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CHARMONIUM ABSORPTION IN HADRONIC MATTER BY COHERENT MULTIPLE SCATTERING

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Charmonium Absorption in Hadronic Matter by Coherent Multiple Scattering *

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Abstract

A quantum mechanical model for a heavy quark pair as produced in hadronic collisions is presented. It describes the dynamical time evolution of the pair, using a potential model, and simultaneously takes into account the interaction of the pair with surrounding hadronic matter while leaving the reaction zone. The physical concepts involved like coherence effects are studied and an expansion time scale is introduced for bound state formation. The model is applied to hadron–hadron, hadron–nucleus and nucleus–nucleus scattering and can account for a large part of the experimental observations.

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1 Introduction

One of the most exciting perspectives in present nuclear physics is a new state of matter emerging at high energies: Ordinary nuclear matter, clustered into hadrons, is expected to undergo a transition to a quasi free gas of quarks and gluons, dubbed quark–gluon–plasma [1]. Origin of this expectation are lattice QCD calculations predicting this phase transition to occur at a critical temperature of the order of 200 MeV [2]. Ultrarelativistic nucleus-nucleus collisions provide our experimental access to this high temperature range. A series of fixed target experiments at CERN using oxygen and sulfur beams has been carried out recently [3]. The energy of 200 GeV/n achieved might be sufficient to reach the critical energy density needed to form the plasma. Now, what could be a possible signal of plasma formation?

Among several characteristic signals suggested, the suppression of the J/Ψ -meson was expected to be one of the cleanest. Preceding the first experimental run, depletion of J/Ψ by the plasma phase was predicted as an effect of color screening of the heavy quark pair potential in the deconfined medium [4]. Since then, this idea has attracted considerable interest [5] - [14]. J/Ψ –production was recognized to proceed in two steps: Fast, quasi pointlike creation of the $c\bar{c}$–pair in a hard process, $\tau_{\text{prod}} \approx \hbar/M_{J/\Psi}$, and subsequent slow formation of the extended bound state, whose size is affected by the surrounding deconfined medium, $\tau_{\text{form}} \approx r_{\text{bound}}(T)/v_{\text{rel}}$ being of the order of 1 fm/c. Therefore only sufficiently fast $c\bar{c}$–pairs are liberated by the plasma as bound states, generating a kinematic-dependent suppression pattern.

In the μ-spectrometer experiment NA38, depletion of J/Ψ 's in central collisions with respect to peripheral ones was seen [15], showing a suppression of about 50% in the most violent reactions and a pattern similar to the one predicted by plasma models, except for a linear decrease of suppression with $p_\perp$ rather than a cut–off.

Though plasma models are very successful, some conceptual difficulty remains: Without considering the interaction of an evolving $c\bar{c}$–pair with its surrounding, the presence of a certain bound state is determined immediately by the creation of the pair (classically, by the $c\bar{c}$ relative momentum, as well as quantum mechanically by the time independent projection ⟨ bound state | initial $c\bar{c}$ state ⟩), leaving no room for a formation time. This has been noted already [12, 13, 14]. For a $c\bar{c}$–pair that interacts with its hadronic surrounding where it is produced it looks different, but the effect of the external interaction on the formation of bound states is yet unclear.

Alternative explanations leaving the hadrons confined, but keeping a very high density have been worked out [16] – [20] as well as hybrid models [21] and can account for the data also. The typical breakup–reaction of hadron gas models is $J/\Psi\,\pi \rightarrow D\bar{D}X$ being inserted in a phenomenological way, leaving open a more microscopic description as performed in [22].

In nuclear matter of normal density, stronger absorption of the J/Ψ as compared to the quasi not interacting continuum μ-pairs is also expected. This however can account only for a 10% suppression [23]. Models dealing with nuclear matter are applicable to pA collisions as well. In fact, some unexpected features are found already there [24] – [29], similar to effects seen in open charm production [30, 31]. The J/Ψ absorption cross section per nucleon is reduced for large nuclei and shows a dependence on kinematical variables $p_\perp$ and x_F. The $p_\perp$ dependence [32] as well as the one seen in nucleus–nucleus collisions can be explained satisfactorily by initial gluon scattering [33, 34, 35]. The physical effect causing the energy (x_F) dependence is however not fully understood yet [28, 36, 37]. When used as an input, these datas do not yield a sufficient suppression either [38].

An attempt has been made to tackle both problems by hadronic absorption involving the beforementioned concept of a formation time for the J/Ψ [39]. This raises again the question of the validity of this idea.

Consequently, a detailed treatment of the dominant underlying physical effects is necessary and will be undertaken in this work. The following questions are studied:

1. How does a heavy quark pair evolve in time while interacting with hadronic matter?
2. How do the underlying microscopic processes result in a possibly time dependent 'macroscopic' cross section?
3. What is the effect of multiple scattering?

In order to do so, a transparent quantum mechanical model of the dynamical evolution of the interacting charmonium is presented in the next section. A theoretical study of the model in single and multiple scattering processes is performed in the following two sections, respectively. In the last section, the model is applied to experimentally relevant reactions.

2 Modelling dynamical charmonium

Starting from the point where the heavy quark pair is created, we want to follow its motion along a path while it evolves, interacting simultaneously with hadrons. For a quantum mechanical description of the dynamics, the following ingredients are needed:

– time evolution of a free charmonium

– interaction with hadronic matter

– specification of the initial state $|\Psi_0\rangle$ at the time of the pair creation.

In this order, the model is assembled now.

Free charmonium, i.e. the $c\bar{c}$–pair not interacting with its surrounding, is described in its center-of-mass frame by a nonrelativistic potential model. This is justified by the large charm mass. To keep the calculation treatable analytically while accounting for the dominant physical effects, a realistic description is only required of the abundant low lying states J/Ψ, χ, Ψ' and of confinement. Therefore a harmonic oscillator potential is chosen, $V_{\mathrm{HO}}(\vec{r}) = \frac{m}{2}\omega^2\vec{r}^{\,2}$, providing analytic solutions in contrast to the more realistic Cornell potential $V_{\mathrm{QCD}} = -\frac{\alpha}{r} + \kappa r$. The harmonic potential is adjusted to give the correct ground state width corresponding to the physical size of J/Ψ, $\langle r^2\rangle_{J/\Psi}^{\mathrm{HO}\frac{1}{2}} = \langle r^2\rangle_{J/\Psi}^{\frac{1}{2}} = 0.45$ fm, r being the relative coordinate, describing the system diameter. This leads to a potential constant $\omega = 1.85\frac{c}{\mathrm{fm}}$. The corresponding level distance $\hbar\omega = 370$ MeV matches the low part of the experimental spectrum satisfactorily.

'Absorption' of J/Ψ merely stands for transitions into charmonium or open charm states that do not decay into μ-pairs. Here we focus on one mechanism that can cause such transitions. Since the initial $c\bar{c}$-pair has a very small spatial extension (see below), it is sensitive to gluonic fields inside the hadrons it scatters with. The interaction with gluons originating from the scatterer is considered in the following as the charm pair's dominant interaction mechanism. It can provide excitations of a J/Ψ and in general produce any transition between different charmonium states.

How can the gluonic interaction of the charm pair with one scattering hadron be described realistically ? A J/Ψ detected in any experiment is very fast, so the pair's interaction time with a typical hadron is small, $\tau_{\rm int} \sim \tau_{\rm had}/\gamma c$ in the charmonium frame. We can safely neglect intrinsic hadron dynamics and assume the gluonic field as static during this time. Due to the size of a typical hadron encountered (π's, nucleons) the mean $q\bar{q}$ or qq separation is sufficiently large to allow the intrinsic field to be described by a linear potential, i.e. the gluonic field is constant in the space region charmonium will traverse. Since the time the charm pair interacts with such a field is short, any non-abelian contributions to the interaction are expected to be small. Therefore, we do a step back from QCD and use its abelian counterpart, a 'color'-electric field. That is, we model the gluonic interaction charm-hadron by the analog scenario in QED: An e^+e^--pair, positronium, subject to intrinsic dynamics as described by a potential (which is different here from Coulomb-like) passes through a constant electric field caused by a color-condenser, the scattering hadron.

This treatment of color fields is somewhat analog to the one used in the colorelectric flux tube model of [40], where non-abelian effects have indeed been found to be small [41].

In this way, the scattering of J/Ψ with one hadron is described by the passage of a $c\bar{c}$-pair through a corresponding color condenser, the interaction being $H_{\rm int} = -e\,\vec{E}\cdot\vec{r} = -\kappa\,\hat{n}\cdot\vec{r}$ with the string constant $\kappa = 1$ GeV/fm and a unit vector $\hat{n}$ parallel to the hadron color string axis $\vec{r}_q - \vec{r}_{\bar{q}}$ that is randomly oriented. This gives the Hamiltonian of the model charmonium during its interaction with this particular hadron as

$$H_{c\bar{c}} = \frac{\vec{p}^{\,2}}{2m} + \frac{m}{2}\omega^2\vec{r}^{\,2} - \vec{\kappa}\cdot\vec{r} \tag{1}$$

with the relative coordinate $\vec{r}$, the coupling $e\vec{E} = \kappa\hat{n} = \vec{\kappa}$ and the reduced mass $m = m_c/2 = 0.8$ GeV. This is formulated in the charmonium rest system into which the scattering hadrons have to be transformed. Since the experimentally observed J/Ψ 's have high momenta, intrinsic motion of the scattering hadron can be neglected.

The problem of multiple scattering is formulated as the passage of a heavy quark pair, initially not being in an energy eigenstate, through a number of color condensers providing a field $\vec{\kappa}$ each. The fields of different hadrons are equal in magnitude, but randomly oriented, $e\vec{\kappa} = \kappa\hat{n}_j(\vec{x}_j)$ for the j'th hadron. To obtain physically relevant quantities, we later have to average over all possible random field configurations. During each interaction the pair's dynamics follows the Hamiltonian (1) with the corresponding field orientation. Setting the field to zero during some time allows us to describe scattering with a hadron gas of low density.

Note that the single field Hamiltonian, even including the interaction, still separates in space, allowing a simpler treatment in one dimension that we use in the following to perform the necessary calculations. Introducing a dimensionless variable x and a coupling constant b, $x = \sqrt{\frac{m\omega}{\hbar}}\,r$, $b = \sqrt{\frac{m\omega}{\hbar}}\,\frac{eE}{m\omega^2}$ giving the ground state unit width, the charmonium Hamiltonian can be written as

$$H = \frac{\hbar\omega}{2}\left[-(\frac{d}{dx})^2 + (x-b)^2\right] - \Delta E \qquad \text{with} \quad \Delta E = \frac{(e\,\vec{E})^2}{2m\omega^2}. \tag{2}$$

From this, the effect of the colorelectric field is immediately transparent. The energy eigenstates of free charmonium are just the oscillator eigenfunctions $|\phi_n\rangle$ with the corresponding energy levels, $H_0|\phi_n\rangle = H(b=0)|\phi_n\rangle = E_n|\phi_n\rangle$. The states of interacting charmonium are denoted by $|\overline{\phi}_n\rangle$, $H_{c\bar{c}}|\overline{\phi}_n\rangle = \overline{E}_n|\overline{\phi}_n\rangle$. From the Hamiltonian it is clear that they are just shifted free states with a different zero point energy $\overline{\phi}_n = \varphi_n(x-b)$ and $\overline{E}_n = E_n - \Delta E$. This property, being very helpful for the solution of the problem, is specific for the harmonic potential chosen for the charmonium.

Our treatment of the model charmonium implies the use of higher excited $c\bar{c}$–states above the $D\bar{D}$ threshold by the confining potential. This is justified by the different time scales of the processes: The passage of the $c\bar{c}$–pair even through a big nucleus proceeds much faster than the decay of a typical state like $\Psi(4415)$ into open charm.

There are other mechanisms that can cause a transition of J/Ψ into undetected states. The colorelectric field we used to describe the gluons can also interact magnetically, causing transitions like $J/\Psi \rightarrow \eta_c$. This is however not considered in the present work.

A color string flip process as used in hadron gas models is also a source of J/Ψ–absorption, yielding open charm mesons $D\bar{D}$ directly without an intermediate charmonium state. This process has been calculated in a related work [22]. The basic quantity determining the breakup probability is found to be the colorelectric moment $\mu \cdot \vec{r}$ of the charm pair with the coupling μ and the system radius $\vec{r}$. For the physical scattering processes however, the charmonium size is usually small. For this reason, we neglect in the following the contribution to absorption from the breakup mechanism, restricting ourselves to absorption by radial excitation only.

As mentioned in the beginning, the creation of a $c\bar{c}$–pair is an almost pointlike process. Even taking into account that NA38 is sensitive to J/Ψ's with an average $\bar{\gamma} = 10$, the length scale of $c\bar{c}$–production is $l_{\rm prod} \approx \hbar c\bar{\gamma}/M_{J/\psi} = 0.7$ fm since $\hbar/M_{J/\psi}$ is the time the pair can stay off–shell before turning into real partons. This justifies the view of a pointlike process. The $c\bar{c}$–pair, so created, will further not be in an eigenstate, but in some very narrow initial state $|\Psi_0\rangle$.

The most simple description of $|\Psi_0\rangle$ is a Gaussian with an appropriate width σ, $\Psi_0(x) = (\frac{1}{\pi\sigma^2})^{\frac{1}{4}}\, e^{-\frac{(x)^2}{2\sigma^2}}$. We will use Ψ_0 in the following as a rough approximation of the wave function of the initial $c\bar{c}$–pair. Adjusting σ to $l_{\rm prod}$ leads to $\sigma = 0.14$ in units of the ground state width.

The possibility to fix σ from experiment are rather limited. The ratio Ψ' to Ψ as measured by NA38, $r_{\Psi'/\Psi} = 0.13$ (corrected for branching ratios), is consistent with zero width of the initial state. Using the experimental ratios of open to hidden charm production

does not help either since these cross sections are measured from different experiments [42] that are not properly comparable.

Some care has to be taken when using highly excited states as contained in the wave function Ψ_0. Within a nonrelativistic treatment as ours, the use of these states is dubious since they correspond to high frequency components expanding faster than c. In addition, the use of a confining potential without including particle production is not justified for states above a certain threshold since these correspond to physical $c\bar{c}$–pairs that later break up into open charm mesons, escaping the $c\bar{c}$ confinement. These deficiencies can be tolerated since the contribution of Ψ_0 to the states in question is very little.

A δ–like function such as the Gaussian Ψ_0 can not perform a complete description of the initial state since it only contains even parity states like Ψ', Ψ that are produced by three gluon fusion. In addition, there is however an around 45% contribution to the $c\bar{c}$ production due to two gluon fusion that yields odd parity states like χ and has to be included also [43, 44, 45]. A better description of the initial state has to include all charmonium eigenstates. The condition that even and odd parity states must not interfere then requires a density matrix formulation.

The use of the Gaussian Ψ_0 only provides a schematic description of the initial state. For a more realistic treatment, a calculation of the charmonium production process is necessary to define the initial state more precisely.

3 Single Scattering Process

In this section, the scattering of a heavy quark pair with a single hadron is studied in detail. The use of different initial states for the pair will illustrate the concepts of coherence and the origin of a corresponding time dependence of the absorption cross section. The classically motivated concept of a wave function expanding in space is found to be meaningful and adequately an expansion time is introduced to substitute the misleading term formation time.

The model presented above can be applied straightforward to the collision process. For the J/Ψ 's registered experimentally, the longitudinal momentum dominates, $p_L \gg p_\perp$. Since absorption depends on the length travelled within hadronic matter and the contribution of $p_\perp$ to this is very little geometrically, $p_\perp$–effects are neglected in the following. Distances along the path of the $c\bar{c}$–pair are described timelike, i.e. as time intervals transformed from the hadronic into the $c\bar{c}$ rest system.

Consider now the following collision scenario: A $c\bar{c}$–pair is created at $t = 0$ as some initial state $|\Psi_0\rangle$ and propagates freely up to t_1, when it enters a hadronic scattering field. It stays in this for a time τ, leaving it at $t_2 = t_1 + \tau$. Repetition of this process, multiple scattering, will be treated in the next section.

When charmonium leaves the hadronic field at t_2, the quantity of interest is the probability to obtain the J/Ψ –state $|\phi_0\rangle$ at $t \geq t_2$, $P_{\Psi_0 \to \phi_0} = |\langle\phi_0|\Psi(t)\rangle|^2$.

To treat the scattering process, we first use the obvious expansion in energy eigenstates. Free propagation before entering the field, $0 \leq t \leq t_1$, is described by the trivial time evolution of the free charmonium eigenstates $|\phi_m\rangle$,

$$|\Psi(t)\rangle = \sum_{m=0}^{\infty} \langle\phi_m|\Psi_0\rangle e^{-iE_m t} |\phi_m\rangle . \tag{3}$$

The orthogonal eigenstates $|\phi_m\rangle$ propagate with a time dependent phase $e^{-iE_m t}$ but the probability of a single eigenstate to occur is at all times still $P_{\Psi_0 \to \phi_m} = |\langle\phi_m|\Psi_t\rangle|^2 = |\langle\phi_m|\Psi_0\rangle|^2$. Without an interaction, no formation time can be associated with this free propagation.

An interaction, here the onset of the hadronic field at t_1, produces transitions between different states, transferring charmonium into new eigenstates $|\overline{\phi_n}\rangle$. These again propagate independently through the field with a new phase each. The relative contribution of the old states to the $|\overline{\phi_n}\rangle$'s however depends on their actual phase at the beginning of the field. In this way, an interaction 'freezes' the relative phase relations at this moment t_1, mixing the old states coherently.

To see this, the free eigenstates are expanded in terms of the field eigenstates, $|\phi_m\rangle = \sum_{n=0}^{\infty} \langle\overline{\phi_n}|\phi_m\rangle\, |\overline{\phi_n}\rangle$, to give the time evolution for $t_1 \leq t \leq t_2$ in the field,

$$|\Psi(t)\rangle = \sum_{m=0}^{\infty} \langle\phi_m|\Psi_0\rangle e^{-iE_m t_1} \sum_{n=0}^{\infty} \langle\overline{\phi_n}|\phi_m\rangle\, e^{-i\overline{E_n}(t-t_1)}\, |\overline{\phi_n}\rangle .$$

Leaving the field at t_2 again causes a mixture of different states. At this point, the field eigenstates are expanded correspondingly again in terms of the free charmonium eigenstates. The free propagation for $t \geq t_2$, after the scattering process, is now described by the reexpansion of the field states in terms of the free ones. Here we stop the procedure, since for the moment we are interested in a single scattering process only, and project the wave function on the J/Ψ –state,

$$\langle\phi_0|\Psi(t)\rangle = e^{i\chi} \sum_{m=0}^{\infty} \langle\phi_m|\Psi_0\rangle e^{-i\omega m t_1} \underbrace{\sum_{n=0}^{\infty} \langle\phi_0|\overline{\phi_n}\rangle\, \langle\overline{\phi_n}|\phi_m\rangle\, e^{-i\omega n\tau}}_{w_{m\to 0}} ,$$

where the constant phase factors add up to the overall phase $e^{i\chi}$. $w_{m\to 0}$ defines transition amplitudes that describe the propagation of the free charmonium state $|\phi_m\rangle$ through the field into the J/Ψ–state $|\phi_0\rangle$, $w_{m\to 0} = \langle\varphi_0|e^{-iH_{\rm field}\tau}|\phi_m\rangle e^{i\chi'}$. The propagation of the initial state is composed in an obvious way of its expansion into the charmonium eigenstates and the transition of these states through the hadronic field. The resulting transition probability, the quantity of our interest, can be written in the compact form

$$P_{\Psi_0 \to \phi_0} = \left| \sum_{m=0}^{\infty} \langle\phi_m|\Psi_0\rangle e^{-i\omega m t_1}\, w_{m\to 0}(\tau) \right|^2 . \tag{4}$$

The algebra to compute the transition probabilities, eq.(4), can be carried out analytically. The mixing matrix elements can be expressed as a series in powers of the coupling constant b, with which the transition amplitudes can be figured out to be

$$w_{m\to 0} = \frac{1}{\sqrt{m!}} e^{-\frac{b^2}{2}(1-e^{-i\omega\tau})} \frac{b^m}{\sqrt{2^m}} [1 - e^{-i\omega\tau}]^m . \tag{5}$$

Note that each term of this sum corresponds to the result of a perturbative calculation of the respective order. However, there is no need to do this since the calculation can be handled in closed form. Now the central eq.(4) can be evaluated.

Consider first a J/Ψ as initial state. Its probability to survive the collision with the hadron as a J/Ψ is

$$P_{\phi_0\to\phi_0} = |e^{-iE_0 t_1}\, w_{0\to 0}(\tau)\,|^2 = e^{-b^2(1-\cos(\omega\tau))} \qquad (6)$$

which is clearly not dependent of t_1, the time passed before the collision. For a realistic scattering process, this can be expanded (note that τ gives the length of the hadronic field in the fast moving charmonium frame and is therefore small) as $P_{\phi_0\to\phi_0} \approx 1 - \frac{1}{2}b^2\omega^2\tau^2$. The coupling parameter b can be written in the form $b^2 = \frac{2}{3}\frac{\kappa^2}{(\hbar\omega)^2}\langle r^2\rangle_{J/\Psi}$. Field strength κ and the operation length τ of the field have to be transformed from the hadronic into the charmonium frame via $\tau \to t/\gamma$ and $\kappa^2 \to \gamma^2\kappa^2$ in the two transverse directions, the longitudinal one can be neglected since it is not transformed and therefore small compared to the transverse ones. This leads to the relativistically invariant expression for the absorption cross section $\sigma^{abs}_{J/\Psi\, h}$ of a J/Ψ with one hadron of geometrical transverse area A_{had},

$$\sigma^{abs}_{J/\Psi\, h} = A_{had}\, P^{abs}_{J/\Psi} = \pi\,\langle r^2\rangle_h\,(1 - P_{\phi_0\to\phi_0}) = \frac{2\pi}{9}(\frac{\kappa t}{\hbar})^2\langle r^2\rangle_{J/\Psi}\,\langle r^2\rangle_h\,. \qquad (7)$$

This result relates the absorption cross section to the geometric size of the scattering hadrons and the product of field strength and penetration time. Is this of physical relevance? Povh and Hüfner found an empirical relation between hadronic radii and their mutual cross sections, $\sigma^{abs}_{h_1 h_2} = k\,\langle r^2\rangle_{h_1}\,\langle r^2\rangle_{h_2}$ with the constant k determined from experiment [46]. There is good agreement with the result obtained from the scattering model, where the constant k can be related to the physical quantities involved. This makes us confident that we are working on a realistic description of the scattering process.

How does one have to understand the time dependence of the absorptive cross section? Its origin is not an expansion of the initial state, but simply the time charmonium is affected by the hadronic field, causing a quadratic increase of the cross section with time in the approximation used here. For larger hadronic fields, the cross section starts to oscillate according to $\sigma^{abs}_{J/\Psi\, h} = A_{had}\,(1 - P_{\phi_0\to\phi_0})$ with the exact expression, eq.(6) for the survival probability. But since the size of real hadrons falls in the validity range of the approximation for small τ, this is of no further physical significance.

In a realistic collision, charmonium is not produced as a J/Ψ eigenstate, but as some spatially strongly localized initial state. Seen in a semiclassical picture, it will expand subsequently, leading to an increase of its size and cross section that is often parametrized in terms of a characteristic time τ, dubbed formation time. Since no 'formation' of charmonium states in time occurs, a better term for the scale τ is expansion time. Now, is there any meaning of this classical picture?

To study this, we model a narrow initial state using a Gaussian wave packet $|\Psi_0\rangle$ with an appropriate small width σ (in units of the ground state width). Before scattering, the free state $|\Psi_0\rangle$ spreads out in time, to be more exact: its width oscillates forced by the confining potential as $\langle\sigma^2\rangle(t) = [\frac{1}{\sigma^2}\sin(\omega t)^2 + \sigma^2\cos(\omega t)^2]$.
As long as the evolving state does not collide, its distribution in terms of charmonium eigenstates remains fixed. A scattering field causes a mixture of eigenstates and consequently a dependence of the cross section on the time t_1 when the scattering occurs. Using the results of this section, this cross section $\sigma^{abs}_{J/\Psi\, h}(t_1,\tau)$ can be evaluated. The expansion coefficients needed are

$$\langle\phi_{2m}|\Psi_0\rangle = \sqrt{\frac{2\sigma}{1+\sigma^2}}\,\frac{\sqrt{(2m)!}}{2^m m!}\,[\frac{\sigma^2-1}{\sigma^2+1}]^m\,,$$

the odd ones vanish since $|\Psi_0\rangle$ contains only even parity states. The transition probability for the physical threedimensional wave function is

$$P_{\Psi_0\to\phi_0}(t_1,\tau) =$$

$$(\frac{2\sigma}{1+\sigma^2})^3 e^{-b^2(1-\cos(\omega\tau))} e^{-\frac{b^2}{2}(1-\frac{2\sigma^2}{1+\sigma^2})[(\cos(\omega t_2)-\cos(\omega t_1))^2-(\sin(\omega t_2)-\sin(\omega t_1))^2]}$$

and is converted into a cross section as

$$\sigma^{abs}_{J/\Psi\, h}(t_1,\tau) = A_{had}\,(1 - \frac{P_{\Psi_0\to\phi_0}}{|\langle\Psi_0|\phi_0\rangle|^2})\,. \qquad (8)$$

This is normalized to the geometrical hadron cross section $A_{\rm had}$ when all charm component is absorbed.

Charmonium will expand even during the collision. To stay close to the classical picture, we have consequently to compare the cross section $\sigma^{\rm abs}_{J/\Psi\,h}(t_1,\tau)$ with the size at the middle of the collision, $\langle r^2\rangle(t_1+\frac{\tau}{2})$. This comparison for a realistic collision with a proton is shown in Fig. 1. Note that no relativistic effects are taken into account, corresponding to a very slow J/Ψ and leading a large cross section. One finds an approximate proportionality,

$$\sigma^{\rm abs}_{J/\Psi\,h}(t) \approx k\,\langle r^2\rangle_{J/\Psi}(t)\,\langle r^2\rangle_h$$

that confirms (approximately) the classical image of an expansion. The time to reach the full cross section, the expansion time, is $\tau_{\rm exp} = \frac{\pi}{2\omega} = 0.85$ fm which agrees well with the estimates of this time scale mentioned before, being of the order of 1 fm.

The initial increase of $\sigma^{\rm abs}_{J/\Psi\,h}$ is quadratic which justifies the assumption in the model of Blaizot and Ollitrault [39] of a cross section increasing as $\sigma^{\rm abs}_{J/\Psi\,N}(\tau) = \sigma_0\,(\frac{\tau}{\tau_{J/\Psi}})^2$ that was based on the semiclassical picture of the increasing spatial extension of the pair in time. As we will see at the end of the next section, their assumed cross section behavior agrees with the one resulting from our model also for larger times. But before, we have to treat now the multiple collision process.

4 Coherent Multiple Scattering

Now we come to the central part of our study: How does charmonium scatter with several hadrons? To describe this, we first develop a transparent treatment and subsequently consider the scattering of a pre-fabricated J/Ψ and of a more realistic $c\bar{c}$ initial state. We will see how the results of the so calculated coherent scattering differ from the view of an independent collision process.

As before, we describe hadrons by their colorelectric field with a given length, associated with the hadron's size. Charmonium traversing a number of these fields will feel a sudden onset and end of each field. During the time the pair stays in the k'th field, $\tau_k = t_{k+1} - t_k$, its interaction follows a single field Hamiltonian H_k with orientation $\hat{n}_k$: the single collision process, eq.(1). Since we will consider scattering with only one type of hadrons in a specific reaction (nucleons or pions), the length τ_k will be equal for all fields, $\tau_k = \tau$, expressed in the charm frame. The entire scattering with n hadrons is described by a sum over these individual Hamiltonians, being switched on and off, to give the total time dependent Hamiltonian

$$H_{\rm tot}(t) = \sum_{k=0}^{n} H_k(t) \qquad \text{with} \qquad H_k(t) = H_k \cdot \Theta(t_k < t < t_{k+1}) \tag{9}$$

that is now employed to evaluate the entire scattering process.

The expansion into eigenstates, as we used in the previous section to illustrate the concepts of the model, is not appropriate to treat several fields. Instead, a direct calculation can be performed using a Green's function $G(x,x',t,0)$ that allows to evaluate the time evolution of the charmonium wave function in closed form, $\Psi(x,t) = \int dx' G(x,x',t,0)\Psi(x',0)$. The Green's function needed for the free charmonium oscillator potential is derived from the Schrödinger equation using appropriate boundary conditions to be

$$G(x,t,x',0) = \frac{1}{\sqrt{2\,\pi\,i\,\sin(\omega\,t)}}\,e^{\frac{i}{2\sin(\omega t)}\left[(x^2+x'^2)\cos(\omega t)-2\,x\,x'\right]}. \tag{10}$$

The effect of an external hadronic field is a shift of the state versus the potential and a lowering of the total energy. Since this can be described by an additional momentum, the qualitative behavior of a general wave function will remain unchanged. The wave function of interacting charmonium can therefore be written in terms of the free one as

$$\Psi_{field}(x,t) = \Psi_0(x-\overline{x}(t),t)\,e^{-i\overline{p}(t)(x-\overline{x}(t))}e^{i\phi(t)} \tag{11}$$

with the time dependent expectation values for position and momentum, $\overline{x}(t)$ and $\overline{p}(t)$. The time dependent phase $\phi(t)$, giving the action along the $c\bar{c}$ path, is of no further concern since we are only interested in the probability to obtain a certain state.

From this, the time evolution of the pair can be seen to be a superposition of an oscillation in width as in the free case and a shift of position and momentum caused by the fields. Since we are only interested in finding J/Ψ's in the end, we can avoid part of this labor, keeping track of the state width, by looking at the process in reversed time direction: $t \to -t$ results in

$$\langle\phi_0|e^{-iH_{tot}t}|\Psi_0\rangle \rightarrow \langle\phi_0|e^{iH_{tot}t}|\Psi_0\rangle = \langle\Psi_0|e^{-iH_{tot}t}|\phi_0\rangle \tag{12}$$

and corresponds to an interchange of initial and final state: The J/Ψitself is now scattered through the hadrons and projected on the initial state. This allows a more transparent treatment analog to the evolution of coherent states since ϕ_0, when scattered, always keeps its initial width.

Keeping track of the time evolution is now rather simple. We only need to follow the expectation values for position and momentum that obey the classical equations

$$\overline{x}_{k+1} = \overline{x}(t_{k+1}) = (\overline{x}_k - b_{k+1})\cos(\omega\tau) + \overline{p}_k \sin(\omega\tau) + b_{k+1} \qquad \text{and}$$

$$\overline{p}_{k+1} = \frac{1}{\omega}\cdot\frac{\partial\,\overline{x}_{k+1}}{\partial t} = -(\overline{x}_k - b_{k+1})\sin(\omega\tau) + \overline{p}_k\cos(\omega\tau)\,.$$

As before, the quantity of interest is the probability to find a J/Ψ after the interaction. It is obtained by projecting a state with unit width and expectation values $\overline{x}_n$ and $\overline{p}_n$ acquired by the interaction on the initial state Ψ_0. To keep track of the contributions from the different eigenstates, we write this in analogy to eq.(4) from a single collision as an eigenstate expansion

$$P_{\Psi_0\rightarrow\phi_0}(n) = |\langle\Psi_0|e^{-iH_{tot}t}|\phi_0\rangle|^2 = |\sum_{m=0}^{\infty}\langle\Psi_0|\phi_m\rangle e^{-i\omega m t_1}\, a_{m\rightarrow 0}(n)\,|^2 \tag{13}$$

where the multi transition amplitudes defined as $a_{m\rightarrow 0}(n) = \langle\phi_m|\prod_{k=0}^{n} e^{-iH_k\tau}|\phi_0\rangle$ contain the effect of all n fields. t_1 is the time before the first collision; from the structure of (13) it can be seen that the time dependence is identical to the one discussed for a single scattering process.

The amplitudes are readily evaluated in terms of the above expectation values with the result

$$a_{m\rightarrow 0}(n) = \frac{1}{\sqrt{m!}}\frac{\beta^m}{\sqrt{2^m}}e^{-\frac{|\beta|^2}{2}}e^{i\chi} \tag{14}$$

with an overall phase χ and $\beta = \overline{x}_n + i\overline{p}_n$, the values for position and momentum after the n'th field.

So far, the calculation was performed in one dimension. The extension of these results to the physical three dimensions is straightforward by using the corresponding 3–d–expectation values $\overline{\vec{r}_n^{\,2}} = \overline{x_n^2} + \overline{y_n^2} + \overline{z_n^2}$ and the momentum $\overline{\vec{p}_n^{\,2}}$.

The probability of a transition from the m'th eigenstate ϕ_m into ϕ_0 or, in the time-reversed view, of exciting a J/Ψ into the m'th eigenstate by the interaction takes the form

$$P_{\phi_m\rightarrow\phi_0}(n) = \frac{1}{m!}\epsilon_n^m e^{-\epsilon_n} \tag{15}$$

with the (dimensionless) energy picked up from the fields, $\epsilon_n = \frac{1}{2}[\overline{\vec{r}_n^{\,2}} + \overline{\vec{p}_n^{\,2}}]$. This is a Poisson distribution as expected for a coherent state.

With the results obtained so far, we turn now to a study of the physically relevant scattering processes.

J/Ψ -scattering.

First we will consider the scattering of a (pre-fabricated) J/Ψ. Its probability to survive the collision with n hadrons reads

$$P_{\phi_0\rightarrow\phi_0}(n) = e^{-\epsilon_n} \tag{16}$$

which reduces to eq.(6) for a single scattering process.

The energy transfer ϵ_n from the fields is still a function of the orientation $\{\vartheta_k,\varphi_k\}$ of all the n fields charmonium passed through. To describe the physically relevant quantity, we have to average (16) over all configurations with randomly oriented fields to obtain

$$\langle P_{\phi_0\rightarrow\phi_0}(n)\rangle = \frac{1}{(4\pi)^n}\int\prod_{k=1}^{n} d\Omega_k P_{\phi_0\rightarrow\phi_0}(n)\,. \tag{17}$$

The average energy transferred by the fields can be obtained in the same way with the result

$$\langle E(n)\rangle = \frac{\hbar\omega}{2}[\,3 + \langle\epsilon_n\rangle\,] = \frac{\hbar\omega}{2}[\,3 + n\cdot\epsilon\,]\,. \tag{18}$$

The average energy picked up increases linearly with the number of fields. The energy transfer parameter is $\epsilon = \frac{b^2}{2}(\omega\tau)^2 = \frac{(\kappa\tau)^2}{2m\hbar\omega} = \frac{1}{3}(\frac{\kappa\tau}{\hbar})^2\langle r^2\rangle_{J/\Psi}$. In the expression (16) for the probability, the energy transfer enters however in the exponential and prevents us so from performing the average (17) for more than one field analytically.

The transition probability $\langle P(n)\rangle$ shows a weak dependence on the charmonium momentum, caused by the term $\kappa \cdot \tau$ that enters via the energy transfer ϵ_n. Colorelectric field strength $\vec{\kappa}$ and the time τ to traverse a hadron have to be transformed to the fast moving $c\bar{c}$ frame via $\tau \to t/\gamma$ and $\vec{\kappa} \to \gamma\vec{\kappa}_\perp + \kappa_L$ in the directions transverse and parallel to the $c\bar{c}$–motion with t and κ expressed in the target frame. In the limit of high momenta, $\kappa\tau$ reduces to $\vec{\kappa}_\perp \cdot t$ and we only need to average the field orientations in the two transverse directions. This high energy limit is reached for $\gamma \geq 5$ and gives a transition probability independent of charmonium momentum. The contribution of the longitudinal field component causes a decrease of the survival probability, corresponding to an increase of absorption, towards lower momenta. The resulting dependence of absorption on the charmonium momentum is however rather weak (see Fig. 2, below).

Note that towards very low charm momenta, the model in the form presented here is not applicable for reasons of energy conservation. We used static classical fields to describe scattering hadrons that can in principle supply an unlimited amount of energy to the charmonium. At high energies (large x_F), where in addition the energy transfer is limited by the time the field can act upon the $c\bar{c}$–pair, sufficient energy is provided by the relative motion of J/Ψ and hadron. Towards low energies however there is few kinetic energy available, the hadrons are in their ground state before scattering and cannot supply energy for the $c\bar{c}$ excitation either. The model therefore overestimates the absorption at low momenta. We can estimate the applicability range of the model by requiring the average energy pickup to be lower than the charmonium kinetic energy, $\frac{\hbar\omega}{2}(\bar{n}\epsilon) \leq T(c\bar{c})$ where $\bar{n}$ is the average number of scattering fields (from the target nucleus, for instance). This leads to the limit of $\gamma \geq 2$ for our model to be applicable. For the 'real world physics' this plays no further role since the corresponding low momentum range is hardly reached experimentally.

The results for the survival probability of the J/Ψ with the average over all possible random field configurations performed numerically are shown in Fig. 2 for different final state momenta. It describes the (hypothetical) scenario of scattering a pre-fabricated J/Ψ with a pion gas of nuclear density. Note how the high energy limit is approached quickly with increasing charmonium momentum.

In contrast to the coherent picture, one can view the scattering as a sequence of separated single collisions without taking any interference into account. This would correspond to a survival probability decreasing exponentially, shown in Fig. 2 as the dashed line. This incoherent scattering is clearly faster in depleting J/Ψ 's since once the ground state is excited, it is lost. In contrast to this, the coherent model takes the 'feeding' from a state excited in one collision back to the ground state by means of a subsequent collision into account. For this reason, the resulting transition probability in the coherent picture is larger, corresponding to a lower value of the absorption.

The analytic form of the survival probability in the limit of high energies can be found to be

$$\langle P(n)\rangle = \frac{1}{1 + \epsilon_{\mathrm{HE}} \cdot n} \qquad (19)$$

where $\epsilon_{\mathrm{HE}} = \frac{2}{3} \cdot \epsilon = \frac{2}{9}(\frac{\kappa t}{\hbar})^2\langle r^2\rangle_{J/\Psi}$ since in this limit, only the transverse fields contribute. This is equivalent to a result found previously [47] in the eikonal approximation using a 'frozen' wave function, i.e. constant during the total interaction time. The high energy limit performed here corresponds to neglecting the intrinsic charmonium dynamics and the results therefore agree.

Scattering a realistic initial state.

Here we come to the physically most relevant part of our study: the multiple scattering of a narrowly formed $c\bar{c}$–pair while evolving in time. For a description of the initial state, we use again the schematic Gaussian $|\Psi_0\rangle$.

In which aspects will this more realistic scattering differ from a pure J/Ψinitial state as discussed before? As we have seen, the radius of the $c\bar{c}$ wave packet oscillates in time with a maximum at the expansion time $\tau_{\exp}$ in the charm frame. Increase of size corresponds to an increase of absorption. We will therefore intuitively expect a stronger decrease of the J/Ψ transition probability on the time scale $\tau_{\exp}$ in addition to the absorption caused by the pure interaction as we just discussed.

With the results obtained so far, the transition probability of the $c\bar{c}$–pair turning into a J/Ψ after multiple collision is readily evaluated to be

$$P_{\Psi_0\to\phi_0}(n) = (\frac{2\sigma}{1+\sigma^2})^3 e^{-\frac{\overline{\vec{r}_n^{\,2}}+\sigma^2\overline{\vec{p}_n^{\,2}}}{1+\sigma^2}} . \quad (20)$$

The first factor here is simply the overlap of initial and final state, $(\frac{2\sigma}{1+\sigma^2})^3 = |\langle\Psi_0|\phi_0\rangle|^2$. The second factor corrects this projection for the shift in position and momentum caused by the interaction, lowering the transition probability. For $\sigma = 1$ (i.e. J/Ψ as initial state), the result reduces to the simple form (16) discussed before. In contrast, here we cannot write it any more in terms of the energy transfer. Since $\sigma^2 \ll 1$, we can instead approximate it as $P(n) \approx |\langle\Psi_0|\phi_0\rangle|^2 \cdot e^{-\overline{\vec{p}_n^{\,2}}}$: $\overline{\vec{p}_n^{\,2}}$ is the quantity that determines now the transition probability.

Note that we calculated the scattering in backward time direction, following a J/Ψthrough the scattering hadrons. A dependence on the charm size can only occur when we project the unit width state with the information on the scattering process on the initial $c\bar{c}$ state. This dependence is contained in the quantity $\overline{\vec{p}_n^{\,2}}$.

For the average energy transfer $\langle\epsilon_n\rangle = \frac{1}{2}\langle[\overline{\vec{r}_n^{\,2}} + \overline{\vec{p}_n^{\,2}}]\rangle$ we found a linear increase with the number of fields. This is however true only for the sum of kinetic and potential energy. $\langle\overline{\vec{p}_n^{\,2}}\rangle$ only (the angular average of the momentum expectation value $\overline{\vec{p}_n^{\,2}}$) is some trigonometric function that causes a strong increase and subsequent flat behavior in steps of $\tau_{\exp}$, generating so an effective $\langle r^2\rangle$ dependence on this scale.

To obtain the transition probability corresponding to the physical scattering process, we have again to average $P(n)$ (numerically) over all possible field configurations. The result of a calculation for the scattering with nucleons, each providing a field during a time $t = 1$ fm/c in the nucleon frame, is displayed in Fig.'s 3 for different charmonium momenta.

The first, Fig. 3a, shows the transition probability, normalized to $P(0)$, in function of the number of fields passed through. Fig. 3b gives the corresponding absorption cross section, $\sigma_{\text{abs}} = A_{\text{Nuc}}[1 - \frac{P(n)}{P(0)}]$. This is normalized to the value of the geometrical nucleon cross section A_{Nuc} for the case that all J/Ψ contribution is absorbed in the collision.

We realize that the behavior of transition probability and cross section is dominated by the expansion of charmonium in size. Increase to maximum size occurs on the scale $\tau_{\exp} =$ 0.85 fm/c that has to be transformed to the nucleon frame with the corresponding factor γ. During this time, $0 \leq t \leq \gamma \cdot \tau_{\exp}$, the transition probability drops rapidly. For the subsequent period of recontracting in size, the shrinking (causing a decrease of absorption cross section) is balanced by the effect of the interaction (cross section increasing with the number of fields), resulting in a saturation–like, almost flat behavior of $P(n)$ and the cross section for times larger than $\gamma \cdot \tau_{\exp}$. As in the previous section, we find for the multiple scattering process that the intuitive, 'semiclassical' view of the $c\bar{c}$–pair expanding in time, causing the absorption cross section to follow the time dependence of the system size, is (approximately) justified by the outcome of the quantum mechanical calculation.

Now we can come back again to the calculation of Blaizot and Ollitrault (see note at the end of the last section) that was based on the assumption of a J/Ψ absorption cross section increasing quadratically and saturating after some time scale (see eq.(2) and Fig. 1 from [39]). The corresponding cross section here, now a result based on the intrinsic charm dynamics, is shown in Fig. 3b as the upper curve and agrees well with the behavior they assumed intuitively.

Towards high energy, dilatation of the expansion also slows down the decrease of the transition probability. Again, we can find a limit of high energies generating a linear behavior equivalent to the result found in [47],

$$\langle P_{\Psi_0\to\phi_0}(n)\rangle = \frac{|\langle\Psi_0|\phi_0\rangle|^2}{1+s\cdot n} \qquad \text{with} \qquad s = \frac{2\sigma^2}{1+\sigma^2}\cdot\epsilon_{\text{HE}} \quad (21)$$

with the energy transfer parameter at high energy, ϵ_{HE} from before. In contrast to the scattering of a J/Ψ, however, this limit is only approached for $\gamma \geq 40$ where the interaction dominates over the dilated expansion in size, restricting the applicability of (21) to a small range of very high momenta only.

5 Application to hA and AA' collisions

What can the scattering model so far achieve? To get some idea of it, we turn now from the theoretical study of the model to its application in hadron–nucleus and nucleus–nucleus collisions (hadron–hadron processes were already discussed in section 3). Calculating the J/Ψ yield when accounting for absorption, we will find that the model can explain part of the experimental results, but a study of the charm production process is further needed to complete our description.

In a hadronic picture as ours, collisions with nuclei merely consist of the superposition of independent collisions between the individual hadrons. No collective phenomena occur when turning from hadron–hadron to nucleus–nucleus collisions, the range where a hadronic model must be applicable uniformly.

First, let us consider hadron–nucleus (hA) collisions.

To produce a considerable J/Ψ yield, high energy beams are used. High momenta of the projectile hadrons allow us to neglect any energy loss of the projectile before creating a $c\bar{c}$–pair, described by the initial state $|\Psi_0\rangle$. Therefore we further assume the $c\bar{c}$–production to be uniformly distributed over the nuclear volume. Since we did not include a study of the production process yet, the model so far does not generate a dependence on the type of projectile hadrons. Up to variations that can be attributed to the different parton content of the projectiles, this property is supported by the experimental datas.

With this assumption of uniform $c\bar{c}$ production over the hadronic matter volume we can split the J/Ψ inclusive cross section in a production and an absorptive part as

$$\sigma(\mathrm{pA} \to J/\Psi\, X) = \mathrm{A} \cdot \sigma^{\mathrm{prod}}_{\mathrm{pN}\to\Psi_0} \cdot \overline{P_{\Psi_0\to J/\Psi}(\mathrm{A})}. \tag{22}$$

The last factor stands for the probability of the charm pair to end up in the J/Ψ state, averaged over all points of pair creation inside the nuclear volume. Using impact parameter b and the length z that gives the distance from the point of pair creation out of the nucleus along the beam direction, this averaging reads

$$\overline{P_{\Psi_0\to J/\Psi}(\mathrm{A})} = \overline{P(\mathrm{A})} = \frac{1}{\mathrm{V}} \int_0^R d^2b \int_0^{L(b)} dz\, P((n(z)) \tag{23}$$

with $n(z)$ being the number of nucleons encountered along the path z.

For an estimation, this can be approximated using the average number of nucleons $\bar{n}_\mathrm{A}$ hit after the pair is created,

$$\overline{P(\mathrm{A})} \approx P(\bar{n}_\mathrm{A}) \qquad \text{where} \qquad \bar{n}_\mathrm{A} = \frac{\bar{z}_\mathrm{A}}{d} \approx \frac{1}{2}\mathrm{A}^{\frac{1}{3}}$$

with the average path length $\bar{z}_\mathrm{A} = \frac{3}{4}R$ and the average nucleon distance d.

In the following we use however the exact form (23). Remember that the probability of J/Ψ yield depends strongly on the charmonium momentum, except in the limit of high energies where it reaches a momentum independent value. In this limit, the average can be performed analytically using $n(z) = [\frac{z}{d}]$ (integer) and gives the result

$$\overline{P(\mathrm{A})} = |\langle\Psi_0|\phi_0\rangle|^2 \cdot [1 - \frac{3}{8} s A^{\frac{1}{3}}] \tag{24}$$

with the constant $s = \frac{2\sigma^2}{1+\sigma^2} \cdot \epsilon_{\mathrm{HE}}$ from eq.(21).

Towards lower charmonium momenta, the average over the target nucleus is performed numerically. The dependence on charm momentum is expressed in terms of $x_F = p_{in}/p_{out}$. To compare the results for charmonium absorption with experimental datas, we use the common parametrization $\sigma(pA \to J/\Psi\, X) = \sigma_0 \cdot A^\alpha$. The model predictions are expressed in terms of this α, being a function of x_F, as deduced from

$$\sigma(\mathrm{pA} \to J/\Psi\, X) = \mathrm{A} \cdot \sigma^{\mathrm{prod}}_{pN\to\Psi_0} \cdot \overline{P(\mathrm{A})}$$

$$= \mathrm{A}^{\alpha_{th}} \cdot \sigma(\mathrm{pN} \to J/\Psi\, X) = \mathrm{A}^{\alpha_{th}} \cdot \sigma^{\mathrm{prod}}_{pN\to\Psi_0} \cdot P(0) \tag{25}$$

with $P(0) = |\langle\Psi_0|\phi_0\rangle|^2$. The result for the 'absorptive' α then reads

$$\alpha_{th} = 1 + \frac{\ln(\frac{\overline{P(\mathrm{A})}}{P(0)})}{\ln \mathrm{A}}. \tag{26}$$

Since $\overline{P(A)} < P(0)$, the value of α_{th} is always smaller than one, corresponding to a reduction of the absorption cross section per nucleon (as compared to a proton target) by means of the absorption.

In the high energy limit (23) this can be written, using the approximation $A^{\frac{1}{3}} \approx \ln A$ valid in our A range, in the form $\alpha_{th}^{HE} = 1 - \frac{3}{8}s$ with s from eq.(21). A calculation performed for Pt (A = 195) is shown in Fig. 4. The γ-dependence was translated for an initial beam momentum of 200 GeV in a dependence on x_F. The figure shows the parameter α as a function of x_F. Towards high momenta the high energy limit is applicable and the absorption approaches the constant value $\alpha_{th}^{HE} = 0.98$ as $x_F \rightarrow 1$. Towards lower x_F, the absorption increases, corresponding to a reduction of the cross section per nucleon.

The behavior of the absorption agrees with the intuitive picture that $c\bar{c}$–pairs will be less and less affected by the surrounding nuclear matter as their momentum increases.

The experimental results on inclusive charm (J/Ψ) production are usually parametrized in the same form,

$$\sigma(pA \rightarrow J/\Psi\, X) = A^{\alpha_{exp}} \cdot \sigma(pN \rightarrow J/\Psi\, X) = \sigma_0 \cdot A^{\alpha_{exp}}\,. \tag{27}$$

α_{exp} is determined by comparing the cross section measurements for two different target nuclei. Its value is measured to be lower than one and in addition is found by most experiments to depend on kinematical variables x_F and $p_\perp$ [24] – [29].

We compare our results with the detailed measurements of NA3 [27] who determined $\alpha_{exp} = 0.94 \pm 0.03$, averaged over all x_F. The measurement for the dependence of α_{exp} for a beam momentum of 200 GeV are shown in Fig. 4 for incident pions and protons on targets of hydrogen and platinum. For low x_F, a rather flat behavior of α_{exp} is seen which then drops towards high x_F, the decrease being stronger for protons than for pion projectiles.

As we see from Fig. 4, the results of the coherent absorption calculation are partially in disagreement with the experimental data. The values for α agree for low momenta, but the trend of the x_F–dependence towards large x_F is opposite for theoretical and experimental results.

The behavior of α, or the cross section per nucleon, approaching a constant value towards large x_F, is a general feature of absorption models of the form presented here. Taking, for instance, string breaking into account as has been done in [22], generates an identical dependence on charmonium momentum.

What can be the reason for this discrepancy? Up to now, we still did not consider the production process of the initial $c\bar{c}$–pair. Producing a $c\bar{c}$ at high x_F requires the availability of partons carrying a large momentum fraction. We know the parton (gluon and quark) distribution to be different in nuclei (here Pt) from protons. The difference for π and p projectile can be taken as a hint for the influence of the initial parton distribution: The large $\bar{q}$–content of the π can produce a $c\bar{c}$ via a Drell-Yan-like process that is experimentally known to follow an A^1-dependence, leading so to a flatter behavior in x_F than for proton projectiles with a low $\bar{q}$ content from sea quarks only.

Here we emphasize that the resulting behavior of α is a direct consequence of the assumptions of this kind of absorption models and is not caused by some insufficient calculation, i.e. this property can not be altered by an improvement of the model. So we have to look for the reasons of this discrepancy outside of the framework of the present model, calculating the differences in the x_F-dependence of the $c\bar{c}$-pair production in proton and nuclei caused by the different initial parton distributions. Since this has not been done quantitatively yet we must leave open further the possibility that the absorption model presented here does not provide an appropriate description of the entire process. Some of our assumptions, like the concept of a locally pointlike produced $c\bar{c}$–pair spreading out in time subsequently, might simply fail to describe the process correctly.

Other attempts, invoking a different (nonperturbative) production mechanism towards large x_F, fail to explain the data at high energies as well [36, 37].

We should mention here that the x_F dependence is not the only possible comparison with experiment. The results of coherent scattering depend on the total final state momentum. Since experimentally in most cases $p_L \gg p_\perp$, the parameter x_F is equivalent to the total momentum. For the small p_L range however, corresponding to scattering at large angles. $p_\perp$ dominates the total momentum. In this case, the dependence of α on x_F is identical to the one seen in Fig. 4, now as a function of $p_\perp$ instead of x_F. The corresponding range of p_L and $p_\perp$ is however hardly accessible by experiments.

At last, we consider nucleus–nucleus collisions (AA'), the most interesting application of the model.

When describing hA reactions, we found some discrepancy with experimental measurements in the cross section behavior at large x_F. Therefore, we cannot dare to perform a detailed study of nucleon–nucleon reactions at this stage. For this reason, we restrict ourselves to estimate the properties of the J/Ψ -suppression mentioned in the beginning as we expect it from absorption. This is rather safe since here the measured J/Ψ 's all stem from the low x_F region, where the description of the hA data from our absorption model is satisfactory.

J/Ψ suppression, the most prominent measurement of J/Ψ properties in AA' collisions, is performed by NA38 who determined the depletion of J/Ψ 's in central collisions with respect to peripheral ones [15]. Central and peripheral 'hits' are distinguished via the transverse energy released in the collisions.

The suppression is expressed as the ratio of the measured ratios J/Ψ signal versus continuum for different impact parameters,

$$R^{\mathrm{exp}} = \frac{\frac{J/\Psi}{\mathrm{continuum}}(\mathrm{central})}{\frac{J/\Psi}{\mathrm{continuum}}(\mathrm{peripheral})} \tag{28}$$

and the measured ratios for the most extreme energy bins are

$$R^{\mathrm{exp}}_{\mathrm{O-Cu}} = 0.82 \pm 0.22 \tag{29}$$

for Oxygen on Copper and

$$R^{\mathrm{exp}}_{\mathrm{O-U}} = 0.52 \pm 0.10 \quad \text{and} \quad R^{\mathrm{exp}}_{\mathrm{S-U}} = 0.50 \pm 0.08 \tag{30}$$

for Oxygen and Sulfur on Uranium [15].

To calculate the rate of suppression expected from absorption, we apply now the scattering model, following the experimental analysis. In a purely hadronic view as presented here, the projectile nucleus consists of A nucleons that scatter quasi independently with the target A', i.e. the effect is the same as performing A different pA' collisions with all at approximately the same impact parameter (the projectile nuclei used are small, A $\ll$ A'). We therefore use the same assumptions and an analysis identical to treating hA collisions. As there the model does not yield a dependence on the type of projectile nucleus (as long as it is small). This property is supported by the data that show nearly the same suppression rate for Oxygen as for Sulfur projectiles.

Experimentally, the distinction in central and peripheral collisions is performed using E_T, the transverse energy produced, as a trigger. To translate this in an impact parameter, we assume the released transverse energy to be proportional to the interaction volume of the respective collision. The ratio of $E_T^{\mathrm{max}}/E_T^{\mathrm{min}}$ is found to be equal for all reaction types, the corresponding ratio of the interaction volume

$$\frac{V^{\mathrm{cent}}}{V^{\mathrm{per}}} = \frac{E_T^{\mathrm{max}}}{E_T^{\mathrm{min}}} = 2.5 \tag{31}$$

allows us to estimate the impact parameter to $b^{\mathrm{cent}} = 0$ and $b^{\mathrm{per}} \simeq R - \frac{r}{4}$ with target radius R and projectile radius r.

In analogy to the experimental analysis, we define the ratio S as the number of produced J/Ψ's divided by the number of continuum μ pairs produced in a particular reaction. Dividing by the background, the numbers of projectile and target nucleons participating in a collision cancel. S still contains the unknown production cross sections $\sigma^{\mathrm{prod}}_{\mathrm{NN}\to\Psi_0}$ and $\sigma^{\mathrm{prod}}_{\mathrm{NN}\to\mu\mu}$. Taking the ratio $R^{\mathrm{th}} = \frac{S^{\mathrm{cent}}}{S^{\mathrm{per}}}$, these quantities drop out as well and we are left with

$$R^{\mathrm{th}}_{\mathrm{A-A'}} = \frac{\overline{P(\mathrm{A}'^{\mathrm{cent}})}}{\overline{P(\mathrm{A}'^{\mathrm{per}})}}, \tag{32}$$

the ratios of transition probabilities averaged over the interaction volume. To calculate these probabilities, we use the mean number of target nucleons hit by a projectile nucleon in a specific collision, $\bar{n}_{\rm U}^{\rm cent} = 9$, $\bar{n}_{\rm Cu}^{\rm cent} = 6$ for central and $\bar{n}_{\rm U}^{\rm per} \simeq \bar{n}_{\rm Cu}^{\rm per} = 3$ for peripheral collisions. The coordinates of $c\bar{c}$–pair production are assumed as before to be equally distributed over the interaction volume or the mean number of target nucleons hit, respectively. Averaging the transition probability for the specific reaction (t) then reads

$$\overline{P(A'^t)} = \frac{1}{\bar{n}_t} \cdot \sum_{k=0}^{\bar{n}_t} P(k)$$

which serves as an approximation for expression (23).

We use the results of calculating the absorption of a $c\bar{c}$–pair from section 4 to evaluate (32). The experimental J/Ψ acceptance peaks rather sharply at $x_F = 0.15$, according to an average $\bar{\gamma} = 10$ of the observed charm states. Using the corresponding values for the transition probability at this momentum (see the central curve in Fig. 3a) gives for the values of the J/Ψ suppression

$$R^{\rm th}_{\rm O-Cu} = 0.73 \qquad \text{and} \qquad R^{\rm th}_{\rm O-U} = R^{\rm th}_{\rm S-U} = 0.55 . \qquad (33)$$

Considering that this is a rough estimation only, the values for the suppression calculated from the absorption model agree well with the experimentally observed ones.

Note the strong dependence of the J/Ψ suppression predicted by the absorption mechanism on the final state momentum, x_F. Towards higher x_F, dilatation of the charmonium expansion in size leads to a reduction of absorption cross section, resulting in a lower suppression. In the same way, depletion of J/Ψ 's will increase towards low x_F. This dependence of the suppression on the charm momentum is a peculiarity of the absorption mechanism and allows for a straightforward experimental test about its physical significance.

6 Conclusions

The concern of the present work is a quantitative study of the effects present in the production of the heavy J/Ψ meson in hadronic collisions. To describe the relevant processes while simultaneously allowing for a simple and transparent treatment that can be handled largely analytically guided us to the formulation and evaluation of the absorption model presented. It consists of a quantum mechanical description of the dynamical evolution of an initially pointlike produced $c\bar{c}$–pair interacting simultaneously with hadrons, causing absorption via radial excitation. The study of the model, when describing different scattering scenarios, lead us to a better understanding of the dominating physical effects involved.

Considering the scattering with a single hadron, the absorption cross section of a pre-fabricated J/Ψ was evaluated. The dependence on the time in the hadronic field agrees with the behavior assumed previously, motivated from a semiclassical view. For an initially strongly localized $c\bar{c}$–pair, absorption and the momentary system size were found to be approximately proportional. This correlation causes the probability of bound state formation to depend on the time scale describing the spatial expansion of the charm pair.

Coherence in a multiple collision, allowing for the transition from a previously excited state back to the ground state in a later collision, reduces the absorption as compared to an equivalent number of independent single collisions. The scattering of a realistic $c\bar{c}$–pair is mainly governed by its system size increasing in time as compared to a pure J/Ψ state, whose absorption is determined merely by its hadronic interactions. The resulting time dependence of the absorption cross section agrees approximately with the one expected from a semiclassical view of the collision process.

A component still missing in the model is a description of the $c\bar{c}$ production process to define the initial state more precisely. Further, a calculation of the production yield as a function of final state momentum has to decide whether the discrepancy found when describing the A^α dependence at high x_F can be fully attributed to the different parton distributions in proton and nucleus targets or are due to an inadequate representation of some of the processes involved in the absorption model.

The J/Ψ suppression experimentally observed can be accounted for satisfactorily. The absorption mechanism predicts further a strong dependence of the suppression on final state momentum (x_F) that makes it experimentally differentiable from other models accounting for the suppression.

The applicability of the model presented here is not restricted to J/Ψ absorption in nuclear matter. It can be used to calculate the scattering properties of other bound states as long as they are well described by a nonrelativistic potential. Finally, the model is able to describe the interactions of mesons with a plasma phase also and will so allow to distinguish between the effects of a purely hadronic and a plasma-like phase quantitatively.

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A Figure Captions

Fig. 1a

Size and cross section of charmonium that evolves freely during a time t and then scatters with a hadron. The upper curve gives the size $\langle r^2 \rangle_{charm}$ at the middle of the collision in arbitrary units (scaled to allow for comparison) in function of the time before the collision. The lower curve is the absorption cross section of this collision. Both quantities are approximately proportional and reach their maximum at the expansion time $\tau_{exp} = 0.85$ fm/c.

Fig. 1b

Scattering scenario identical as for Fig. 1a. For better comparison it gives the charm cross section as a function of charmonium size when it is scattering. The proportionality between actual system size and absorption cross section as expected classically is seen to hold only approximately.

Fig. 2

Probability of a J/Ψ surviving a scattering process with pions of nuclear density is plotted versus the number of scattering pions, n. Full lines: Result for the coherent model for different charm momenta. The limit of high energies is approached quickly with increasing charm momentum. Dashed line: Incoherent scattering with $\gamma = 2$. Since scattering occurs only with integer number of fields, the points are connected by straight lines.

Fig. 3a

Multiple scattering of a realistic initial state (a narrow Gaussian) with nucleons. The transition probability, normalized to $P(0)$, is plotted versus the number of nucleon fields for different charm momenta. Note the strong decrease of $P(n)$ on the expansion scale in the nuclear frame, $\gamma\tau_{exp}$.

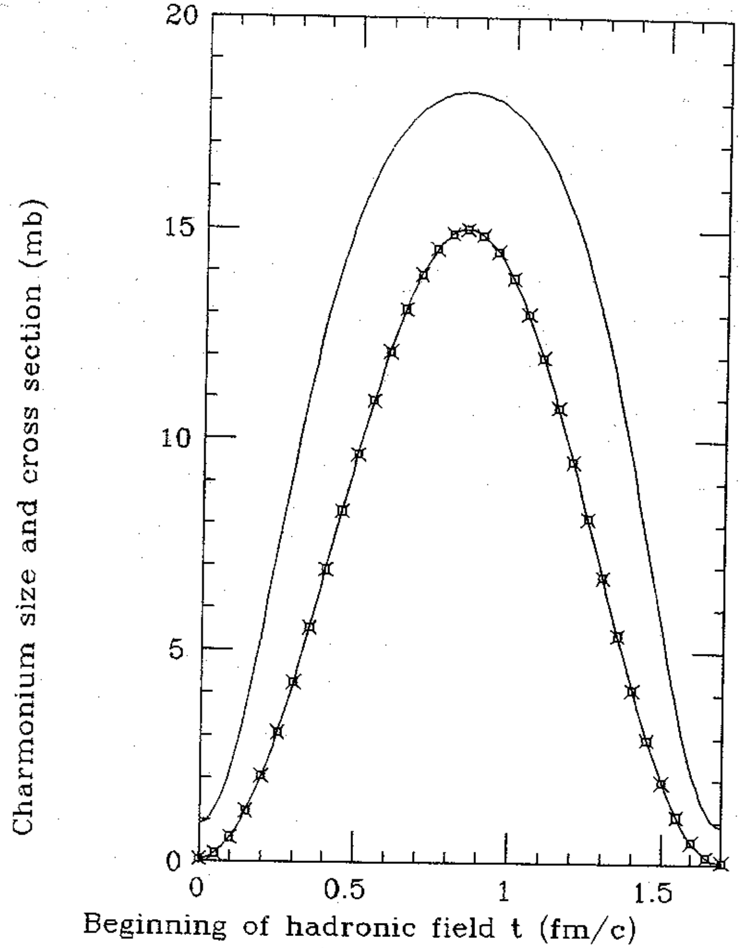
Fig. 3b

Absorption cross section of the $c\bar{c}$–pair scattering with nucleons, normalized to the geometrical nuclear cross section when all J/Ψ contribution is absorbed. The strong dependence of the transition probability on charm final state momentum (on the scale $\gamma\tau_{exp}$) from Fig. 3a appears here in terms of an absorptive cross section, causing larger absorption towards lower charmonium momentum.

Fig. 4

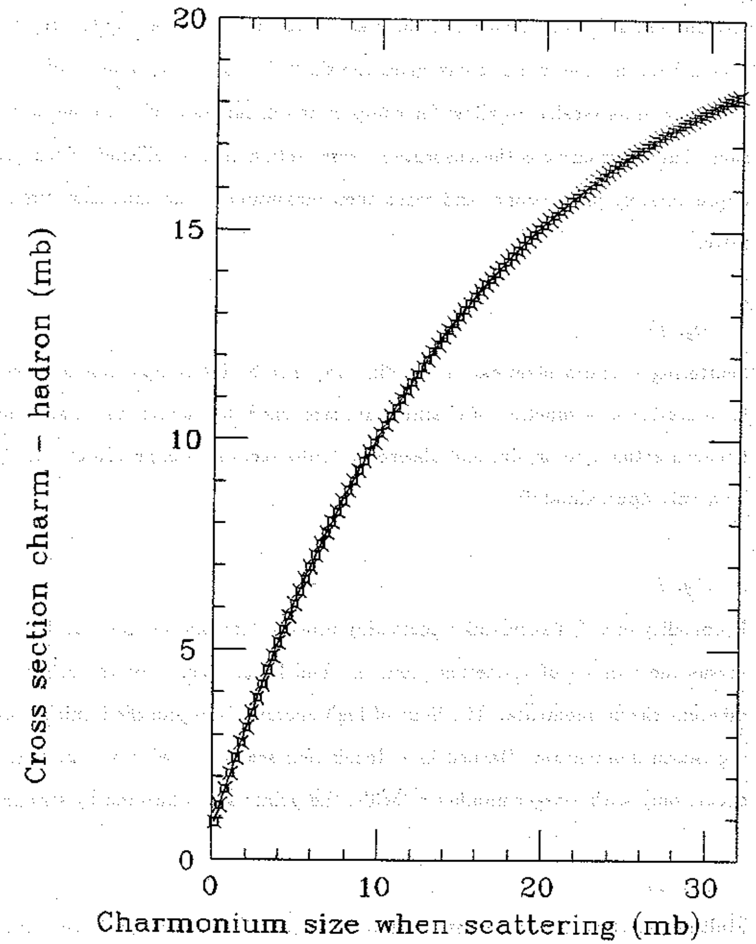
J/Ψ cross section per nucleon for a Pt nucleus (A = 195) as compared to the proton cross section, expressed in form of the parameter α being plotted here versus x_F. Results from experiment and the coherent scattering model are compared for a beam energy of 200 GeV/c. πA and pA scattering results are from experiment NA3 [27].

Fig. 1a



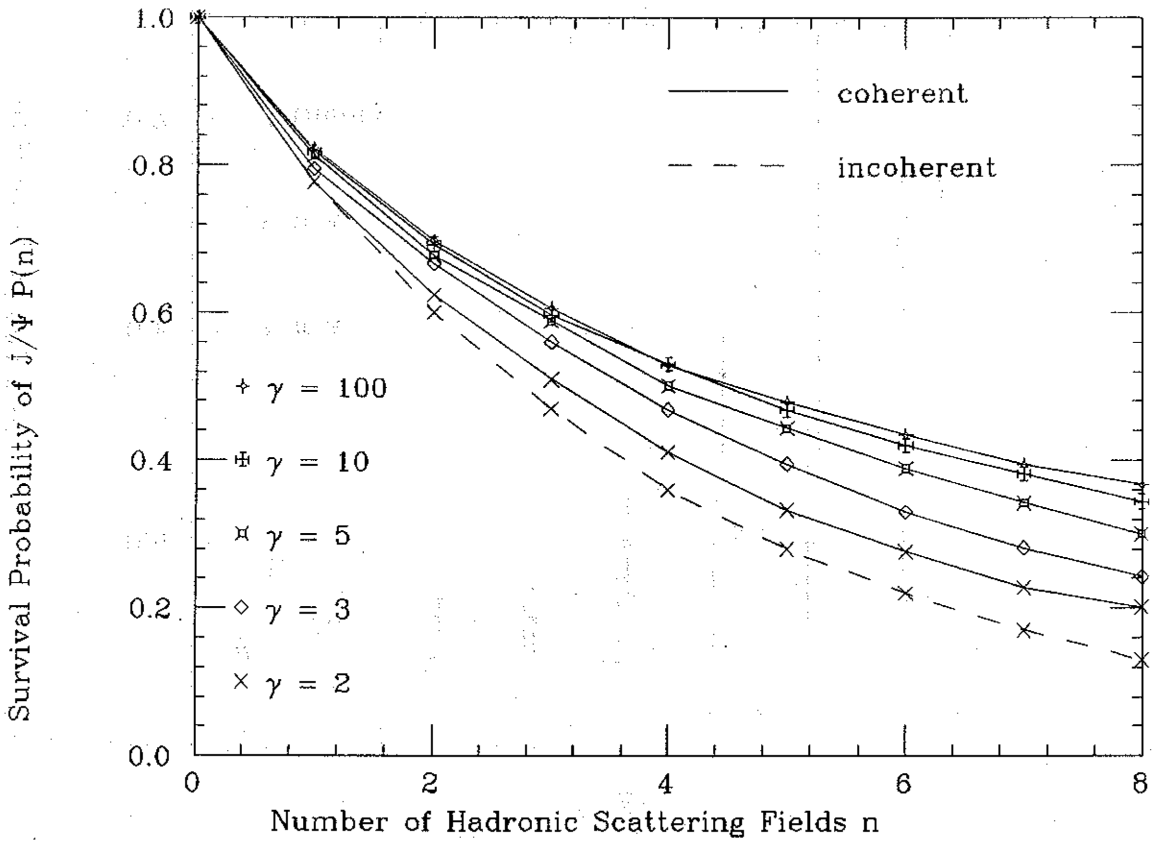
20
15
10
5
0
0
0.5
1
1.5
Charmonium size and cross section (mb)
Beginning of hadronic field t (fm/c)

Fig. 1b



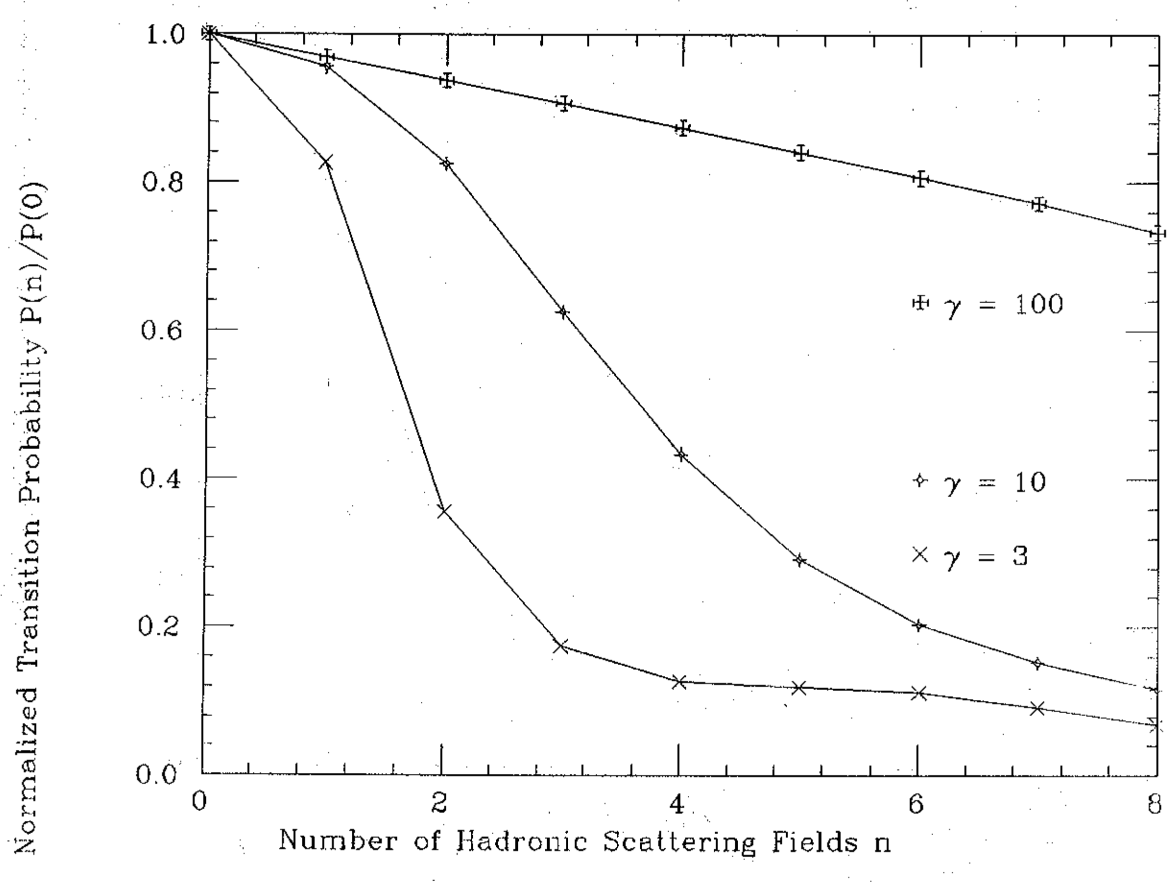
20
15
10
5
0
0
10
20
30
Cross section charm – hadron (mb)
Charmonium size when scattering (mb)

Fig. 2



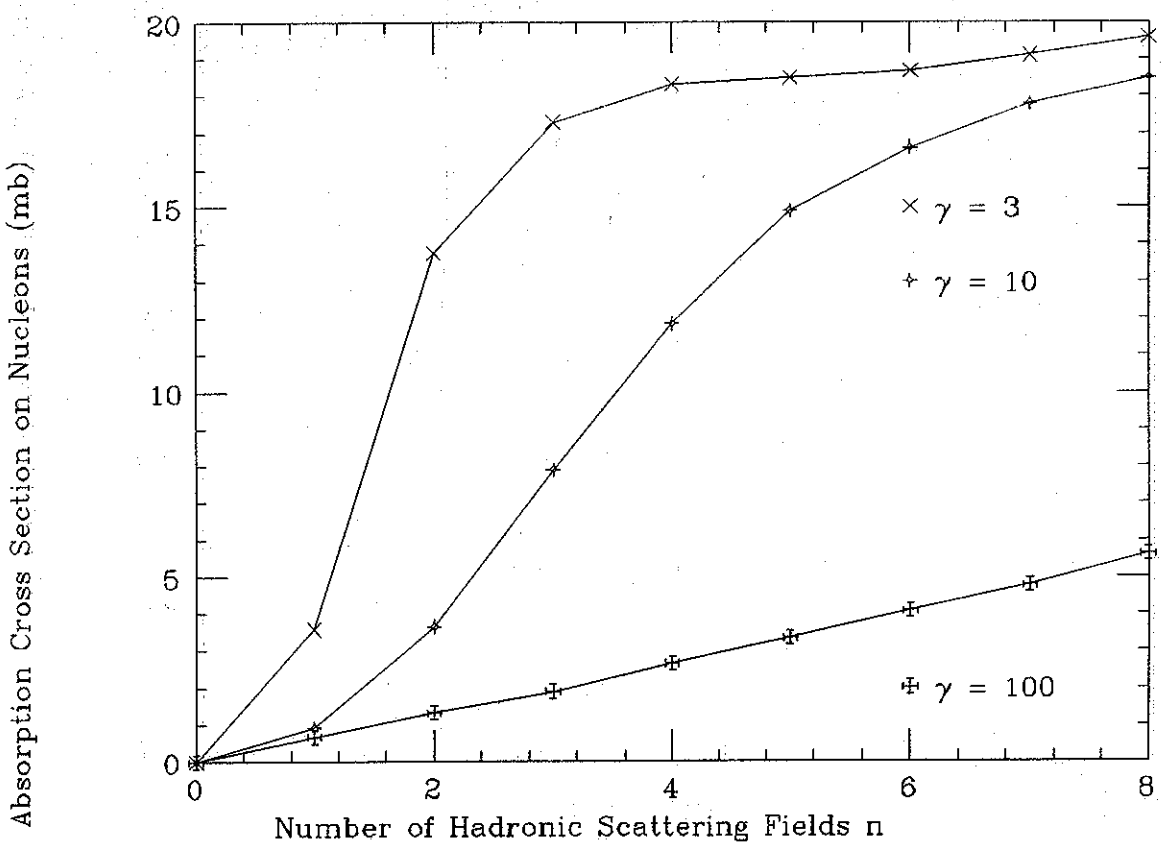
coherent
incoherent
γ = 100
γ = 10
γ = 5
γ = 3
γ = 2
Survival Probability of J/Ψ P(n)
Number of Hadronic Scattering Fields n

Fig. 3a



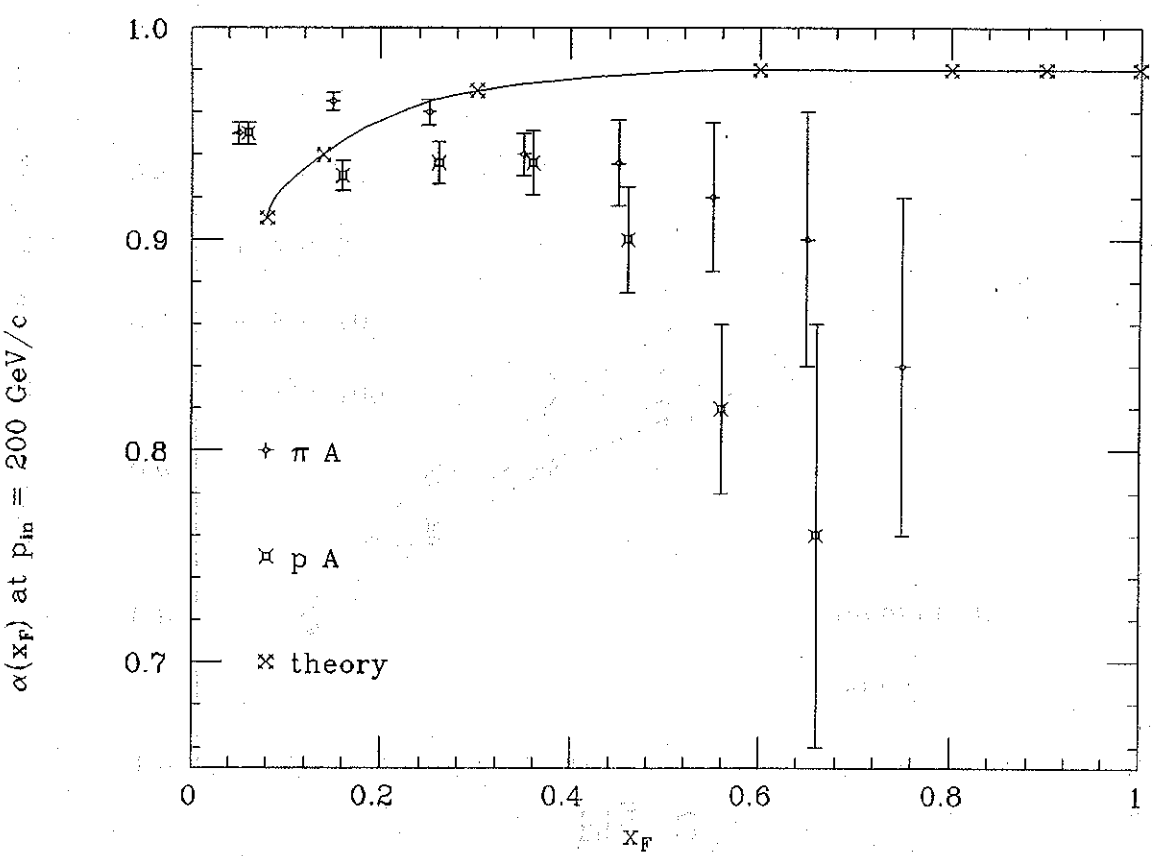
γ = 100
γ = 10
γ = 3
Normalized Transition Probability P(n)/P(0)
Number of Hadronic Scattering Fields n

Fig. 3b



Absorption Cross Section on Nucleons (mb)
Number of Hadronic Scattering Fields n
× γ = 3
+ γ = 10
± γ = 100
0
5
10
15
20
0
2
4
6
8

Fig. 4



α(x_F) at p_in = 200 GeV/c
x_F
π A
p A
theory
0.7
0.8
0.9
1.0
0
0.2
0.4
0.6
0.8
1