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A MODEL OF CP VIOLATION HAVING A COSMOLOGICAL ORIGIN

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ABSTRACT

CP is spontaneously broken through a Higgs field conformally coupled to the open Friedmann spacetime. The effect grows as we go toward the origin of the universe, resisting to its equilibrium temperature.

We dedicate this paper to our friend Jayme Tiomno and wish to continue learning physics from him for still a long time.

1. INTRODUCTION

It was first pointed out by T.D. Lee[1] that CP symmetry could be violated spontaneously. This has many virtues and one problem: the broken symmetry is usually restored at a high enough temperature[2]. Two difficulties arise in connection with cosmology: 1) The presence of CP non invariance in the early, hot universe, is needed to explain the present baryonic asymmetry[3]; 2) As pointed out by Zel'dovich et al.[4], the symmetry restoration favors the formation of domain walls separating regions where a mesonic scalar (or pseudoscalar) field has vacuum expectation values (VEV) of different values. In a previous attempt[5] to solve, these difficulties we proposed a model for spontaneous CP violation which used the curvature of the cosmological spacetime as, grossly speaking, a "wrong signed mass term" which generated a doubly degenerate vacuum which performed the job. The most welcome result was that no transition between symmetric and asymmetric phases could occur caused by the equilibrium temperature T of the universe (assumed to be of the open Friedmann type), as the ratio T/T_c turned out to be a constant, T_c being the critical temperature for the would–be transition. So, if the CP violation is due to this mechanism, the fact that it is present today[6] guarantees its existence in the past, when it was badly needed.

A problem with the model of Ref.(1) was that CP was violated through a spontaneous violation of P, so that both P and T were broken. This allows for an electric dipole moment for the neutron[8], which is severely restricted by experiments. To avoid that we, in this paper, choose C, instead of P, to be spontaneously broken.

2. A SIMPLE CASE

The essence of the model is the following: a zero mass scalar meson ϕ is coupled to a fermion by a Yukawa interaction, to itself by a $\lambda\phi^4$ term and, conformally, to the spacetime by a term $\frac{R}{12}\phi^2$. To start with the simplest case, take ϕ to be real. Neglecting the other fields the Lagrangian is

$$\mathcal{L} = \tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\mu \phi + \tfrac{1}{12} R\phi^2 - \tfrac{\lambda}{12}\phi^4 \tag{1}$$

the conventions being those of Ref.(9). The metric is written

$$ds^2 = dt^2 - a^2(t)\left[dr^2/(1-kr^2) + r^2(d\theta^2+\sin^2\theta d\phi^2\right] \tag{2}$$

where k equals 1, 0 or -1 for the closed, flat or open universes, respectively. The function $a(t)$ is the scale factor of the universe, also called its "radius". The scalar curvature is given by

$$R = -6\,\ddot{a}/a - 6(\dot{a}/a)^2 - 6k/a^2 \tag{3}$$

where the dot stands for differentiation with respect to t. The action being given by

$$S = \int d^4x\,\sqrt{-g}\,\mathcal{L}\ , \tag{4}$$

the Euler–Lagrange equation for ϕ are

$$\Box\phi - \tfrac{1}{6} R\phi + \tfrac{1}{3}\lambda\phi^3 = 0\ . \tag{5}$$

Putting

$$<0|\phi(x)|0> = \rho \tag{6}$$

eq.(5) gives, in the tree approximation(10,11)

$$\Box\rho - \tfrac{1}{6} R\rho + \tfrac{1}{3}\lambda\rho^3 = 0 \tag{7}$$

and ρ can, in principle, depend on the time. Equation (7) then reads

$$\ddot{\rho} + 3(\dot{a}/a)\dot{\rho} + (\ddot{a}/a + \dot{a}^2/a^2 + k/a^2)\rho + \tfrac{1}{3}\lambda\rho^3 = 0\ . \tag{8}$$

Changing variables to η and f, defined by

$$\frac{d}{d\eta} = a\frac{d}{dt} \tag{9}$$

$$\rho = (3/\lambda)^{1/2}\,(f/a) \tag{10}$$

Eq.(8) simplifies to

$$f'' + kf + f^3 = 0 \tag{11}$$

where

$$f' \equiv df/d\eta\ .$$

To find out which of the solutions of (11) correspond to the lowest energy, consider the quantity

$$<0|T^0{}_0|0> \equiv \varepsilon(\eta) = \frac{3}{\lambda a^4}\left[\tfrac{1}{2}f'^2 + \tfrac{1}{2}kf^2 + \tfrac{1}{4}f^4\right] \tag{12}$$

where $T^\alpha{}_\beta$ is the energy–momentum tensor associated to (1), viz.

$$T^{\alpha}{}_{\beta} = \partial^{\alpha}\phi\,\partial_{\beta}\phi + \left[R^{\alpha}{}_{\beta} - \nabla^{\alpha}\nabla_{\beta} + \delta^{\alpha}{}_{\beta}\,\Box\right]\frac{1}{6}\phi^2 - \delta^{\alpha}{}_{\beta}\,\mathcal{L}\ . \tag{13}$$

From (11) one readily gets

$$f'^2 + kf^2 + \frac{1}{2}f^4 = C \tag{14}$$

so that

$$\varepsilon(\eta) = \frac{3C}{2\lambda a^4}\ . \tag{15}$$

For $k = 0$ or 1 the lowest possible value of C is zero, so that the energy density of the vacuum is zero. Also, $f = 0$, that is to say, $\rho = 0$. The interesting case is $k = -1$ (open universe). The general solution of (14) is, then,

$$\frac{f^2}{1+B} = 1 - 2B\,\mathrm{sn}^2\left\{\left[\frac{1}{2}(1+B)\right]^{1/2}\eta, [2B/(1+B)]^{1/2}\right\} \tag{16}$$

$$B \equiv (1 + 2C)^{1/2}$$

$$C \geq -\frac{1}{2} \tag{17}$$

where sn stands for the jacobian elliptic function called sine amplitude. The lowest energy solution corresponds to $C = -\frac{1}{2}$ and reads

$$f = \pm 1 \tag{18}$$

$$\langle 0|\phi(x)|0\rangle = \rho = \pm\left[\frac{3}{\lambda}\right]^{1/2}\frac{1}{a} \tag{19}$$

$$\varepsilon = -\frac{3}{4\lambda\, a^4}\ . \tag{20}$$

So, we have the typical double–well structure associated to the spontaneous breakdown of a discrete, two–valued symmetry.

Let us take temperature now into account. Assume that the universe was in the radiation–dominated era, so that

$$p = \frac{\varepsilon_M}{3} \tag{21}$$

where ε_M stands for the density of the dominating matter (the one which causes the curvature). The entropy constancy leads to

$$\varepsilon_M\ a^4 = \text{constant} \tag{22}$$

and, because of (21),

$$\varepsilon_M \propto T^4 \tag{23}$$

so that

$$T = \frac{B}{a(t)} \tag{24}$$

B being a constant.

The energy difference between the symmetric "vaccum" and the asymmetric one is

$$\Delta E \propto a^3\,\varepsilon(t)\ . \tag{25}$$

At the critical temperature T_c for symmetry restoration the thermal energy just balances this difference, so that

$$T_c \approx \left[\frac{3}{\lambda}\right]^{1/2}\frac{1}{a}\ . \tag{26}$$

(For a more conventional estimate see Ref.(12)). Therefore

$$\frac{T_c}{T} \propto \frac{1}{\sqrt{\lambda}} = \text{constant} \quad , \tag{27}$$

and no phase transition ever happens: either the symmetry prevails or, if broken, never restores.

3. CP VIOLATION

The ideas exposed above in a simple situation will now be used to exhibit a model of CP violation via spontaneous breakdown of C. To avoid the well–known problems connected with the invariance of the Lagrangian under a phase translation of all fields[13], we introduce a non–hermitian zero–spin field which is electrically neutral but has a "charge" of some other type (like, for instance, that which distinguishes K^o from $\bar{K}^o$). We write the action as

$$S = \int d^4x \sqrt{-g}\, \mathcal{L} \tag{28}$$

with

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_{int} \tag{29}$$

$$\mathcal{L}_\phi = \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi^* \partial_\nu \phi + \partial_\nu \phi^* \partial_\mu \phi) + \frac{R}{6} (\phi^* \phi) - \frac{\lambda}{6} (\phi^* \phi)^2 \tag{30}$$

$$\mathcal{L}_\psi = \bar{\psi} (i \gamma^\mu D_\mu - m) \psi \tag{31}$$

$$\mathcal{L}_{int} = \alpha \bar{\psi} \psi (\phi + \phi^*) \tag{32}$$

α and λ being real and dimensionless, $\lambda > 0$. We will break C and T spontaneouly, so that $\mathcal{L}$ must be invariant under these transformations. This will happen if ϕ transforms as

$$C\, \phi\,(x,t)\, C^{-1} = \phi^*(x,t) \tag{33}$$

$$P\, \phi\,(x,t)\, P^{-1} = \phi\,(-x,t) \tag{34}$$

$$T\, \phi\,(x,t)\, T^{-1} = \phi\,(x,-t) \quad . \tag{35}$$

The equations of motion for the scalar field which are relevant for the computation of its VEV are

$$\Box\, \phi\,(x) - \frac{R}{6}\, \phi\,(x) + \frac{\lambda}{3}\, \phi^*(x)\, \phi(x)\, \phi(x) = 0 \quad . \tag{36}$$

Among the solutions of this equation we look for those which are of lowest energy and complex.

The energy–momentum tensor associated to $\mathcal{L}_\phi$ reads

$$T^\mu{}_\nu = \partial^\mu \phi^* \partial_\nu \phi + \partial_\nu \phi^* \partial^\mu \phi - \delta^\mu{}_\nu \mathcal{L} + \frac{1}{3} \left[R^\mu{}_\nu - \nabla^\mu \nabla_\nu + \delta^\mu{}_\nu \Box \right] (\phi^* \phi) \tag{37}$$

and the energy density ε

$$\varepsilon = \langle 0 | T^0{}_0 | 0 \rangle = \langle 0 | \partial^0 \phi^* \partial_0 \phi + \frac{\lambda}{6} (\phi^* \phi)^2 + \frac{1}{3} \left[R_{00} - \frac{R}{2} + \Box - \partial_0^2 \right] (\phi^* \phi) | 0 \rangle \tag{38}$$

Putting now

$$\phi(x) = \varphi(x) + \rho \tag{39}$$

$$\rho \equiv \langle 0 | \phi(x) | 0 \rangle \tag{40}$$

one has

$$\varepsilon = |\dot{\rho}|^2 + \frac{1}{3}\left[R_{00} - \frac{R}{2} + \Box - \partial_0^2\right]\left[|\rho|^2 + \frac{T^2}{6}\right] + \frac{\lambda T^2}{9}|\rho|^2 + \frac{\lambda}{6}|\rho|^4 + \xi(T) \quad . \tag{41}$$

where we have used

$$\begin{aligned} <0|\varphi|0> &= 0 \\ <0|\varphi^* \varphi|0> &= \frac{T^2}{6} \end{aligned} \tag{42}$$

$$<0|\varphi^2|0> = <0|\varphi^* \varphi^*|0> = <0|\varphi^* \varphi^2|0> = <0|\varphi \varphi^* \varphi^*|0> = 0$$

which are discussed in Ref.(2). $\xi(T)$ depends only on the temperature and has no relevance for the dynamics of φ. As ρ can only depend on time,

$$\varepsilon = |\dot{\rho}|^2 + \left[\frac{k^2 + a^2}{a^2} + \frac{\dot{a}}{a}\partial_0\right]|\rho|^2 + \frac{\lambda T^2}{9}|\rho|^2 + \frac{\lambda}{6}|\rho|^4 + \xi(T) \quad . \tag{43}$$

Changing variables as in the precedent section we arrive at

$$\varepsilon(\eta,T) = \frac{3}{\lambda a^4}\left[|f'|^2 + k\,|f|^2 + \frac{1}{2}|f|^4 + \frac{\lambda T^2 a^2}{9}|f|^2 + \gamma(T)\right] \tag{44}$$

with

$$\gamma(T) = \frac{\lambda a^4}{3} <\dot{\varphi}^* \dot{\varphi}> + \frac{\lambda^2 a^2}{18} <(\varphi^* \varphi)^2> + \frac{\lambda T^2 a^2}{18}\left[k + \left[\frac{a'}{a}\right]^2\right] + \frac{\lambda}{9} a'a\, T^2 \quad , \tag{45}$$

which contains no f.

On the other hand, the equation of motion (36) leads to

$$f'' + \left[k + \left[\frac{T}{T_c}\right]^2\right] f + |f|^2 f = 0 \tag{46}$$

where we have written

$$T_c = \frac{3}{\sqrt{\lambda}}\,\frac{1}{a} \tag{47}$$

anticipating its physical meaning. From (46) one easily gets

$$\frac{d}{d\eta}\left\{|f'|^2 + \left[k + \frac{T^2}{T_c^2}\right]|f|^2 + \frac{1}{2}|f|^4\right\} = 0 \tag{48}$$

or yet

$$|f'|^2 + \left[k + \left[\frac{T}{T_c}\right]^2\right]|f|^2 + \frac{1}{2}|f|^4 = C \tag{49}$$

so that, using (44),

$$\varepsilon(\eta,T) = \frac{3C}{\lambda a^4} \quad , \tag{50}$$

neglecting γ. As the space is homogeneous, the minimum ε will give the minimum energy. So, we have to look for the solution of (46) which gives, in (49), the smallest value for C.

For $k = 0$ or 1 it is clear from (49) that the lowest possible value for C is zero, and this corresponds to $f = 0$. The VEV of $\varphi(x)$ vanishes and no breakdown of symmetry occurs. For $k = -1$, however, interesting things occur. As (43) shows that the energy density does not depend on the phase of ρ (or f), it is enough to search among the *real* solutions of (49). The analysis of the previous section is therefore still valid and the lowest possible value for C is

$$C = -\frac{1}{2}\left[1 - \frac{T^2}{T_c^2}\right]^2 \tag{51}$$

corresponding to

$$|f|^2 = 1 - \frac{T^2}{T_c^2} . \qquad (52)$$

Notice that we must have $T < T_c$. For $T = T_c$, both f and C vanish and the breaking disappears, justifying the use of T_c to denote $\frac{3}{\sqrt{\lambda}}\frac{1}{a}$.

Another important feature is that T/T_c is independent of a, that is, is not affected by the evolution of the universe. This is, perhaps, the most attractive feature of this model.

The VEV of ϕ is, therefore, given by

$$\rho = \sqrt{\frac{3}{\lambda}}\, \frac{\sqrt{1 - \frac{T^2}{T_c^2}}}{a(t)}\, e^{i\theta} , \qquad (53)$$

for any value of the phase θ. The best choice to enhance the T–violating terms is to take ρ as an imaginary number. Introducing, at the same time, the eigenstates of C

$$\chi_1 = \frac{1}{\sqrt{2}}(\varphi + \varphi^*)$$

$$\chi_2 = \frac{1}{2\sqrt{2}}(\varphi - \varphi^*) \qquad (54)$$

which have $C = T = +1$ and $C = T = -1$ respectively, the Lagrangian can be put in the following form:

$$\mathcal{L} = \mathcal{L}_{\chi_1} + \mathcal{L}_{\chi_2} + \mathcal{L}_{\chi_3} + \mathcal{L}_{\chi_1,\chi_2} + \mathcal{L}_{\psi} + \mathcal{L}_{int} \qquad (55)$$

where

$$\mathcal{L}_{\chi_1} = \frac{1}{2} g^{\mu\nu}\, \partial_\mu \chi_1\, \partial_\nu \chi_1 - \frac{1}{2}\mu_1^2 \chi_1^2 - \frac{\lambda}{24}\chi_1^4 \qquad (56)$$

$$\mathcal{L}_{\chi_2} = \frac{1}{2} g^{\mu\nu}\, \partial_\mu \chi_2\, \partial_\nu \chi_2 - \frac{1}{2}\mu_2^2 \chi_2^2 - \frac{\lambda}{24}\chi_2^4 - \sqrt{2}\,\xi^2 |\rho|\, \chi^2 - $$

$$- \frac{\lambda}{3\sqrt{2}} |\rho|\, \chi_2^3 + \sqrt{2}\, |\dot{\rho}|\, \partial_0\, \chi_2 \qquad (57)$$

$$\mathcal{L}_{\chi_1,\chi_2} = -\frac{\lambda}{12}\chi_1^2 \chi_2^2 - \frac{\lambda}{3\sqrt{2}} |\rho|\, \chi_2\, \chi_1^2 \qquad (58)$$

$$\mathcal{L}_{\psi} = \bar{\psi}\,(i\gamma^\mu D_\mu - m)\, \psi$$

$$\mathcal{L}_{int} = \alpha \sqrt{2}\, \bar{\psi}\, \psi\, \chi_1$$

$$\xi^2 = \frac{\lambda}{3}|\rho|^2 - \frac{R}{6}$$

$$\mu_1^2 = \xi^2$$

$$\mu_2^2 = \frac{2\lambda}{3}|\rho|^2 + \xi^2$$

$$\rho = i\sqrt{\frac{3}{\lambda}}\, \frac{\sqrt{1 - \left(\frac{T}{T_c}\right)^2}}{a} .$$

This Lagrangian explicitely violates C and T, as there are vertices with even and odd powers of χ_2. It is clear that these effects cannot be made to vanish by an overall redefinition of phases.

4. CONCLUSIONS

The model for spontaneous breakdown of CP presented here has the interesting aspect of resisting, in the broken–symmetric phase, to the high temperatures of the universe. Actually, the breaking increases with the temperature, as $a(t)$ then decreases.

It is therefore to be expected that large CP violation can be obtained in the early universe notwithstanding the diminute violation observed nowadays.

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