UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-874

BRST QUANTIZATION OF RELATIVISTIC SPINNING PARTICLES WITH A CHERN-SIMONS TERM

V.O. Rivelles and L. Sandoval Jr.
Instituto de Física, Universidade de São Paulo

Outubro/1990

BRST QUANTIZATION OF RELATIVISTIC SPINNING PARTICLES WITH A CHERN-SIMONS TERM

V.O.Rivelles *
and
L.Sandoval Jr.
Instituto de Física, Universidade de São Paulo
C.Postal 20516, 01498, S.Paulo, SP, Brasil

October, 1990

Abstract

We perform the BRST-BFV quantization of the theory of a relativistic spinning particle with $N = 2$ extended supersymmetry and a Chern-Simons term. We calculate the transition amplitude and show that it is proportional to the propagator of the field strength of an antisymmetric tensor. The local internal $SO(2)$ symmetry of the theory without the Chern-Simons term turns into a local external $O(2)$ symmetry when the Chern-Simons term is added to the action.

*Partially supported by CNPq

The step for going from a quantum mechanical system to a field theory has been recently shown to be a non-trivial one when local symmetries are involved. An outstanding example of this is string theory for which no satisfactory field theory was found starting from its first quantized version. This has led to the study of topological field theories as an alternative to find out the fundamental symmetries of string field theory.

The same situation seems to happen for particle theories. An action was proposed to describe a relativistic spinning particle with N extended local worldline supersymmetry and local worldline internal $O(N)$ symmetry [1]. Upon quantization it describes a particle with spin $\frac{N}{2}$. The quantization was performed through the Dirac technique of quantization for constrained systems [1] as well as through the BRST-BFV technique [2]. In both cases the relevant field wich appears associated to the spinning particle is the field strength for spin $\frac{N}{2}$ and hence in the massless case the gauge invariance for $N \geq 2$ is not manifest in those formulations. This had been noticed also in ref.[3]. As we will show, even with the addition of a Chern-Simons term to the action this problem remains.

The need for a Chern-Simons term arises because the theory has an anomaly for the local internal $O(N)$ symmetry and it is well defined only in even space-time dimensions where the anomaly does not spoil the internal symmetry [4]. One way to remove the anomaly for the $N = 2$ case is to add a Chern-Simons term to the action which cancels the anomaly and the theory becomes well defined in any dimension [4,5]. Due to the addition of the Chern-Simons term when the theory is quantized through the Dirac technique it is found out that it describes antisymmetric tensors [4]. In this paper we will quantize the theory in the BRST-BFV framework [6] which showed up to be a systematic way to obtain the propagator for a field theory starting from the point particle theory, as it was done for the relativistic and spinning particles [7] and the chiral particles [8].

Antisymmetric tensor field theories are typical examples of reducible gauge theories, i.e., theories which need ghosts for ghosts for ghosts, etc.. The most adequate formalism to treat such class of theories is the BRST formalism for reducible systems [9] since it provides the right counting for all ghosts and ghost numbers. It also provides an easy way to show the equivalence of antisymmetric tensor field theories to usual field theories at a given dimension [10]. Although being a reducible theory its mechanical counterpart which will be studied here is irreducible. This is so because, as we will show, the spinning particle transition amplitude (with the Chern-Simons term) is proportional to the propagator for the field strength of the antisymmetric tensor, which is gauge invariant, in a similar way to the spinning particle without the Chern-Simons term. Therefore the gauge symmetries responsible for the tower of ghosts for ghosts remains hidden in this formulation.

We start with the action [4]

$$
\begin{aligned}
S = & \int_{t_1}^{t_2} dt[\frac{1}{2V}(\dot{X}^\mu - i\chi_i\psi_i^\mu)(\dot{X}_\mu - i\chi_i\psi_{\mu i}) - \frac{1}{2}i\psi_i^\mu\dot{\psi}_{\mu i} - \\
& -f(\frac{i}{2}\epsilon_{ij}\psi_i^\mu\psi_{\mu i} + q - \frac{D}{2})] + \frac{i}{2}\psi_i^\mu(t_2)\psi_{\mu i}(t_1)
\end{aligned} \quad (1)
$$

where $X^\mu(t)$ is the particle coordinate, $\mu = 1 \ldots D$, $\psi_i^\mu(t)$ is the fermionic coordinate, $i, j = 1, 2$, $V(t)$ is the worldline einbein, $\chi_i(t)$ are the gravitinos, $f(t)$ is the $SO(2)$ connection, q is an

integer, $q-1$ being the rank of the antisymmetric tensor as will be shown latter. The action (1) is the usual action for the relativistic massless spinning particle with two supersymmetries (which is well defined only in even space-time dimensions) plus the Chern-Simons term $f(q - \frac{D}{2})$ which makes the theory well defined for any dimension D. It can also be viewed as an worldline $N = 2$ supergravity theory coupled to matter fields, with V, χ_i and f forming the supergravity multiplet. The boundary term in (1) is needed so that we can perform the variational principle on the fermionic coordinates having only one boundary condition for them.

The action (1) is invariant by worldline reparametrizations

$$
\begin{aligned}
\delta X^\mu &= \epsilon\dot{X}^\mu \\
\delta\psi_i^\mu &= \epsilon\dot{\psi}_i^\mu \\
\delta V &= (\epsilon V)^{\cdot} \\
\delta\chi_i &= (\epsilon\chi_i)^{\cdot} \\
\delta f &= (\epsilon f)^{\cdot}
\end{aligned} \quad (2)
$$

if the parameter of the transformation ϵ satisfies the boundary conditions $\epsilon(t_1) = \epsilon(t_2) = 0$. The action (1) is also invariant by local worldline $N = 2$ supersymmetry transformations

$$
\begin{aligned}
\delta X^\mu &= i\alpha_i\psi_i^\mu \\
\delta\psi_j^\mu &= -\frac{\alpha_j}{V}(\dot{X}^\mu - \chi_i\psi_i^\mu) \\
\delta V &= 2i\chi_i\alpha_i \\
\delta\chi_i &= \dot{\alpha}_i - \epsilon_{ij}\alpha_j f \\
\delta f &= 0
\end{aligned} \quad (3)
$$

if the supersymmetry transformation parameter α_i satisfies the boundary conditions $\alpha_i(t_1) = \alpha_i(t_2) = 0$. Finally, the action

(1) is invariant by a local worldline $SO(2)$ transformation

$$
\begin{aligned}
\delta X^\mu &= 0 \\
\delta \psi_i^\mu &= \epsilon_{ij} b \psi_j^\mu \\
\delta V &= 0 \\
\delta \chi_i &= \epsilon_{ij} b \chi_j \\
\delta f &= \dot{b}
\end{aligned}
\tag{4}
$$

Quite differently from the usual theories with internal symmetry where the gauge parameter does not satisfy any boundary condition, in this case the action (1) is invariant only if the $SO(2)$ parameter b satisfies the boundary conditions $b(t_1) = b(t_2) = 0$. This, of course, is due to the addition of the Chern-Simons term which makes it responsible for the changing of the internal $O(2)$ symmetry (in the absence of it) to an external $SO(2)$ symmetry. This is a rather interesting mechanism to interchange internal and external symmetries.

The above boundary conditions for the gauge parameters indicate that we must choose the gauge conditions $\dot{V} = \dot{\chi}_i = \dot{f} = 0$ in order to fully determine the gauge parameters.

To show that the theory has no anomaly we consider the path integral over the coordinates X^μ, ψ_i^μ of the exponential of the action (1) considering the gauge fields as background constant fields. They can be taken as constants in view of the gauge choice made above. The integral over the fermionic coordinates gives $\det^{D/2}(i\partial_t \delta_{ij} - \frac{f'}{\Delta T}\epsilon_{ij})$, where $f' = \Delta T(f + i\chi_1\chi_2/V)$ and $\Delta T = t_2 - t_1$. Using periodic boundary conditions we can calculate this determinant which after a regularization yields $\cos^D \frac{f'}{2}$. Then we get the path integral

$$
\begin{aligned}
W[f', \chi_i, V] &= \int \mathcal{D}X^\mu \cos^D \frac{f'}{2} \times \\
&\times e^{\int_{t_1}^{t_2} dt \frac{1}{2}\frac{\dot{X}^2}{V} - if'(q-\frac{D}{2}) - \frac{\chi_1\chi_2}{V}(q-\frac{D}{2})\Delta T}
\end{aligned}
\tag{5}
$$

Now upon a large gauge transformation in which $f' \rightarrow f' + 2\pi$ we have $\cos^D \frac{f'}{2} \rightarrow (-1)^D \cos^D \frac{f'}{2}$ and the exponential of the Chern-Simons term $e^{-if'(q-\frac{D}{2})} \rightarrow (-1)^D e^{-if'(q-\frac{D}{2})}$ so that $W[f', \chi_i, V] \rightarrow (-1)^{2D} W[f', \chi_i, V]$ and there is no restriction on the space-time dimension D. Then the theory is well defined in any dimension. This justifies the choice of constants in the Chern-Simons term.

From the action (1) we can derive, by standard techniques the first class constraints which give rise to the local symmetries (2) (3) (4); they are respectively

$$
\begin{aligned}
\mathcal{H} &= P^2 \\
\phi_i &= P^\mu \psi_{\mu i} \\
\phi &= \epsilon_{ij} \psi_i^\mu \psi_{\mu j} + 2q - D
\end{aligned}
\tag{6}
$$

where P_μ is the momentum canonically conjugated do X^μ and the fermionic coordinate satisfies the Poisson bracket

$$\{\psi_{\mu i}, \psi_{\mu j}\} = -i\eta_{\mu\nu}\delta_{ij} \tag{7}$$

The constraints (6) close the following Poisson bracket algebra

$$
\begin{aligned}
\{\phi_i, \phi_j\} &= \delta_{ij}\mathcal{H} \\
\{\phi, \phi_i\} &= \phi_i
\end{aligned}
\tag{8}
$$

with all the other brackets vanishing. It is easily seen that there is no linear combination of the constraints which vanishes without use of the constraint equations. Then the constraints (6) are ireducible.

Following the BRST-BFV prescription [6] we now extend the phase space of the theory introducing canonical momenta for the gauge fields (which act as Lagrange multipliers for the constraints): p_v for V, Π_i for χ_i and p for f. These momenta are now considered as new constraints. Now for each constraint we associate a pair of canonically conjugated ghosts $\overline{\mathcal{P}}$ and η for $\mathcal{H}$, $\mathcal{P}$ and $\overline{\eta}$ for p_v, $\overline{P_i}$ and c_i for ϕ_i, P_i and $\overline{c_i}$ for Π_i, $\overline{\mathcal{P}}'$ and η' for ϕ and $\mathcal{P}'$ and $\overline{\eta}'$ for p. The ghosts have opposite statistics to the constraints.

Then the BRST charge can easily be found using the algebra (8). It is given by

$$
\begin{aligned}
Q &= \eta\mathcal{H} + c_i\phi_i - \eta'\phi + \mathcal{P}p_v + P_i\Pi_i + \\
&\quad + \mathcal{P}'p + \epsilon_{ij}\overline{P_i}c_j\eta' - \frac{i}{2}\overline{\mathcal{P}}c_ic_i
\end{aligned} \tag{9}
$$

and it is easily seen to be nilpotent $\{Q,Q\} = 0$

To calculate the transition amplitude we choose the following boundary conditions which are invariant by the BRST transformations generated by (9)

$$
\begin{aligned}
X^\mu(t_1) = X_1^\mu &\;, X^\mu(t_2) = X_2^\mu \\
\frac{1}{2}(\psi_i^\mu(t_1) &+ \psi_i^\mu(t_2)) = \gamma_i^\mu \\
p_v(t_1) &= p_v(t_2) = 0 \\
\Pi_i(t_1) &= \Pi_i(t_2) = 0 \\
p(t_1) &= p(t_2) = 0 \\
\eta(t_1) &= \eta(t_2) = 0 \\
\overline{\eta}(t_1) &= \overline{\eta}(t_2) = 0
\end{aligned}
$$

$$
\begin{aligned}
c_i(t_1) &= c_i(t_2) = 0 \\
\overline{c_i}(t_1) &= \overline{c_i}(t_2) = 0 \\
\eta'(t_1) &= \eta'(t_2) = 0 \\
\overline{\eta}'(t_1) &= \overline{\eta}'(t_2) = 0
\end{aligned} \tag{10}
$$

where X_1^μ, X_2^μ and γ_i^μ are constants. It should be stressed the importance of the boundary conditions. If we had made another choice of boundary conditions for the fermionic coordinates then the boundary term in (1) would have to be changed and the resulting transition amplitude would be in another representation. For example, for N even, if the complex fermionic coordinate $\xi_a^\mu = \frac{1}{\sqrt{2}}(\psi_a^\mu + i\psi_{N/2+a}^\mu), a = 1,\ldots,N/2$, satisfy the boundary conditions $\xi_a^\mu(t_1) = \overline{\xi}_a^\mu(t_1) = 0$ then we get the transition amplitude in a coherent state representation [11]. With the boundary conditions (10) we will get the propagator in the usual momentum representaion.

We now have to choose the gauge fixing fermion Ψ which implements the above mentioned gauge choices $\dot{V} = \dot{\chi}_i = \dot{f} = 0$. If we choose the conventional gauge fixing fermion $\Psi = \overline{\mathcal{P}}V + \overline{P_i}\chi_i + \overline{\mathcal{P}}'f$ we end up with a complicated ordinary integral over $f(0)$, involving trigonometric functions, which we were not able to solve. So we looked for another Ψ which could make the functional integrations feasible. Since by the Fradkin-Vilkovisky theorem [6] the transition amplitude is independent of Ψ we can choose it at will. We then chose Ψ to be

$$
\Psi = \overline{\mathcal{P}}V(\frac{1}{2}\epsilon_{ij}\gamma_i\gamma_j)^{-(q-1)} + i\overline{P_i}\chi_i \tag{11}
$$

which is dependent on the fermionic boundary conditions as well as on the parameter q. The absence of the term $\overline{\mathcal{P}}'f$ in (11) implies that the effective action will not have the constraint ϕ in the standard form $f\phi$ so it is necessary that the parameter q (as well as $\epsilon_{ij}\psi_i\psi_j$) should be introduced in another place.

Then the effective action is given by

$$\begin{aligned} S_{eff} &= \int_{t_1}^{t_2} dt[P\dot{X} + p_v\dot{V} - \frac{i}{2}\dot{\psi}_i\psi_i - i\Pi_i\chi_i + p\dot{f} + \\ &+\overline{\mathcal{P}}\dot{\eta} + \mathcal{P}\dot{\overline{\eta}} + P_i\dot{\overline{c_i}} + \overline{P_i}\dot{c}_i + \mathcal{P}'\dot{\overline{\eta}}' + \overline{\mathcal{P}}'\dot{\eta}' - \\ &-\{Q,\Psi\}] + \frac{i}{2}\psi_i^\mu(t_2)\psi_{\mu i}(t_1) \end{aligned} \qquad (12)$$

where

$$\begin{aligned} \{Q,\Psi\} &= -V\mathcal{H}(\frac{1}{2}\epsilon_{ij}\gamma_i\gamma_j)^{-(q-1)} - \mathcal{P}\overline{\mathcal{P}}(\frac{1}{2}\epsilon_{ij}\gamma_i\gamma_j)^{-(q-1)} + \\ &+i\chi_i\phi_i + \epsilon_{ij}\overline{P_i}\chi_j\eta' - \overline{\mathcal{P}}c_i\chi_i + P_i\overline{P_i} \end{aligned} \qquad (13)$$

The transition amplitude is given by

$$Z[X_1, X_2, \gamma_i] = \int \mathcal{D}\mu e^{-iS_{eff}} \qquad (14)$$

where S_{eff} is given by (12) and the measure is

$$\begin{aligned} \mathcal{D}\mu &= \mathcal{D}X^\mu\ \mathcal{D}P_\mu\ \mathcal{D}\psi_i^\mu\ \mathcal{D}V\ \mathcal{D}p_v\ \mathcal{D}\chi_i\ \mathcal{D}\Pi_i\ \mathcal{D}f\ \mathcal{D}p\ \mathcal{D}\overline{\eta}\mathcal{D}\mathcal{P} \\ &\mathcal{D}\eta\ \mathcal{D}\overline{\mathcal{P}}\ \mathcal{D}\overline{c_i}\ \mathcal{D}P_i\ \mathcal{D}c_i\ \mathcal{D}\overline{P_i}\ \mathcal{D}\eta'\ \mathcal{D}\overline{\mathcal{P}}'\ \mathcal{D}\overline{\eta}'\ \mathcal{D}\mathcal{P}' \end{aligned} \qquad (15)$$

We now perform the functional integrals over the momenta of the gauge fields. The functional integral on p_v gives $\delta[\dot{V}]$ which leaves an undetermined factor det ∂_t which can be absorbed in the overall normalization of Z. Then the functional integral becomes an ordinary integral over $V(0)$, whose integration limits are taken from 0 to ∞ as required by causality. Analogously the functional integral over Π_i gives $\delta[\dot{\chi}_i]$ which leaves us with an ordinary (Berezin) integral over $\chi_i(0)$. We now make a

change of variables $\mathcal{F} = \frac{f}{\Delta T}$ so that $\mathcal{F}$ is dimensionless. Then the functional integral over p gives $\delta[\dot{\mathcal{F}}]$ which turns out into an ordinary integral over $\mathcal{F}(0)$. Being the gauge field of the $O(2)$ symmetry the integration limits are taken from 0 to 2π.

The functional integration over the fermionic ghosts $\mathcal{P}, \overline{\mathcal{P}}, \eta$ and $\overline{\eta}$ leaves us with a det $\partial_t^2 = \Delta T$. The integration over the bosonic ghosts $P_i, \overline{P_i}, c_i$ and $\overline{c_i}$ leaves us with a $\det^{-2}\partial_t^2 = (\Delta T)^{-2}$. Finally the integration over the fermionic ghosts $\mathcal{P}'$, $\overline{\eta}'$, $\overline{\mathcal{P}}'$ and η' leaves us with an undetermined factor of $\det^2 \partial_t$ which can also be absorbed in the overall normalization of Z.

With all these functional integrals performed the effective action (12) reduces to

$$\begin{aligned} S_{eff} &= \int_{t_1}^{t_2} dt[P\dot{X} - \frac{i}{2}\dot{\psi}_i\psi_i + \frac{V\mathcal{H}}{(\gamma_1\gamma_2)^{q-1}} + i\phi_i\chi_i] + \\ &+\frac{i}{2}\psi_i(t_2)\psi_i(t_1) \end{aligned} \qquad (16)$$

and the transition amplitude (14) to

$$\begin{aligned} Z[X_1, X_2, \gamma_i] = \int_0^\infty dV(0) \int d\chi_i(0) \int_0^{2\pi} d\mathcal{F}(0) \times \\ \times \int \mathcal{D}X_\mu\ \mathcal{D}P_\mu\ \mathcal{D}\psi_i^\mu(\Delta T)^{-1}\ e^{iS_{eff}} \end{aligned} \qquad (17)$$

We now perform the folowing change of variables (whose Jacobian is equal to one)

$$\begin{aligned} X^\mu(t) &= X_1 + \frac{\Delta X^\mu}{\Delta T}(t_2 - t_1) + Y^\mu(t) \\ \psi_i^\mu(t) &= \gamma_i^\mu + \tilde{\psi}_i^\mu(t) \end{aligned} \qquad (18)$$

where $\Delta X^\mu = X_2^\mu - X_1^\mu$ and with the following boundary conditionn for Y^μ and $\tilde{\psi}_i^\mu$

$$Y^\mu(t_1) = Y^\mu(t_2) = 0$$

$$\tilde{\psi}_i^\mu(t_1) \; + \; \tilde{\psi}_i^\mu(t_2) = 0 \tag{19}$$

The functional integral over Y^μ reduces the functional integral over P^μ to an ordinary integral over $p^\mu = P^\mu(0)$. The functional integration over $\tilde{\psi}_i^\mu$ gives a $\det \partial_t$ which can be calculated with the periodic boundary conditions (19) and the result, after regularization, is independent of ΔT.

Finally, performing the ordinary integrals over $V(0), \chi(0)$ and $\mathcal{F}(0)$ we obtain the transition amplitude

$$Z[X_1, X_2, \gamma_i] = \int dp \frac{e^{ip\Delta X}}{p^2} p_\mu \gamma_1^\mu p_\nu \gamma_2^\nu (\gamma_1^\rho \gamma_{2\rho})^{q-1} \tag{20}$$

with all factors of ΔT cancelling out. We will now show that this transition amplitude corresponds to the propagator of the field strength of an antisymmetric tensor of rank $q-1$.

First we take a representation for the γ_i^μ in terms of Dirac gamma matrices such that (7) is satisfied

$$\{\gamma_i^\mu, \gamma_j^\nu\} = \eta^{\mu\nu}\delta_{ij} \tag{21}$$

Such a representation for the γ_i^μ is [1]

$$\begin{aligned} \gamma_1^\mu &= \gamma^\mu \otimes 1 \\ \gamma_2^\mu &= \gamma^\star \otimes \gamma^\mu \end{aligned} \tag{22}$$

where $\gamma^\star = \gamma_1 \dots \gamma_D$. For odd dimensions we take a reducible representation of the gamma matrices so that $\gamma^\star$ anticommutes with all γ^μ. Now we have to take into account the ordering of the γ_i^μ in (20). Since the γ_i^μ do not anticommute we antisymmetrize all γ_1^μ and all γ_2^μ so that (20) becomes

$$\begin{aligned} &Z[X_1, X_2, \gamma_i] = \\ &= \int dp \frac{e^{ip\Delta X}}{p^2} p_\mu \gamma_1^{[\mu} \gamma_1^{\rho_1} \dots \gamma_1^{\rho_{q-1}]} p^\nu \gamma_{2[\nu} \gamma_{2\rho_1} \dots \gamma_{2\rho_{q-1}]} \end{aligned} \tag{23}$$

Using (22) we find

$$\begin{aligned} Z[X_1, X_2, \gamma_i] = \int dp \frac{e^{ip\Delta X}}{p^2} \times \\ \times p_\mu (\gamma^{[\mu} \gamma^{\rho_1} \dots \gamma^{\rho_{q-1}]} (\gamma^\star)^{q-1})^{\beta_1}_{\alpha_1} p^\nu (\gamma_{[\nu} \gamma_{\rho_1} \dots \gamma_{\rho_{q-1}]})^{\beta_2}_{\alpha_2} \end{aligned} \tag{24}$$

and multiplying (24) by $((\gamma^\star)^{q-1} \gamma^{[\mu_1} \dots \gamma^{\mu_q]})^{\alpha_1}_{\beta_1} (\gamma_{[\nu_1} \dots \gamma_{\nu_q]})^{\alpha_2}_{\beta_2}$ we obtain, up to numerical factors,

$$\int dp \frac{e^{ip\Delta X}}{p^2} p_\mu p^\nu \delta^{\mu\mu_1 \dots \mu_q}_{\nu\nu_1 \dots \nu_q} = < F^{\mu_1 \dots \mu_q}, F_{\nu_1 \dots \nu_q} > \tag{25}$$

where $\delta^{\mu_1 \dots \mu_q}_{\nu_1 \dots \nu_q} = \delta^{\mu_1}_{\nu_1} \delta^{\mu_2}_{\nu_2} \dots \delta^{\mu_q}_{\nu_q} + permutations$. Of course (25) is precisely the propagator of $F_{\mu_1 \dots \mu_q} = \partial_{[\mu_1} A_{\mu_2 \dots \mu_q]}$.

Like in the case without the Chern-Simons term the gauge symmetries of the corresponding field theory is not manifest in this formulation. The same happens in the Dirac quantization where the gauge symmetry is hidden in the Bianchi identities. This seems also to happen in string theory. In the path integral formulation of the Ramond-Ramond sector of the spinning string the massless fields in the expansion of the wave functional appears through their field strengths rather than their gauge potentials [12]. This shows that we do not have still an appropriate formalism to find out the gauge symmetries of a second quantized theory starting from its first quantized version. May be we need to introduce more symmetries at the classical level in order to generate the gauge symmetries of the corresponding field theory. How this can be done however is not known at present.

Another interesting point which should be stressed is the conversion of the internal $O(2)$ symmetry into an external

$SO(2)$ symmetry when the Chern-Simons term is added to the action. To each external symmetry we can associate a a local transformation on the manifold in which the symmetry is defined. Reparametrization symmetry is associated to a local transformation of the proper-time $t \to t'(t)$. Local supersymmetry is associated to a local transformation in a Grassmannian direction (a sort of fermionic time) which allowed to rewrite the theory in a $N=2$ worldline superspace whose coordinates are t and θ_i [4]. Now the new $O(2)$ external symmetry allows the introduction of a new bosonic coordinate which would enlarge the $N=2$ worldline superspace to have two bosonic coordinates and two fermionic coordinates. A superfield formulation on this superspace is presently under investigation. A $N=1$ worldline superspace has allowed the derivation , e.g., of the Atiyah-Singer index formula [13] and the Weyl character formula [14] in an elementary way and it may be possible that this new $N=2$ wordline superspace be helpful in proving other mathematical theorems.

As a last remark we would like to mention that the massive case can be also treated by the BRST-BFV technique. We add to the action (1) the mass term

$$S_m = \int_{t_1}^{t_2} dt(-\frac{i}{2}\psi_i^*\dot{\psi}_i^* + \frac{i}{2}f\epsilon_{ij}\psi_i^*\psi_j^* + im\chi_i\psi_i^*) + +\frac{i}{2}\psi_i^*(t_1)\psi_i^*(t_2) \tag{26}$$

where γ^* is a new fermionic coordinate. Then the constraints (6) become

$$\begin{aligned} \mathcal{H} &= P^2 - m^2 \\ \phi_i &= P^\mu\psi_{\mu i} - m\psi_i^* \\ \phi &= \epsilon_{ij}(\psi_i^\mu\psi_{\mu j} - \psi_i^*\psi_j^*) + 2q - D \end{aligned} \tag{27}$$

and the new fermionic variable satisfies $\{\psi_i^*, \psi_j^*\} = i\delta_{ij}$. The algebra of the constraints (27) close as in (8) and the BRST charge remains unchanged as in (9) with the constraints (27) instead of (6). For the new fermionic variables we take the boundary conditions $\frac{1}{2}(\psi_i^*(t_1) + \psi_i^*(t_2)) = \gamma_i^*$. Now the gauge fixing fermion is choosen to be

$$\Psi = \overline{\mathcal{P}}V(\frac{1}{2}\epsilon_{ij}\gamma_i\gamma_j)^{-(q-2)}(\frac{1}{2}\epsilon_{kl}\gamma_k^*\gamma_l^*)^{-1} + i\overline{P_i}\chi_i \tag{28}$$

After the change of variables $\psi_i^* = \gamma_i^* + \tilde{\psi}_i^*$ we get the transition amplitude

$$\begin{aligned} Z[X_1, X_2, \gamma_i, \gamma^*] &= \int dp \frac{e^{ip\Delta X}}{p^2 - m^2} \times \\ &\times (p_\mu\gamma_1^\mu - m\gamma_1^*)(p_\nu\gamma_2^\nu - m\gamma_2^*)(\gamma_1^\rho\gamma_{2\rho})^{q-2}\gamma_1^*\gamma_2^* \end{aligned} \tag{29}$$

Notice that the mass terms disappear due to the last factor $\gamma_1^*\gamma_2^*$. Taking the following representation for γ_i^μ and γ_i^* in terms of Dirac gamma matrices in $D=1$ dimensions

$$\begin{aligned} \gamma_1^\mu &= \gamma^*\gamma^\mu \otimes \gamma^* \\ \gamma_2^\mu &= \gamma^* \otimes \gamma^\mu \\ \gamma_1^* &= \gamma^*\gamma^{D+1} \otimes \gamma^* \\ \gamma_2^* &= \gamma^* \otimes \gamma^{D+1} \end{aligned} \tag{30}$$

where now $\gamma^* = \gamma^1 \ldots \gamma^{D+1}$ we obtain the propagator for the field strength of a massive antisymmetric tensor of rank $q-1$ after antisymmetrizing (29) with respect to γ_i^μ and γ_i^* and multiplying it by an appropriated combination of gamma matrices as in the massless case.

Acknowledgments

We would like to thank Dr.P.K.Townsend for pointing ref.[3] to us. L.S.Jr. acknowledges financial help from CNPq

References

[1] V.D.Gershern and V.I.Tkach, JETP Lett. 29 (1979) 288
P.S.Howe, S.Penati, M.Pernici and P.K.Townsend, Phys.Lett. 215B (1988) 555

[2] M.Pierri and V.O.Rivelles, "BRST Quantization of Spinning Relativistic Particles with Extended Supersymmetries", preprint IFUSP/P-838 (1990) (to be published in Phys.Lett.B)

[3] L.Mezincescu, R.I.Nepomechie and P.K.Townsend, Nucl.Phys.B 322 (1989) 127

[4] P.S.Howe, S.Penati, M.Pernici and P.K.Townsend, Class. Quantum Grav. 6 (1989) 1125

[5] P.S.Howe and P.K.Townsend, preprint CERN/TH-5519/89 (1989)

[6] E.S.Fradkin and G.Vilkovisky, Phys.Lett. 55B (1975) 224
L.A.Batalin and G.Vilkovisky, Phys.Lett. 69B (1977) 309

[7] M.Henneaux and C.Teitelboim, Ann.Phys.(NY) 143 (1982) 127

[8] M.Gomes, V.O.Rivelles and A.J.da Silva, Phys.Lett. 218B (1989) 63

J.Gamboa and V.O.Rivelles, Phys.Lett. 241B (1990) 45

[9] I.A.Batalin and E.S.Fradkin, Phys.Lett. 122B (1983) 157
I.A.Batalin and G.A.Vilkovisky, Phys.Rev.D 28 (1983) 2567

[10] V.O.Rivelles and L.Sandoval Jr., preprint IC/90/158 (1990)

[11] J.C.Henty, P.S.Howe and P.K.Townsend, Class. Quantum Grav. 5 (1988) 807
G.Papadopoulos, Class. Quantum Grav. 6 (1989) 1745

[12] J.Polchinski and Y.Cai, Nucl.Phys.B 269 (1988) 91

[13] D.Friedan and P.Windey, Nucl.Phys.B 235 (1984) 395

[14] O.Alvarez, I.M.Singer and P.Windey, Nucl.Phys.B 337 (1990) 467