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SEMI-INCLUSIVE RAPIDITY DISTRIBUTIONS AND A CRITICAL ANALYSIS OF THE CONCEPT OF PARTITION TEMPERATURE

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Abstract

An analytical computation has been performed of the partition temperature of a n-particle system of total invariant mass M with a transverse-momentum cutoff. The result has been used to compare with the previously obtained fitted values at $\sqrt{s}$=540 GeV, which show a complete disagreement with the now available ones, calculated by starting from the definition. The effect of the center-of-mass motion of the system on the pseudo-rapidity distributions is also discussed.

I. INTRODUCTION

Several years ago, Chou, Yang and Yen have proposed to describe a high-energy hadron-hadron collision at a given energy as an incoherent superposition of collisions with different partition temperatures[1]. The model is as follows. Consider, for instance, $\bar{p}p$ collision at a sufficiently high incident energy $\sqrt{s}$ and events with n (*non-leading*) forward-going particles. The total center-of-mass energy of this n-particle system $W = h\sqrt{s}$, is assumed to be stochastically distributed among n particles with some conveniently parametrized transverse-momentum cutoff factor $g(p_T)$. In other words, the exclusive probability distribution for non-leading particles in one hemisphere is described by a *microcanonical ensemble*, *i.e.*,

$$\text{Probability} = \prod_j \frac{d^3\vec{p}_j}{E_j} g(p_{Tj}) \delta\left(\sum_i E_i - h\sqrt{s}\right), \tag{1.1}$$

where all the quantities are given in the center-of-mass frame.

In this case, the single-particle distribution turns out to be given by the *canonical ensemble*

$$\text{Probability} = \alpha \frac{d^3\vec{p}}{E} g(p_T) \exp\left(-\frac{E}{T_p}\right), \tag{1.2}$$

where α is a normalization constant and the parameter T_p, the so called *partition temperature*, is evidently a uniquely defined function of W and n, once $g(p_T)$ is chosen[2].

At this point, the authors of Ref. 1) perform a fit of semi-inclusive pseudorapidity-distribution data[3] in 540-GeV $\bar{p}p$ collisions, by using (1.2) where an empirically determined factor $g(p_T)$ is replaced. All the experimental points for $n_{\text{obs}} > 10$ are surprisingly well reproduced in the entire η range where the distributions have been measured and they conclude that (1.2), and consequently also (1.1), is in excellent agreement with experiment.

Another report has been given[4] where, with an additional assumption that the inelasticity h is a function of the impact parameter b, fairly good results have been obtained also at different energies of $\bar{p}p$ collider.

However, notwithstanding a good fit of the data, we think we need something more before concluding that (1.1) and (1.2) are really in good agreement with experiment, since those authors have not shown the precise relationship among T_p, W and n that we have mentioned above. The main purpose of the present paper is thus to obtain this relationship and then, by using W and n determined experimentally, to compute T_p and compare with the semi-inclusive (pseudo-)rapidity distribution data.

In what follows, starting from (1.1) we derive in the next section the single-particle momentum distribution (1.2) and thereby the functional form of $T_p(W, n)$. A comparison with data is carried out in Sec. III, where firstly we consider the CYY analysis[1] and then an independent work by Takagi and Tsukamoto[5], where the implicitly implied forward-backward symmetry by Ref. 1) has been removed but two uncorrelated leading particles are now assumed. Conclusions are drawn in Sec. IV.

II. DERIVATION OF THE SEMI-INCLUSIVE MOMENTUM DISTRIBUTION

Consider a system of n particles, having a total invariant mass $M = \sqrt{W^2 - \vec{P}^2}$, where $(W, \vec{P})$ is the energy-momentum four-vector of the system. For simplicity, we assume all the particles in the system to be pions of mass m and neglect the statistics. As far as the single-particle distribution is concerned, we believe that this is a reasonable approximation.

Divide the phase space into small boxes with volumes $\Delta V_1, \Delta V_2, \ldots, \Delta V_N$. We shall start from a discrete phase space and obtain the continuum limit by letting $\Delta V_i \to 0$ and $N \to \infty$. The probability of finding n_ℓ particles in ΔV_ℓ ($\ell = 1, 2, \ldots, N$) is written

$$\mathcal{P}(\{n_\ell\}) = \frac{n!}{n_1! \ldots n_N!} q_1^{n_1} \cdots q_N^{n_N} \delta_{n, \sum n_\ell} \delta(W - \sum_{\ell=1}^{N} n_\ell E_\ell) \delta(P_L - \sum_{\ell=1}^{N} n_\ell p_{L\ell}) \delta^2(\vec{P}_T - \sum_{\ell=1}^{N} n_\ell \vec{p}_{T\ell}), \tag{2.1}$$

where $\{n_\ell\}$ stands for any set $\{n_1, \ldots, n_N\}$ and the probability that a produced particle be found in ΔV_k can be written in terms of the probability density $f(y, \vec{p}_T)$ as

$$q_k \equiv f(y_k, \vec{p}_{Tk}) \Delta V_k \overset{\Delta V_k \to 0}{\underset{N \to \infty}{\longrightarrow}} f(y, \vec{p}_T)\, dy\, d\vec{p}_T, \tag{2.2}$$

with the normalization

$$\int f(y, \vec{p}_T)\, dy\, d\vec{p}_T = 1. \tag{2.3}$$

Here, y is the rapidity of the particle.

The single-particle momentum distribution is, then, written in this discrete version as

$$< n_k > \equiv \left\langle \frac{d^3 n}{dy\, d\vec{p}_T}\Big|_{y_k, \vec{p}_{Tk}} \Delta V_k \right\rangle_{n, W, \vec{P}} = \frac{\sum_{\{n_l\}} n_k \mathcal{P}(\{n_l\})}{\sum_{\{n_l\}} \mathcal{P}(\{n_l\})} = \frac{A}{B}. \tag{2.4}$$

In the continuum limit, A and B are given (see Appendix **A**) by

$$A = \frac{-n}{(2\pi)^4} f(y,\vec{p}_T) dy\, d\vec{p}_T \int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds \int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt \int d\vec{u}_T [F(s,t,\vec{u}_T)]^{n-1}$$
$$\times \exp\Big[(W - \sqrt{\vec{p}_T^2+m^2}\,\mathrm{ch}\,y)s - (P_L - \sqrt{\vec{p}_T^2+m^2}\,\mathrm{sh}\,y)t - i(\vec{P}_T - \vec{p}_T)\cdot\vec{u}_T\Big] \tag{2.5}$$

and

$$B = -\frac{1}{(2\pi)^4} \int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds \int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt \int d\vec{u}_T [F(s,t,\vec{u}_T)]^n \exp\Big[Ws - P_L t - i(\vec{P}_T - \vec{p}_T)\cdot\vec{u}_T\Big] \ , \tag{2.6}$$

where we have introduced a definition

$$F(s,t,\vec{u}_T) \equiv \int dy\, d\vec{p}_T f(y,\vec{p}_T)\, e^{-\sqrt{\vec{p}_T^2+m^2}(s\,\mathrm{ch}y - t\,\mathrm{sh}y) + i\vec{p}_T\cdot\vec{u}_T} \ . \tag{2.7}$$

To avoid unessential complexity, let us forget in this paper the $\vec{p}_T$ conservation, since as well known $< p_T >$ is small compared with $< p_L >$, which will be assured by $g(p_T)$ of (1.1) and defined in (2.11) below. Then, $F(s,t,\vec{u}_T) \to F(s,t)$ in (2.7) and $\vec{u}_T$ integrations are suppressed in (2.5) and (2.6). So, (2,4) reduces to

$$< n_k > \simeq n\, f(y,\vec{p}_T)\, dy\, d\vec{p}_T \frac{C}{D} \ , \tag{2.8}$$

with

$$C = \int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds \int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt\ [F(s,t,)]^{n-1} \exp\Big[(W - \sqrt{\vec{p}_T^2+m^2}\,\mathrm{ch}\,y)s - (P_L - \sqrt{\vec{p}_T^2+m^2}\,\mathrm{sh}\,y)t\Big] \tag{2.9}$$

and

$$D = \int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds \int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt\ [F(s,t,)]^n \exp\Big[Ws - P_L t\Big] \ . \tag{2.10}$$

In order to proceed with the computation of the semi-inclusive distribution $< n_k >$, we parametrize the probability density $f(y,\vec{p}_T)$ (cf. (2.1,2.2)) as

$$f(y,\vec{p}_T) = \alpha\, e^{-\beta\sqrt{\vec{p}_T^2+m^2}\,\mathrm{ch}\,y} e^{-\gamma\sqrt{\vec{p}_T^2+m^2}} \ , \tag{2.11}$$

with

$$\iint f(y,\vec{p}_T)\, dy\, d\vec{p}_T = 1 \ , \tag{2.12}$$

where y is measured with respect to the rest frame of M ($y = y_{cm} - Y$, Y = rapidity of M).

This ansatz is more general than the one utilized in Ref. 1) insofar as it includes the case of a longitudinal momentum dependence. That is to say, for $\beta > 0$ it yields an approximate Gaussian in rapidity, which is what one gets for the inclusive particle distribution in Landau's hydrodynamical model[6)]. The pure (longitudinal-) phase-space ansatz which appears in (1.1) is recovered in the limit of $\beta \to 0$. We emphasize that due to the constraint of energy conservation, the resulting semi-inclusive distribution (2.4) will be *independent of* β, as can readily be seen by substituting (2.11) into (2.1), (2.4)

Another minor difference between our ansatz and that of Ref. 1) is that here, $g(p_T)$ is an exponential not in p_T but in transverse energy $\sqrt{\vec{p}_T^2+m^2}$, which is better suited for fitting the data.

Let us now compute $F(s,t)$ by replacing in (2.7) $f(y,\vec{p}_T)$ parametrized as (2.11). We have

$$F(s,t) = \alpha \int_{-\infty}^{\infty} dy \int d\vec{p}_T^2 \exp\Big[-\{\gamma + (\beta+s)\,\mathrm{ch}\,y - t\,\mathrm{sh}\,y\}\sqrt{\vec{p}_T^2+m^2}\Big] \ , \tag{2.13}$$

where the $\vec{p}_T$ integration can easily be done and gives

$$F(s,t) = 2\pi\alpha \int_{-\infty}^{\infty} dy \left[\frac{m}{\gamma + (\beta+s)\,\mathrm{ch}\, y - t\,\mathrm{sh}\, y} + \frac{1}{\{\gamma + (\beta+s)\,\mathrm{ch}\, y - t\,\mathrm{sh}\, y\}^2} \right] \times \exp\left[-m\{\gamma + (\beta+s)\,\mathrm{ch}\, y - t\,\mathrm{sh}\, y\} \right] \ . \tag{2.14}$$

The last integral will be evaluated in Appendix **B** and reads

$$F(s,t) \simeq 4\pi\alpha \left(\frac{m}{\gamma} + \frac{1}{\gamma^2} \right) e^{-\gamma m} K_0\left(m\sqrt{(\beta+s)^2 - t^2} \right) \ , \tag{2.15}$$

in the limit of small argument of the Bessel function.

By introducing this $F(s,t)$ into (2.9) and (2.10), we have as will be shown in Appendix **C**

$$C \simeq -\frac{2\pi^2(n-1)(n-2)(4\pi\alpha)^{n-1}}{M'^2} \left(\frac{m}{\gamma} + \frac{1}{\gamma^2} \right)^{n-1} \left(\ln\frac{M'}{m} \right)^{n-3} \times e^{-\beta(W - \sqrt{p_T^2+m^2}\,\mathrm{ch}\, y) - (n-1)m\gamma} \tag{2.16}$$

and

$$D \simeq -\frac{2\pi^2 n(n-1)(4\pi\alpha)^n}{M^2} \left(\frac{m}{\gamma} + \frac{1}{\gamma^2} \right)^n \left(\ln\frac{M}{m} \right)^{n-2} e^{-\beta W - nm\gamma}, \tag{2.17}$$

where

$$\begin{aligned} M'^2 &= \left(W - \sqrt{p_T^2+m^2}\,\mathrm{ch}\, y \right)^2 - \left(P_L - \sqrt{p_T^2+m^2}\,\mathrm{sh}\, y \right)^2 \\ &= M^2 \left[1 - \frac{2}{M}\sqrt{p_T^2+m^2}\,\mathrm{ch}(y-Y) + \frac{p_T^2+m^2}{M^2} \right] \end{aligned} \tag{2.18}$$

is the squared mass of the remaining system after the subtraction of the single particle that is being observed.

We are now ready to calculate $< n_k >$ by introducing C and D given above into (2.8). First, let us approximate

$$\begin{aligned} \left(\ln\frac{M'}{m} \right)^{n-3} &\simeq \left[\ln\left\{ \frac{M}{m} \left[1 - \frac{\sqrt{p_T^2+m^2}\,\mathrm{ch}(y-Y)}{M} \right] \right\} \right]^{n-3} \\ &\simeq \left(\ln\frac{M}{m} \right)^{n-3} \exp\left[-\frac{(n-3)}{M\ln\frac{M}{m}} \sqrt{p_T^2+m^2}\,\mathrm{ch}(y-Y) \right] \ , \end{aligned} \tag{2.19}$$

where we have assumed

$$\sqrt{p_T^2+m^2}\,\mathrm{ch}(y-Y) \ll M' < M \ . \tag{2.20}$$

Then,

$$\begin{aligned} \frac{d^3 n}{dy\, d\vec{p}_T} &\simeq \frac{(n-2)e^{m\gamma}}{4\pi\left(\frac{m}{\gamma} + \frac{1}{\gamma^2} \right) \ln\frac{M}{m}} e^{-\gamma\sqrt{p_T^2+m^2}} \\ &\quad \times \exp\left[-\left\{ \frac{(n-3)}{M\ln\frac{M}{m}} - \frac{2}{M} \right\} \sqrt{p_T^2+m^2}\,\mathrm{ch}(y-Y) \right] \ . \end{aligned} \tag{2.21}$$

This expression is identical to (1.2), provided

$$T_p = \frac{M\ln\frac{M}{m}}{(n-3) - 2\ln\frac{M}{m}} \overset{\text{large } n}{\simeq} \frac{< E > \ln\frac{M}{m}}{1 - \frac{4}{3n_{ch}}\ln\frac{M}{m}} \ . \tag{2.22}$$

Here, however, E is the particle energy in the rest frame of M. The last term of the denominator of (2.22) is usually a small number, so in the most of the cases it may be neglected in the lowest order approximation.

To complete the derivation, we have checked the large-n approximation which, starting from (2.8,11,16 and 17), has lead to the exponential form (2.21) and verified that, in the entire range of multiplicity and $0 < \eta \simeq y < 5$ where data exist, the error is less than 5%. Moreover, the asymptotic form of the one-dimensional phase space as argued by Chao[7)] is correctly reproduced by our method.

III. COMPARISON WITH DATA

Let us now compare the result obtained in the preceding section with the data. We shall first examine the CYY analysis, main results of which are summarized in Table I (columns 2 – 4). In that analysis only one hemisphere has been considered with the exclusive probability given by (1.1). This is equivalent to taking

$$\begin{cases} M = W = 2\,h\sqrt{s} \quad \text{and} \\ \vec{P} = 0 \end{cases} \tag{3.1}$$

in our notation. So, with an additional assumption of $n = 3n_{ch}/2$, T_p is readily obtained by introducing the experimental parameters of Ref. 1) in (2.22). The results are shown in the last column of Table I, where a huge discrepancy with the fitted values is evident. Thus, going back to the question we raised in the introduction, we are forced to conclude that ***although nice fits have been obtained with (1.2), the parameter T_p thus determined has nothing to do with the partition temperature.*** Pseudorapidity distributions with the correct partition temperatures, within the physical hypothesis of symmetrical multiplicity distributions are too low and too broad to reproduce the data.

One may think at this point that the phase space calculation is nonsense and the concept of partition temperature is completely meaningless when treating hadronic systems. However, one may also be a little less categorical and try to see whether it can be used meaningfully, once some precaution is taken. It is clear from the forward-backward asymmetry observed in the multiplicity distribution,[8] that event-by-event fluctuation is not at all negligible when computing the energy partition among the n central particles or, in other words, the hypothesis embodied by (3.1) is valid only for (semi-)*inclusive* distribution

but too strong for applying an energy partition on it as given by (1.1). Evidently, if one takes the center-of-mass motion of the n-particle system into account without changing the event multiplicity, particles will appear more concentrated, *i. e.*, with smaller T_p.

In Ref. 5), such a calculation has been done under a simplified assumption of a constant $m_T = \sqrt{p_T^2 + m^2}$ and with two uncorrelated leading particles with a flat x-distribution, which is the standard picture of the hadronic multiparticle production. As can be seen in their comparison (Fig. 4 of that paper), the agreement is still not satisfactory. The difficulties arise especially in the low multiplicity data ($n_{ch} \leq 20$), where the maximum in the large-η values cannot be reproduced, but also in higher multiplicity data where the overall width seems to be systematically wider than the experimental trends.

In a previous work,[9] we have used a fragmentation model and obtained a quite good description of the semi-inclusive pseudorapidity distributions in the low and intermediate multiplicity region ($n_{ch} < 35$). In that model, one or both of the incident particles were excited into high-temperature states, with a subsequent expansion and decay according to a one-dimensional hydrodynamical model.[6] If we apply the phase-space calculation or equivalently the partition temperature concept to such objects, the results are probably not far from the earlier ones. The distinct ingredient here as compared to Refs. 1) and 5) is the account of the so called *"diffractive"* component which is usually assumed to be excluded from the *"non-diffractive"* data. However, as the mass of such an excited object grows large enough, it becomes hard to recognize this kind of event as *"diffractive"*, even though the forward-backward asymmetry still remains. Thus, the combination of large fluctuation in the forward-backward multiplicity distributions and the semi-inclusive pseudo-rapidity spectra with large-$|\eta|$ peaks seems to indicate that incident-particle fragmentation as described above plays an important rôle in multiparticle production.

IV. CONCLUSIONS

Starting from the transverse-momentum-cut phase space (1.1), we have derived in this paper the single-particle momentum spectra which, as expected, turned out to be an exponential in particle energy. The inverse of the coefficient in the exponent is to be identified with the previously introduced partition temperature.

A comparison of this result with the experimentally fitted values showed that care must be exercised when (1.2) is used to determine the partition temperature. The event-by-event fluctuation in forward-backward multiplicity distributions, which in general show large asymmetry, is one of the fundamental features of multiparticle production and cannot be neglected.

An inclusion of the fluctuation mentioned above through uncorrelated leading particles is not enough to correctly reproduce the semi-inclusive data. We find that a possible way to giving a better account of the existing data is the consideration of particle fragmentation process which clearly contributes to large fluctuation and gives the momentum distribution a more asymmetrical form. Whether this mechanism is important or not may be decided experimentally through a study of pseudorapidity distributions with fixed multiplicities n_{ch} and fixed forward-backward multiplicity ratios $R = n_F/n_B$.

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APPENDIX A

In this appendix, we shall give a detailed derivation of (2.5) and (2.6), which have appeared in Sec. II.

First, following Giffon, Hama and Predazzi (GHP)[10,11], let us rewrite (2.1) by replacing the energy-momentum-conservation δ functions by Fourier-Laplace representations and also the multiplicity-fixing Kronecker δ by its Fourier representation:

$$\begin{aligned}\mathcal{P}(\{n_\ell\}) = \frac{1}{(2\pi)^3}\frac{n!}{(2\pi i)^2}\int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds\; e^{(W-\sum_{\ell=1}^{N} n_\ell E_\ell)s}\int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt\; e^{-(P_L-\sum_{\ell=1}^{N} n_\ell p_{L\ell})t} \\ \times\int d\vec{u}_T e^{-i(\vec{P}_T-\sum_{\ell=1}^{N} n_\ell \vec{p}_{T\ell})\cdot\vec{u}_T}\int_0^{2\pi} dv\; e^{-i(n-\sum_{\ell=1}^{N} n_\ell)v}\left(\frac{q_1^{n_1}}{n_1!}\right)\cdots\left(\frac{q_N^{n_N}}{n_N!}\right)\ ,\end{aligned} \tag{A.1}$$

where $\epsilon_0 > \epsilon_1$ in order to ensure the convergence of the integral (2.7).

Then, the numerator A of (2.4) can easily be handled and written

$$\begin{aligned}A = \frac{-n!}{(2\pi)^5}\int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds\int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt\int d\vec{u}_T\int_0^{2\pi} dv\; e^{(W-E_k)s}e^{-(P_L-p_{Lk})t} \\ \times e^{-i(\vec{P}_T-\vec{p}_{Tk})\cdot\vec{u}_T}e^{-i(n-1)v}q_k\prod_{\ell=1}^{N}\sum_{n_\ell=0}^{\infty}\frac{(q_\ell\, e^{-(E_\ell s-p_{L\ell}t)+i\vec{p}_{T\ell}\cdot\vec{u}_T+iv})^{n_\ell}}{n_\ell!}\ .\end{aligned} \tag{A.2}$$

In the limit of $\Delta V_\ell \to 0$ and $N \to \infty$, this may be rewritten, on account of (2.2) and

$$E_\ell = \sqrt{\vec{p}_{T\ell}^{\,2}+m^2}\;\mathrm{ch}\, y_\ell\ \ ,\ \ p_{L\ell} = \sqrt{\vec{p}_{T\ell}^{\,2}+m^2}\;\mathrm{sh}\, y_\ell\ . \tag{A.3}$$

as

$$\begin{aligned}A = \frac{-n!}{(2\pi)^5}f(y,\vec{p}_T)\,dy\,d\vec{p}_T\int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds\int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt\int d\vec{u}_T\int_0^{2\pi} dv \\ \times\exp\Big[(W-\sqrt{\vec{p}_T^{\,2}+m^2}\;\mathrm{ch}\, y)s-(P_L-\sqrt{\vec{p}_T^{\,2}+m^2}\;\mathrm{sh}\, y)t-i(\vec{P}_T-\vec{p}_T)\cdot\vec{u}_T-i(n-1)v\Big] \\ \times\exp\Big[\int dy\,d\vec{p}_T f(y,\vec{p}_T)\, e^{-\sqrt{\vec{p}_T^{\,2}+m^2}(s\,\mathrm{ch}\,y-t\,\mathrm{sh}\,y)+i\vec{p}_T\cdot\vec{u}_T+iv}\Big]\ .\end{aligned} \tag{A.4}$$

Recalling that

$$\exp\left[e^{iv}\right] = \sum_{\ell=0}^{\infty} \frac{e^{i\ell v}}{\ell!} \quad , \tag{A.5}$$

the integration in v may easily be effected, giving (2.5) of Sec. II.

The denominator B of (2.4) is computed in a similar way, resulting in (2.6).

APPENDIX B

In this appendix, we shall evaluate the y-integral in (2.14).

By making a change of parameters

$$\begin{cases} \beta + s = \eta\, \mathrm{ch}\, \zeta, \\ \quad t = \eta\, \mathrm{sh}\, \zeta, \end{cases} \longrightarrow \begin{cases} \eta = \sqrt{(\beta+s)^2 - t^2}\ , \\ \zeta = \mathrm{th}^{-1} \dfrac{t}{\beta+s} \end{cases} \tag{B.1}$$

and of the integration variable

$$y = x + \zeta \quad , \tag{B.2}$$

we have

$$F(s,t) = 2\pi\alpha \int_{-\infty - i\,\mathrm{Im}\zeta}^{\infty - i\,\mathrm{Im}\zeta} dx \left[\frac{m}{\gamma + \eta\,\mathrm{ch}\,x} + \frac{1}{(\gamma + \eta\,\mathrm{ch}\,x)^2}\right] \exp\left[-m(\gamma + \eta\,\mathrm{ch}\,x)\right] \quad . \tag{B.3}$$

This integral is convergent, because $\beta > 0$ and $\mathrm{Re}\, s > \mathrm{Re}\, t$. It is not difficult to convince ourselves that the integration path may be deformed to $-\infty$ — ∞, without changing the integral. We have

$$\frac{\partial F}{\partial m} = -2\pi\alpha m\, e^{-m\gamma} \int_{-\infty}^{\infty} \exp\left[-m\eta\,\mathrm{ch}\,x\right] dx = -4\pi\alpha m\, e^{-m\gamma} K_0(m\eta) \quad , \tag{B.4}$$

where the initial condition for F is $F \overset{m\to\infty}{\longrightarrow} 0$. Thus,

$$F(s,t) = 4\pi\alpha \int_{m}^{\infty} z\, e^{-\gamma z} K_0(\eta z)\, dz \quad . \tag{B.5}$$

Now, as will become clear later when computing C and D of (2.9) and (2.10), the precise behavior of $F(s,t)$ is required when $|\eta|$ is small. Since $K_0(\eta z)$ is logarithmic in this limit, a convenient approximation would be to put it out of the integration sign. So,

$$F(s,t) \simeq 4\pi\alpha \left[\frac{m}{\gamma} + \frac{1}{\gamma^2}\right] e^{-\gamma m} K_0(m\eta) \quad . \tag{B.6}$$

Observe that, if $m|\eta|$ were large, we would obtain

$$F(s,t) \sim \frac{e^{-(\gamma+\eta)m}}{\sqrt{\eta}} \quad . \tag{B.7}$$

APPENDIX C

Let us compute D defined by (2.10), with $F(s,t)$ obtained in the preceding Appendix. The evaluation of C is entirely similar, so we shall omit it here.

Define

$$\begin{cases} W = M\,\mathrm{ch}\,Y \ , \quad M > 0 \\ P_L = M\,\mathrm{sh}\,Y \ . \end{cases} \tag{C.1}$$

Then, D may be rewritten

$$D \simeq (4\pi\alpha)^n \left(\frac{m}{\gamma} + \frac{1}{\gamma^2}\right)^n e^{-nm\gamma} \int_{a-i\infty}^{a+i\infty} [K_0(m\eta)]^n \eta\, d\eta \times \int_{C_\pm} \exp\Big[-M\beta\,\mathrm{ch}\,Y + M\eta\,\mathrm{ch}\,(\zeta - Y)\Big] d\zeta \ , \tag{C.2}$$

where a is a small positive constant and integration paths $C_\pm$ are as shown in Fig. 1. The integral in ζ has been computed previously (see Appendix C of Ref. 11)) and reads

$$\int_{C_\pm} \exp\Big[-M\beta\,\mathrm{ch}\,Y + M\eta\,\mathrm{ch}\,(\zeta - Y)\Big] d\zeta = 2\pi i\, e^{-M\beta\,\mathrm{ch}Y}\left[I_0(M\eta) + \frac{1}{i\pi}K_0(M\eta)\right] \ . \tag{C.3}$$

The last term in (C.3) may be dropped because, when integrated over η, it does not give any contribution. Thus,

$$D \simeq 2\pi i\,(4\pi\alpha)^n \left(\frac{m}{\gamma} + \frac{1}{\gamma^2}\right)^n e^{-M\beta\,\mathrm{ch}Y - nm\gamma} \int_{a-i\infty}^{a+i\infty} [K_0(m\eta)]^n I_0(M\eta)\,\eta\, d\eta \ . \tag{C.4}$$

Since $M >> m$, the integrand above is rapidly dominated by $I_0(M\eta)$ when $|\eta| \to \infty$, so as stated below (B.5) only the dominant behavior of K_0 for $m\eta \to 0$ is needed to evaluate this integral. This has already been done by GHP[10] (see Appendix C therein) and in leading order

$$D \simeq -\frac{2\pi^2 n(n-1)(4\pi\alpha)^n}{M^2}\left(\frac{m}{\gamma} + \frac{1}{\gamma^2}\right)^n \left(\ln\frac{M}{m}\right)^{n-2} e^{-\beta W - nm\gamma}. \tag{C.5}$$

TABLE I. Comparison of the partition temperature T_p as given by (2.22) with the experimentally fitted one (fourth column). The data marked with * have been taken from Ref. 1).

$n_{ch,obs}$	$n_{ch,cal}$ *	Energy fraction in forward central region, h *	T_p * (GeV)	Our results T_p (GeV)
≥ 71	99.4	0.451	4.38	29.9
51–70	73.3	0.419	6.25	39.0
41–50	55.0	0.332	6.80	42.2
31–40	44.2	0.316	8.84	52.5
21–30	33.0	0.308	13.8	76.2
11–20	21.2	0.257	23.8	127
≤ 10	10.7	0.197	183	1116

Figure Caption

Fig. 1: Integration paths $C_{\pm}$ which appear in (C.2) of Appendix C

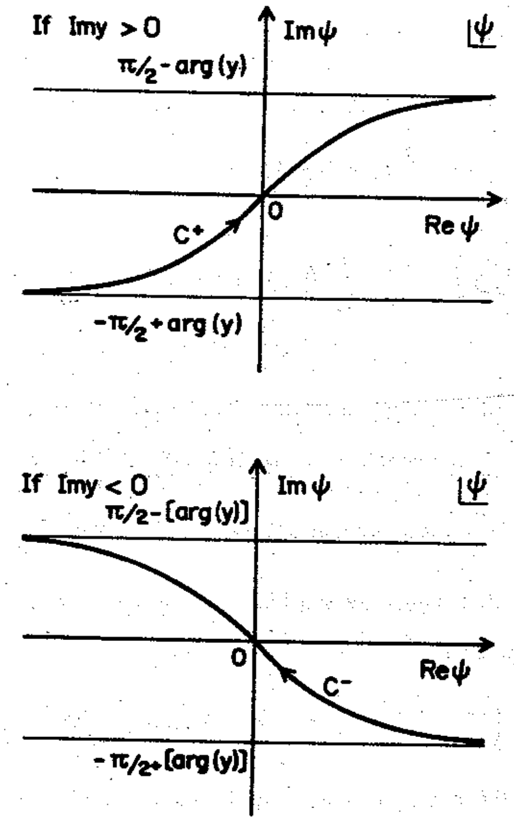

FIG. 1