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Abstract

The statistical properties of matter fields with anomalous magnetic momentum interacting with the Chern-Simons-Maxwell (CSM) field are considered. It is shown, that in the theory with pure Chern-Simons (CS) action the Semenoff gauge results in anyons with selfinteraction. Even in the presence of the Maxwell term there is a particular solution for which anyonic system with (current)×(current) selfinteraction arises.

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Three-dimensional space-time has remarkable geometrical and group-theoretical features, which enrich 3D quantum field theory with new peculiar properties. It allows, for example, the identical particles to possess fractional (anyonic) statistics and spin [1], and gauge field can be topologically massive without loosing gauge invariance [2].

It occurs also that in D=3 not only spinning particles, but scalar ones can possess anomalous magnetic momentum [3,4], which interaction with electromagnetic field results in new effects of spontaneous symmetry breaking manifested in the CS term appearance and the cancelation of the Maxwell term [4].

In the present paper we consider the statistical properties of the matter fields with an anomalous magnetic momentum, interacting with CSM field. It will be shown, that in the theory with pure CS action not only the magnetic flux is associated with field's charge density [1,5] , but the vortex of field's current as well. It contributes to the interaction terms of the Hamiltonian of the system and, hence, in the case considered we get the anyonic vortices with rather complicated selfinteraction. It also terns out that in difference with the conventional case [5] (see, however, [6]) the fields with anomalous magnetic momentum can acquire anyonic properties even in the presence of the Maxwell term for a particular solution of the CSM equations of motion, which has the form of conventional CS equations of motion [2] . For such a solution the dual gauge stress vector is equal to the matter current and anyonic system with (current)×(current) selfinteraction arises.

The gauge interaction of a charged matter field caused by its anomalous magnetic momentum can be introduced in D=3 using the following covariant derivative [4]:

$$\hat{D}_m = \partial_m + iA_m(x) + ilF_m(x), \qquad (1)$$

where m=0,1,2, $A_m(x)$ is the abelian field potential, $F_m(x)=1/2*\varepsilon_{mnl}F^{nl} = \varepsilon_{mnl}\partial^n A^l$ is the dual stress vector and l being the dimension of length parameter characterizing the value of anomalous magnetic momentum. Matter field's charge is chosen to be one.

For example, the action for spinor $\Psi(x)$ and scalar $\Phi(x)$ fields with mass m are written as follows:

$$S^{\Psi}_{matter} = \int d^3x\, (i\bar{\Psi}\gamma^m\hat{D}_m\Psi - m\bar{\Psi}\Psi), \qquad (2a)$$

$$S^{\Phi}_{matter} = \int d^3x\, [(\hat{D}_m\Phi)^*\hat{D}^m\Phi - m^2|\Phi|^2] \qquad (2b)$$

Let us reproduce the energy spectrum of a particle with anomalous magnetic momentum in the external homogeneous magnetic field B :

$$E = [m^2 - (2n+1-2s)B]^{1/2} + lB, \qquad n=0,1,2\ldots,$$

where a particle helicity s determines the "normal" magnetic momentum, which is equal to zero for a scalar particle. Thus, the scalar particles with anomalous magnetic momentum can be regarded as the "elementary" charged vortices.

If in addition to (2a) or (2b) we take into account the CSM terms the action takes the form:

$$S = S_{matter} + \int d^3x(\vartheta/2\ \varepsilon_{mnl}A^mF^{nl} - 1/4e^2\ F_{mn}F^{mn}), \qquad (3)$$

where the coupling constant e has the dimension $l^{-1/2}$ and ϑ is dimensionless.

Note, that if the spontaneous symmetry breaking takes place in the system described by the action (3) and the field Φ vacuum ex-

pectation value $\langle\Phi\rangle^2$ is equal to $1/(2l^2e^2)$, then the Maxwell term is canceled by the symmetry breaking contributions, and, at the same time, the CS term arises, if it was not present in the action (3) from the beginning (ϑ=0) [4].

The CSM equations of motion, derived from (2) and (3) have the symmetric form with respect to dual stress vector F_m and conserved matter current $J^{\Psi}_m = \bar{\Psi}\gamma_m\Psi$ or $J^{\Phi}_m = i(\Phi^*\hat{D}_m\Phi - \Phi(\hat{D}_m\Phi)^*)$:

$$2\vartheta F^n + (-1/e^2)\varepsilon^{nml}\partial_m F_l = J^n + l\varepsilon^{nml}\partial_m J_l\ . \qquad (4)$$

To construct the Hamiltonian of the system considered one has to introduce the canonical momenta, which read as follows:

$$\Pi_{\Psi} = \delta S/\delta(\partial^0\Psi) = i\bar{\Psi}\gamma_0,$$

$$\Pi = \delta S/\delta(\partial^0\Phi) = (\hat{D}_0\Phi)^*, \quad \Pi^* = \hat{D}_0\Phi,$$

$$\Pi_0 = \delta S/\delta(\partial^0 A^0) = 0,$$

$$\Pi_i = \delta S/\delta(\partial^0 A^i) = -(1/e^2)F_{0i} + \vartheta\varepsilon_{ik}A^k - l\varepsilon_{ik}J^k \qquad (5)$$

$(i,k = 1,2)$.

Then, with the use of equations of motion (4) and up to full derivatives the Hamiltonian can be represented in the form:

$$H = H_{matter} + \int d^2x[(1/2e^2)F_{0i}F_{0i} + (1/4e^2)(F_{ik})^2 + + (1/2)J_0\varepsilon_{ik}F^{ik} + l\varepsilon_{ik}J^iF^{0k})]\ , \qquad (6)$$

where for spinor field $\Psi(x)$

$$H_{matter} = \int d^2x\ [i\bar{\Psi}\gamma_i\hat{D}_i\Psi + m\bar{\Psi}\Psi] \qquad (7a)$$

and for scalar field $\Phi(x)$

$$H_{matter} = \int d^2x[|\Pi|^2 + (\hat{D}_i\Phi)^*\hat{D}_i\Phi + m^2|\Phi|^2]. \qquad (7b)$$

The last two terms in eq.(6) spoil, in general, the positive definiteness of the Hamiltonian. This happens, as we will see below, when the field modes with large momenta are generated. But if we restrict ourselves to the long-wave approximation and consider the present model as the effective theory, which has to be modified at critical values of the field momenta, then the positive definiteness of the energy is ensured.

Let us investigate the statistical properties of the matter fields in the model concidered.

As is well known (see [1,5] and references therein) the free anyons arise in a system of matter fields minimally coupled to the Abelian gauge field with pure CS action. The gauge field in such a system causes the statistical interaction of matter fields. It is expressed in terms of matter current and, hence, does not have the independent dynamics, for example:

$$2\vartheta B \equiv 2\vartheta\varepsilon^{ik}\partial_i A_k = J_0. \qquad (8)$$

The expression (8) means that magnetic flux is associated with particle charge. This leads to the Aharonov-Bohm effect when particles move around each other and results in a system of anyons with only statistical interaction among them (see refs. [1,5] for the details).

If the Maxwell kinetic term is not present in our case, the second term in the r.h.s. of eq.(4) vanishes and the gauge stress tensor is fully determined by the matter current and its vortex. For example, one gets the following relation:

$$2\vartheta B - \varepsilon^{ik}\partial_i lJ_k = J_0 , \qquad (9)$$

which means that not only the magnetic flux, but the current vortex as well is associated with field's charge. Hence, the quantum Hamiltonian (6) (with vanishing Maxwell term and F_m replaced by its expression in terms of J_m) describes the system of anyonic vortices with rather complicated selfinteraction of the anyonic currents (the appropriate operator ordering within (6) is implied).

It is interesting, that simpler selfinteracting anyonic system arise as a particular solution of the complete CSM eqs.(4). Let us consider the case, where l is not independent parameter, but is expressed in terms of e^2 and ϑ:

$$l = -1/(2e^2\vartheta),$$

then the eqs.(4) take the form

$$(F_m - (1/(2\vartheta))J_m) - (1/(2e^2\vartheta))\varepsilon_{mnl}\partial^n(F^l - (1/(2\vartheta))J^l) = 0 . \qquad (10)$$

They are solved identically if we choose the first-order ansatz

$$F_m = (1/(2\vartheta))J_m , \qquad (11)$$

which is similar to the CS equations of motion in conventional anyonic model [5]. Thus, following the same reasoning as was proposed in [5], we can obtain the matter fields with anyonic statistics, but since the stress vector (11) (or the matter current) contributes to the Hamiltonian (6) this results in the (current)x(current) interaction of anyons.

For example, the Hamiltonian for the scalar field $\Phi(x)$ takes the form

$$H^{\Phi} = \int d^2x[\,|D_0\Phi|^2 + (D_i\Phi)(D_i\Phi) + m^2|\Phi|^2$$
$$+ (1/(8e^2\vartheta^2))(1 - |\Phi|^2/(2e^2\vartheta^2))(J_0^2 + J_i^2)] , \qquad (12)$$

where $D_m = \partial_m + iA_m$ and A_m causes the statistical interactions of anyons. It is straightforwardly positive definite for the field amplitudes $|\Phi|^2 \leq 2e^2\vartheta^2$. So let us see what happens for $|\Phi|^2 \gg 2e^2\vartheta^2$. When the field Φ amplitude does not depend on x^m the Hamiltonian for the anyonic phase mode $\varphi(x)$ ($\Phi = |\Phi|\exp(i\varphi)$) can be represented as

$$H^{\varphi} \approx \int d^2x(m^2|\Phi|^2 - (2e^2\vartheta^2)[(\partial_0\varphi + A_0)^2 + (\partial_i\varphi + A_i)^2]) .$$

One can see, that it is positive definite for the long-wave exitations of the φ field. In the case of high energies and large momenta the effective theory based on the Hamiltonian (12) has to be modified by some additional terms (for example with higher derivatives and higher l's powers), which could conserve the Hamiltonian positive definiteness of the system in the short-wave limit. Of course, the problems with the quantum theory renormalizability have to be studied as well.

In conclusion, we have obtained the effective theory for selfinteracting anyons, starting with the nonminimal interaction of the charged fermionic or bosonic fields, possessing anomalous magnetic momentum, with CSM gauge field.

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Note added.

When this work was completed the authors became aware of the ref. [7] , where the spontaneous symmetry breaking effects in the presence of anomalous magnetic momentum interactions were studied previously.

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