y}) \int_\Gamma^{\vec{x}} \vec{A}\cdot d\vec{\ell} \quad . \tag{6}$$

If $\vec{\mathcal{E}}_c(\vec{y},\vec{x})$ is the classical electric field at the point $\vec{y}$, produced by a particle localized at the point $\vec{x}$, one can replace the delta function in Eq. (6) by $\frac{1}{e}\vec{\nabla}_{\vec{y}}\cdot\vec{\mathcal{E}}_c(\vec{y},\vec{x})$. And after integration by parts one gets

$$ie\,\theta_\Gamma(\vec{x}) = -i\int d^3\vec{y}\; \vec{\mathcal{E}}_c(\vec{y},\vec{x})\cdot\vec{A}(\vec{y}) \quad . \tag{7}$$

$C^\dagger(\vec{x})$ is therefore an eigenstate of the very electric field operator, like that unitary operator in Dirac's analysis[1].

In some of the paper's equations it is implicit that $\vec{\nabla}\int_\Gamma^{\vec{x}} \vec{F}\cdot d\vec{\ell} = \vec{F}(\vec{x})$. This assumption is made regardless of the nature of the $\vec{F}$ field.

The analysis of the Coulomb field may be supplemented with the consideration of a magnetic mode. Let $\vec{v}$ initially be a classical speed. With the help of the $\vec{T}$ field, I define the Ampère mode

$$a^\dagger(\vec{x},\vec{v}) = \exp ie \int_\Gamma^{\vec{x}} (\vec{v}\times\vec{T})\cdot d\vec{\ell} \quad . \tag{8}$$

In order to provide an immediate indication about the mode's nature, I resort again to a formal procedure, expressing the exponent in Eq. (8) in the form

$$i\int d^3\vec{y}\;\vec{\nabla}_{\vec{y}}\cdot\vec{\mathcal{E}}_c(\vec{y},\vec{x}) \int_\Gamma^{\vec{y}} (\vec{v}\times\vec{T})\cdot d\vec{\ell} = i\int d^3\vec{y}\;\vec{\mathcal{B}}_c(\vec{y},\vec{x};\vec{v})\cdot\vec{T}(\vec{y}) \tag{9}$$

where $\vec{\mathcal{B}}_c = \vec{v}\times\vec{\mathcal{E}}_c$, is the magnetic field at the point $\vec{y}$, produced by a particle moving with speed $\vec{v}$ at the point $\vec{x}$.

To construct an electron field with the flux modes manifest, one first takes a neutral Fermi field $\psi_0(\vec{x})$, which commutes with the electromagnetic field.

Then the magnetic mode must be extended to become suitable for the coupling to a relativistic particle. I introduce the generalized Ampère operator

$$A^\dagger(\vec{x}) = \exp ie \int_\Gamma^{\vec{x}} (\vec{D}\times\vec{T})\cdot d\vec{\ell} \quad , \quad \text{with} \quad \vec{D} = \int d^3\vec{x}\;\vec{J}_0 \quad ; \tag{10}$$

where $\vec{J}_0$ is a current formed with every neutral field capable of receiving flux. In particular it includes the neutral fermion current $:\psi_0^\dagger\,\vec{\alpha}\,\psi_0:$. The operator $\vec{\alpha}$ is formed with the Dirac matrices α_i.

Observing that $A^\dagger(\vec{x})\,\psi_0^\dagger(\vec{x})\,A(\vec{x}) = \psi_0^\dagger(\vec{x})\,a^\dagger(\vec{x},\vec{\alpha})$, I define the electron field as $\psi_e^\dagger = C^\dagger A^\dagger \psi_0^\dagger$; so that the one–electron state $\int d^3\vec{x}\;\psi_e^\dagger\, f_k|\Omega>$, has the register of the electric and magnetic fields of the electron.

In elementary particle reactions, the electron and its neutrino share a conserved lepton number, and at the same time they differ in that the neutrino does not carry flux. Then it is natural to explore the possibility of using the neutral fermion $\psi_0(\vec{x})$ to describe the electron–neutrino.

In order to couple the fields $\psi_e^\dagger$ and ψ_0, one needs to construct a charged boson field, which will receive the electron flux. One needs also a mechanism of local transference of flux.

Given a mixing angle ϕ, one can form a complex linear combination of the real fields $\vec{A}$ and $\vec{T}$: $\vec{Y} = \text{sen}\phi\,\vec{A} - i\cos\phi\,\vec{T}$.

$\vec{Y}(\vec{x})$ is still a neutral field, because it has no flux factor operators. The dynamics of its transverse components is given by the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\int d^3\vec{x}\left[\dot{\vec{Y}}^*\dot{\vec{Y}} + (\vec{\nabla}\times\vec{Y}^*)(\vec{\nabla}\times\vec{Y})\right] \quad . \tag{11}$$

Now, I take the interaction Hamiltonian, coupling the photon to the electron in Quantum Electrodynamics, $-e\,\vec{J}\cdot\vec{A}$, and rewrite it with the $\vec{Y}$ field:

$$-\frac{e}{2\mathrm{sen}\phi}\left[\psi_e^\dagger\,\vec{\alpha}\,(1-\gamma^5)\,\psi_e\,\vec{Y}+\mathrm{h.c.}\right]-e\,\psi_e^\dagger\,\vec{\alpha}\cdot\vec{A}\,\gamma^5\,\psi_e \quad . \qquad (12)$$

The next procedure is to define a charged vector field $\vec{W}^- = A\,C\,\vec{Y}$, designed to absorb the flux of the electron field ψ_e, transforming it in a neutrino field ψ_0.

Products of fields in the Hamiltonian of Eq. (12) can be rewritten with the prescription: $\psi_e\,\vec{Y} = \psi_0\,A\,C\,\vec{Y} = \psi_0\,\vec{W}^-$, and the first two vertex acquire the form

$$-\frac{e}{2\mathrm{sen}\phi}\,\psi_e^\dagger\,\vec{\alpha}\,(1-\gamma^5)\,\psi_0\,\vec{W}^- + \mathrm{h.c.} \quad . \qquad (13)$$

This structure is similar to the charged current coupling of Weinberg–Salam Theory[3,4]. Although the charged vector bosons $\vec{W}^-$ and $\vec{W}^+$ in that theory are independent variables, whereas in the present construction they are not.

The pseudo–vector coupling between the electron and the $\vec{A}$–field in Eq. (12), is also in correspondence with a vertex of the Standard Model: a coupling of the electron to the $\vec{Z}$ field.

In the scheme analyzed here, a vertex $\psi_e^\dagger\,\vec{\alpha}\cdot\vec{F}\,\psi_e$ cannot be transformed into a simple neutrino vertex like $\psi_0^\dagger\,\vec{\alpha}\cdot\vec{F}\,\psi_0$ by dislocations of the flux operators, because the operator $a^\dagger(\vec{x},\vec{\alpha})$ does not commute with the α–matrices.

Replacing the $\vec{Y}$ field by $A^\dagger\,C^\dagger\,\vec{W}^-$, in the Hamiltonian of Eq. (11), one gets an effective Hamiltonian $\mathscr{H}(\vec{W})$ for the charged vector boson, which has certain similitudes with the gauge field dynamics. Specially in the fact that $\mathscr{H}(\vec{W})$ has quartic terms. This is an indication that the $\vec{W}$–field defined above, could be related to the Yang–Mills[5] fields.

Concerning to the mass of the $\vec{W}$–field, I speculate that in the weak interaction regime, that involves short distance effects, some components of the electromagnetic field could eventually develop a vacuum expectation value; and, through the quartic terms, they might lend an effective mass to the $\vec{W}$–field, in a manner analogous to the Higgs mechanism[6].

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