

UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-907

NEUTRINOLESS DOUBLE BETA WITHIN A SINGLE
MODE MODEL

J. Hirsch and F. Krmpotic'
Departamento de Física, Facultad de Ciencias Exactas,
Universidad Nacional de La Plata, 1900 La Plata, Argentina

H. Dias
Instituto de Física, Universidade de São Paulo

Março/1991

PACS numbers; 23.40.Hc, 21.10.Re, 21.60.Jz

Neutrinoless Double Beta Decay within a Single Mode Model

J. Hirsch and F. Krmpotić

Departamento de Física, Facultad de Ciencias Exactas,
Universidad Nacional de La Plata, 1900 La Plata, Argentina

H. Dias

Instituto de Física, Universidade de São Paulo, CP 20516,
01498 São Paulo, Brasil

ABSTRACT

The Vogel-Zirnbauer (VZ) suppression mechanism on the 0ν matrix elements is discussed in the framework of a single mode QRPA model. A multipole expansion of the neutrino potential is performed and the Horie-Sasaki method is employed for the evaluation of radial form factors. The sensitivity to finite nucleon size and short-range correlations is analyzed. We conclude that the VZ effect is operative for all multipole moments and that it is likely that the 0ν and 2ν decay amplitudes behave quite similarly.

The intimate connection between no-neutrino double beta ($\beta\beta_{0\nu}$) decay and the neutrino mass m_ν is well known. Our difficulties begin, however, as soon as we intent to extract the effective m_ν from the measured half-life for the $\beta\beta_{0\nu}$ disintegration. The problem is that this observable depends not only on m_ν , but also on the nuclear matrix element $M_{0\nu} = M_{0\nu}^A - M_{0\nu}^V$, and we do not seem yet to have developed a computational scheme for the vector (V) and axial-vector (A) moments, $M_{0\nu}^V$ and $M_{0\nu}^A$, in which we can have complete confidence.¹ The question can be traced back to the work of Primakoff and Rosen (PR) [1], who argued that $M_{0\nu}$ should be proportional to the corresponding matrix element $M_{2\nu} = M_{2\nu}^A - M_{2\nu}^V$ for the two-neutrino mode ($\beta\beta_{2\nu}$). If so, our ability to calculate the $M_{2\nu}$ becomes a direct test of our ability to calculate the $M_{0\nu}$.

In 1986 Vogel and Zirnbauer (VZ) [2] have shown that, within the framework of the proton-neutron (pn) quasiparticle random phase approximation (QRPA), the particle-particle (pp) interaction plays an essential role in reducing the $\beta\beta_{2\nu}$ decay probability. More recently following the VZ idea, the groups from Tübingen [3] and Heidelberg [4,5] have performed calculations of the $\beta\beta_{0\nu}$ -decay processes which called into question the PR proportionality. Their result is: the pp ground state correlations (GSC) reduce drastically the transition probability for the $\beta\beta_{2\nu}$ -decay, but only relatively slightly for the $\beta\beta_{0\nu}$ -decay. As a consequence, the matrix elements $M_{0\nu}$ and $M_{2\nu}$ would hardly be proportional. Moreover, only for the first one we would have sufficient command over the nuclear structure. On the other hand, the Pasadena group [6] has found that also the amplitude $M_{0\nu}$ is very sensitive to the pp

force, the implication of which is that the limits on m_ν deduced from data are less stringent than commonly thought. The chief aim of the present letter is to explore the differences and the possible similarities between the matrix elements $M_{0\nu}$ and $M_{2\nu}$ within the single mode model (SMM) introduced in ref. [7].

With this purpose in mind we first perform the Fourier-Bessel expansion of the moments $M_{0\nu}^V \equiv M_{0\nu}^V (S=0)$ and $M_{0\nu}^A \equiv M_{0\nu}^A (S=1)$ and get

$$M_{0\nu}^V(S) \equiv \sum_{LJ\pi} m(L,S,J^\pi) = 4\pi R(-)^S \sum_{LJ\pi} \int dq q^2 v(q; \omega_J) \times \langle 0_f^+ \| O(qr; LSJ) \| J^\pi \rangle \langle J^\pi \| O(qr; LSJ) \| 0_1^+ \rangle, \quad (1)$$

where

$$v(q; \omega_J) = \frac{2}{\pi} \frac{1}{q(q+\omega_J)}; \quad \omega_J \approx E_J - (E_1 + E_f)/2, \quad (2)$$

is the neutrino potential,

$$O(qr; LSJ) = i^L J_L(qr) (Y_L \otimes \sigma_S)_J t_+, \quad (3)$$

are one-body operators, and R is the nuclear radius which is introduced to make $M_{0\nu}(S)$ dimensionless.

Within the QRPA formulation done by the present authors [7], the energies ω_J are solutions of the QRPA equation, and the transition matrix elements are given by

$$\langle J^\pi \| O(qr; LSJ) \| 0_1^+ \rangle = - \sum_{pn} \langle p \| O(qr; LSJ) \| n \rangle \Lambda_+(pnJ^\pi), \quad (4)$$

$$\langle 0_f^+ \| O(qr; LSJ) \| J^\pi \rangle = - \sum_{pn} \langle p \| O(qr; LSJ) \| n \rangle \Lambda_-(pnJ^\pi),$$

where the reduced pn form factors are

$$\sqrt{4\pi} \langle p \| O(qr; LSJ) \| n \rangle \equiv W_{pn}^{LSJ} R_{pn}^L(q); \quad R_{pn}^L(q) = \int_0^\infty u_n(r) u_p(r) j_L(qr) r^2 dr, \quad (5)$$

$u(r)$ being the one-particle radial wave functions, and

$$\Lambda_+(pnJ^\pi) = \sqrt{\rho_p \rho_n} [u_p \bar{v}_n X(pnJ^\pi) + \bar{v}_p \bar{u}_n Y(pnJ^\pi)], \quad (6)$$

$$\Lambda_-(pnJ^\pi) = \sqrt{\rho_p \rho_n} [\bar{v}_p \bar{u}_n X(pnJ^\pi) + u_p \bar{v}_n Y(pnJ^\pi)].$$

The barred (unbarred) quantities indicate that quasiparticles and excitations are defined with respect to $|0_f\rangle$ ($|0_i\rangle$), $\rho_p^{-1} = u_p^2 + \bar{v}_p^2$, $\rho_n^{-1} = \bar{u}_n^2 + v_n^2$, and all the remaining notation has the standard meaning [7,8].

Within the SMM the following relation is fulfilled:

$$\Lambda_+(J^\pi) \Lambda_-(J^\pi) = u_p \bar{v}_n \bar{v}_p \rho_p \rho_n \mathcal{G}_{pn}(J^\pi) / \omega_{J^\pi}, \quad (7)$$

where

$$\mathcal{G}_{pn}(J^\pi) = -[G^{\text{pair}}(pp, pp; 0^+) + G^{\text{pair}}(nn, nn; 0^+)] [1 - Z_{pn}(J^\pi)] / 4, \quad (8)$$

and

$$Z_{pn}(J^\pi) = \frac{G(pn, pn; J^\pi)}{G^{pair}(pp, pp; 0^+) + G^{pair}(nn, nn; 0^+)} \quad (9)$$

The quantities G and G^{pair} are the pp matrix elements in the pn channel and in the pairing channels, respectively. We can now easily obtain that

$$m(L, S, J^\pi) = m^0(L, S, J^\pi) \mathcal{F}(J^\pi) / \omega_{J^\pi}, \quad (10)$$

where

$$m^0(L, S, J^\pi) = (-)^{S_u} v_{pn} \bar{u}_p \bar{v}_n \rho_p \rho_n \left[W_{pn}^{LSJ} \right]^2 \mathcal{R}_{pn}^L \quad (11)$$

are the unperturbed moments and

$$\mathcal{R}_{pn}^L = R \int_0^\infty dq q^2 v(q; \omega_J) \left[R_{pn}^L(q) \right]^2 \quad (12)$$

The finite nucleon size (FNS) effects are introduced through the dipole form factor [3] and we get:

$$v_{FNS}(q; \omega_J) = v(q; \omega_J) \left[\frac{\Lambda^2}{\Lambda^2 + q^2} \right]^4; \quad \Lambda = 850 \text{ MeV}. \quad (13)$$

The two-nucleon short range correlations (SRC) arise mainly from the repulsion due to the ω -exchange in the nucleon-nucleon interaction [9]. Thus the correlated potential is constructed as:

$$v_{SRC}(q; \omega_J) = \int \frac{d^3k}{(2\pi)^3} \Omega(q-k) v(q; \omega_J), \quad (14)$$

where

$$\Omega(q) = (2\pi)^3 \delta(q) - \frac{2\pi^2}{q_c^2} \delta(q - q_c), \quad (15)$$

with $q_c \approx 3.93 \text{ fm}^{-1}$ roughly the Compton wavelength of the ω -meson.

Within the approximation $q_c \gg \omega_J$ we get

$$v_{SRC}(q; \omega_J) = v(q; \omega_J) - \Delta v(q); \quad \Delta v(q) = \frac{2\pi}{q q_c} \ln \left| \frac{q+q_c}{q-q_c} \right|. \quad (16)$$

When both the FNS and SRC effects are considered the neutrino potential takes the form:

$$v_{SRC+FNS}(q; \omega_J) = v_{FNS}(q; \omega_J) - \Delta v(q) + \Delta v'(q), \quad (17)$$

with

$$\Delta v'(q) = \frac{\pi}{q q_c} \left[\sum_{n=1}^3 \frac{1}{n} \left(x_-^n - x_+^n \right) + \ln \left[\frac{x_-}{x_+} \right] \right]; \quad x_\pm = \frac{\Lambda^2}{\Lambda^2 + (q \pm q_c)^2}. \quad (18)$$

In the SMM the 2ν matrix elements read:

$$M_{2\nu}(J^\pi) = (-)^{J_u} v_{pn} \bar{u}_p \bar{v}_n \rho_p \rho_n \left[W_{pn}^{0JJ} \right]^2 \mathcal{F}(J^\pi) / \omega_{J^\pi}^2, \quad (19)$$

and the backward going β strengths

$$\mathcal{V}_-(LSJ^\pi) = \hat{J}^{-2} \left[\sum_{pn} \Lambda_{pn} (J^\pi) W_{pn}^{LSJ} \langle p | r^L | n \rangle \right]^2, \quad (20)$$

are also proportional to $\mathcal{G}_{pn}(J^\pi)$. Therefore all the $M_{2\nu}(J^\pi)$ and $m(L, S, J^\pi)$ moments go through zero at the values of $Z_{pn}(J^\pi)$ where the corresponding $\mathcal{V}_-(LSJ^\pi)$ are null.

The formulae (10) and (19) are valid for any type of interaction. In addition, they are remarkable in their simplicity and quite illustrative as shown below. First of all, let us point out that:

(1) the quasiparticle Tamm-Dancoff approximation (QTDA) does not introduce any type of correlations, i.e., $m(L, S, J^\pi) = m^0(L, S, J^\pi)$;

(2) when the pp interaction is not included (i.e., when $Z_{pn}(J^\pi) = 0$), the role of the QRPA correlations on the 0ν moments in the particle-hole (ph) channel is only of minor importance, and the correlated moments $m(L, S, J^\pi)$ turn out to be of the same order of magnitude as the unperturbed ones $m^0(L, S, J^\pi)$.

Thus, the SMM makes evident the VZ suppression mechanism in the case of $\beta\beta_{0\nu}$ -decay.

Usually, the pp coupling constants are chosen in such a way to make $Z_{pn}(J^\pi=1^+) \approx 1$ and $Z_{pn}(J^\pi=0^+) \approx 1$, and as a consequence only the forbidden matrix elements $m(L>0, S, J^\pi)$ turn out to be significant. This is roughly the reason for the picture which emerges from the the calculations performed in refs. [3-5] (i.e., for the enhancement of forbidden processes over the allowed ones, as well as for the insensitivity of the $m(L>0, S, J^\pi)$ to the pp interaction).

There is, however, no prompting motive for fixing the pp coupling in the way described above. Neither there is any fundamental reason for employing the same interaction strength for all J^π . The only real motivation for such a procedure is its simplicity. But it is as much arbitrary as it would be, for instance, to impose the condition $Z_{pn}(J^\pi) \approx 1$ for all J^π , the implication of which is a similar suppression of all 0ν moments. One should bear in mind that the effective interaction drastically relies on the size of the configuration space, which while steadily increasing with the number of single particle states (sps), also depends on the multipolarity J^π . Therefore, it might be reasonable to use different pp couplings for different J^π .

As in our previous studies of the $\beta\beta_{2\nu}$ -decays [7,8] we will base our discussion on the δ -interaction

$$V = -C (\kappa_s P_s + \kappa_t P_t) \delta(r); \quad C \equiv 4\pi \text{ MeV fm}^3, \quad (21)$$

with different strength constants κ_s and κ_t for the ph, pp and pairing channels. In this case, when all the radial integrals are assumed to be equal, and the same pairing strengths for protons and neutrons are considered [$\kappa_s^{\text{pair}}(p) = \kappa_s^{\text{pair}}(n) \equiv \kappa_s^{\text{pair}}$], we get

$$Z_{pn}(J^\pi) = 2\hat{J}^{-2} \left[\hat{J}_p^2 + \hat{J}_n^2 \right]^{-1} \sum_L \left[s \left[W_{pn}^{L0J} \right]^2 \delta_{LJ} + t \left[W_{pn}^{L1J} \right]^2 \right], \quad (22)$$

where $s \equiv \kappa_s^{\text{pp}} / \kappa_s^{\text{pair}}$ and $t \equiv \kappa_t^{\text{pp}} / \kappa_s^{\text{pair}}$. Thus, when $s=1$ (i.e. when the isospin symmetry is not broken) we obtain $Z_{pn}(J^\pi=0^+) \approx 1$ and therefore $M_{2\nu}(J^\pi=0^+) = m(L=0, S=0, J^\pi=0^+) \approx 0$.² Moreover, $M_{2\nu}(J^\pi=1^+) =$

$m(L=0, S=1, J^\pi=1^+) = m(L=2, S=1, J^\pi=1^+) = 0$ when:

- i) $j_p = j_n = j$ and $t = 8j(j+1)/(4j^2+4j+3)$, or
- ii) $j_p = j_n - 1 = j$ and $t = 8(j+1)^2/(4j^2+8j+3)$.

It means that the ratio t varies from 1 (for $j=1/2$) to 2 (for $j \rightarrow \infty$) in the case i) and from 3/2 (for $j=3/2$) to 2 (for $j \rightarrow \infty$) in the case ii). For $J^\pi > 1^+$ the moments $m(L, S, J^\pi)$ become null at larger values of s or t . For example, in the case $j_p = j_n = j$, the matrix element $m(L=2, S=0, J^\pi=2^+)$ will vanish when $s = 16j(j+1)/(4j^2+4j-3)$, with s going from 5 (for $j=3/2$) to 4 (for $j \rightarrow \infty$).

We continue now with the discussion of the SMM by focusing our attention on the $^{48}\text{Ca}(0_1^+) \rightarrow ^{48}\text{Ti}(0_1^+)$ decay within the single $0f_{7/2}$ shell [11]. All the model parameters have been fixed as in our earlier studies [7,8]. In this way we get: $v_s^{\text{pair}}(p) = 59$, $v_s^{\text{pair}}(n) = 46$, $v_s^{\text{pp}} = 52.5$ ($s = s_0 = 1$), $v_t^{\text{pp}} = 100.2$ ($t = t_0 = 21/11$), $v_s^{\text{ph}} = 55$ and $v_t^{\text{ph}} = 92$.

For one-particle wave functions we assume an harmonic oscillator, in which case the pertinent radial integrals $R^L(q)$ are:

$$R^0(z) = (6 - 18z + 9z^2 - z^3)e^{-z}; \quad R^2(z) = \frac{72}{5}z(12 + 8z + z^2)e^{-z};$$

$$R^4(z) = \frac{352}{7}z^2(5 - z)e^{-z}; \quad R^6(z) = \frac{4992}{7}z^3e^{-z}, \quad (23)$$

where $z = q^2/4\nu$ [12]. For $\omega_j = 0$, also the integrals \mathcal{R}^L can be evaluated analytically and we get:

$$\mathcal{R}^0 = \frac{1241}{2240} R \sqrt{\frac{2\nu}{\pi}}; \quad \mathcal{R}^2 = \frac{789}{11200} R \sqrt{\frac{2\nu}{\pi}}; \quad \mathcal{R}^4 = \frac{187}{8720} R \sqrt{\frac{2\nu}{\pi}}; \quad \mathcal{R}^6 = \frac{33}{2240} R \sqrt{\frac{2\nu}{\pi}}. \quad (24)$$

In fig. 1 the integrands of \mathcal{R}^L are displayed within different

approximations and for the following energies: $\omega = 0$, 4.06 MeV and 12.5 MeV. The second and third values of ω correspond, respectively, to the energies of the 1^+ state obtained in the present calculation and to the experimental value for the Gamow-Teller (GT) resonance in ^{48}Sc [13]. One sees that the maximum of the $R^L(q)$ functions, and thus of the integrands of \mathcal{R}^L , are at ≈ 0 , 0.8, 1.2 and 1.8 fm^{-1} for $L=0, 2, 4$ and 6, respectively. In table 1 the corresponding values of \mathcal{R}^L are listed. The results displayed in fig. 1 and in table 1 clearly show the way in which the $\beta\beta_{0\nu}$ moments depend on ω , FNS and SRC. The FNS effects are negligible for the allowed transitions but increase rapidly when L increases. The same remark stands for the SRC when the FNS effects are not included. However, if both corrections are considered simultaneously the net result may not be very different from that it would have been without any SRC at all.

The quantities $\omega_{j\pi}^2$ and $m(L, S, J^\pi)$, as a function of the pp-interaction, are represented in fig. 2. At a certain value of s or t the QRPA breaks down and leads to unphysical complex energies. The moments $m(L, S, J^\pi)$ go to zero before this point is reached.

The different pieces of the total $\mathcal{M}_{0\nu}(S)$ moments are exhibited in table 2. We have used here the so called closure approximation with $\omega_j = 4.06$ MeV, since the integrals \mathcal{R}^L depend on the exact value of ω_j only very weakly. Of course, within the SMM this simplification does not yield any computational advantage. It was used just to emphasize that the main effect of the virtual intermediate states within the QRPA is to take into account the GSC in the initial and final nuclear states. As expected, the largest

unperturbed matrix elements are the allowed ones; in the present case $u_p v_n \bar{u}_n \bar{v}_p \rho_p \rho_n = 4/25$ ($u_p = v_n = 1$, $\bar{u}_n = \bar{v}_p = 1/2$). It can be seen, from the values of $Z_{pn}(J^\pi)$ listed in table 2 as a function of ratios s and t , that the magnitudes of different multipoles of $M_{0\nu}^A$ and $M_{0\nu}^V$ depend critically on the pp force within the $T=0$ and $T=1$ channels, respectively. The following two columns of the same table display given the moments $m(L, S, J^\pi)$ for $s=s_0$ and $t=t_0$, i.e. at the minima of $\mathcal{F}(L=0, S, J^\pi)$. As already mentioned only the moments with $L>1$ make the values of the amplitudes $M_{0\nu}^A$ and $M_{0\nu}^V$ non-zero in this case. This is due to the lack of the spin-orbit interaction in the SMM. When such a force is taken into account in a calculation with several sps, both $M_{2\nu}^A$ and $m(L=0, S=1, J^\pi=1^+)$ turn out to be different from zero even if $t=t_0$. Moreover, it might happen that the allowed 0ν moments dominate on the forbidden ones.⁴ Finally, the quantities $\mathcal{F}(J^\pi)$ were evaluated with the renormalized Kuo-Brown (KB) G-matrix elements [15] and the resulting moments $m^{KB}(\text{LSJ})$ are presented in the last column of table 2. When compared with matrix elements from the penultimate column the main and crucial difference is found between the GT moments, which leads to quite different values for the total 0ν amplitude (see the last row in table 2). It is self-evident from fig.2 that, when the condition (18) is slightly relaxed (for example with $t/t_0=3/4$), the δ -force yields A-moments similar to those provided by the KB G-matrix.

It is clear that the numbers which come out from the SMM (fig.2 and table 2) should not be taken too seriously. However, the important issue is that they qualitatively support the VZ suppression mechanism for all 0ν moments. The question is now

whether we can have sufficient command on this cancellation effect and in this way achieve a reliable estimate for $M_{0\nu}$ and $M_{2\nu}$. Our general feeling is that the answer relies, not only on a better understanding of the pp force, but also on the dominance on the GT moment 0ν over the forbidden ones. This happens, for instance, with the $m^{KB}(\text{LSJ})$ matrix elements. In such a case the PR proportionality: $M_{0\nu}/M_{2\nu} \approx m(L=0, S=1, J^\pi=1^+)/M_{2\nu}^{A, \omega_{GT}}$ (with ω_{GT} roughly the energy of the GT resonance) would be rather well satisfied, and hopefully we will be able to get control on the VZ effect from the experimental data for the $\beta\beta$ -decay. Otherwise, all our understanding of such a fundamental issue as the neutrino mass would hang upon the forbidden 0ν moments, which, besides being as sensitive to the pp force as the allowed ones, are also small in the zero order approximation and are not related with the $\beta\beta$ -decay.

Acknowledgment

One of the authors (HD) acknowledges support from the Argentine-ICTP Scientific Cooperation Program and from the Universidad Nacional de La Plata. Two of us (JH and FK) are fellows of the CONICET, Argentina. We would like to thank A.F.R. Toledo Piza for fruitful discussions and R. Mercader for critical reading of the manuscript.

Footnotes

¹For the axial-vector coupling constant we will always use the value $g_A = -1$ (see ref. [7]).

²It is worth mentioning here that, within the QRPA and when there is no dynamical violation of the isospin symmetry, this is an exact symmetry. In other words, it is not necessary to resort to any particle- and/or isospin projected QRPA in order to restore the isospin symmetry. Moreover, from above results it is also self evident that the particle-number projection is only of minor importance for the $\beta\beta$ -moments; we have verified this statement by carrying out full calculations (see also ref. [10]).

³It is worthwhile to be noted that: i) the Pandya relation is quite well satisfied within the SMM, and ii) $s_0 = 0.6t_0$ has been adopted in ref. [6].

⁴The spin-orbit splitting breaks the SU4 symmetry and plays a key role in the $\beta\beta$ -decay [7,8,14].

References

- [1] H. Primakoff and S.P. Rosen, Phys. Rev. 184 (1969) 1925.
- [2] P. Vogel and M.R. Zirnbauer, Phys. Rev. Lett. 57 (1986) 731.
- [3] T. Tomoda and A. Faessler, Phys. Lett. B 199 (1987) 475.
- [4] K. Muto, E. Bender and H.V. Klapdor, Z. Phys. A 334 (1989) 187.
- [5] A. Staudt, T.T.S. Kuo and H.V. Klapdor-Kleingrothaus, Phys. Lett. B 242 (1990) 17.
- [6] J. Engel, P. Vogel and M.R. Zirnbauer, Phys. Rev. C 37 (1998) 731.
- [7] J. Hirsch and F. Krmpotić, Phys. Lett. B 246 (1990) 6.
- [8] J. Hirsch and F. Krmpotić, Phys. Rev. C 41 (1990) 792; F.

Krmpotić, in "Lectures on Hadron Physics", Ed. by E. Ferreira (World Scientific, Singapore, 1990) p. 205; J. Hirsch, E. Bauer and F. Krmpotić, Nucl. Phys. A 516 (1990) 304.

- [9] G.E. Brown, S.O. Bäckman, E. Oset and W. Weise, Nucl. Phys. A 286 (1977) 191; I.S. Towner, Phys. Reports 155 (1987) 263.
- [10] O. Civitarese, A. Faessler, J. Suhonen and X.R. Wu, Phys. Lett. B 251 (1990) 33.
- [11] J.D. Vergados, Nucl. Phys. A 506 (1990) 482; G. Pantis and J.D. Vergados, Phys. Lett. B 242 (1990) 1.
- [12] H. Horie and K. Sasaki, Prog. Theor. Phys. 25 (1961) 475.
- [13] B.D. Anderson et al, Phys. Rev. Lett. 44 (1980) 1759.
- [14] J. Bernabeu, B. Desplanques, J. Navarro and S. Noguera, Z. Phys. C 46 (1990) 323; B. Desplanques, S. Noguera and J. Bernabeu, preprint FTUV/90-56
- [15] T.T.S. Kuo and G.E. Brown, Nucl. Phys. A 114 (1968) 241.

Table 1. Values of the radial integrals \mathcal{R}^L for the $0f_{7/2}$ orbit in ^{48}Ca ; a) bare, b) with FNS, c) with SRC and d) with FNS+ SRC.

L	$\omega=0$				$\omega=4.06\text{MeV}$				$\omega=12.5\text{MeV}$			
	a	b	c	d	a	b	c	d	a	b	c	d
0	.967	.950	.962	.950	.810	.794	.804	.794	.656	.642	.650	.641
2	.123	.107	.117	.107	.120	.104	.114	.103	.113	.098	.107	.097
4	.049	.034	.043	.034	.048	.034	.042	.033	.046	.033	.040	.032
6	.026	.014	.020	.013	.025	.014	.019	.013	.024	.014	.019	.012

Table 2. Anatomy of the M_{OV} moments within the degenerate model for the $0f_{7/2}$ orbit in ^{48}Ca . We use here the radial integrals \mathcal{R}^L for $\omega=4.06$ MeV and with FNS+ SRC. The quantity $Z(J)$ is defined in eq. (9). ω_j and $m(\text{LSJ})$, listed in following two columns, have been evaluated with $\omega_s^{\text{pair}}=52.3$ and $s=1$ and $t=21/11$. In the last column are shown the results obtained with the Kuo-Brown interaction for $\mathcal{S}(J)$.

L S J	$\left[\frac{W^{\text{LSJ}}}{J}\right]^2$	$m^0(\text{LSJ})$	Z(J)	$\omega_j[\text{MeV}]$	$m(\text{LSJ})$	$m^{\text{KB}}(\text{LSJ})$
0 0 0	1	1.0159	s	7.14	0	0
0 1 1	$\frac{9}{7}$	-1.3061	$\frac{11t}{21}$	4.06	0	-0.412
2 1 1	$\frac{2}{7}$	-0.0378	$\frac{11t}{21}$	4.06	0	-0.012
2 0 2	$\frac{25}{21}$	0.1573	$\frac{5s}{21}$	5.37	0.103	0.030
2 1 3	$\frac{11}{7}$	-0.2077	$\frac{19t}{77}$	4.49	-0.113	-0.074
4 1 3	$\frac{12}{77}$	-0.0066	$\frac{19t}{77}$	4.49	-0.004	-0.002
4 0 4	$\frac{81}{77}$	0.0446	$\frac{9s}{77}$	5.00	0.036	0.015
4 1 5	$\frac{195}{77}$	-0.1073	$\frac{235t}{1001}$	4.50	-0.061	-0.031
6 1 5	$\frac{50}{1001}$	-0.0008	$\frac{235t}{1001}$	4.50	-0.001	-0.000
6 0 6	$\frac{25}{33}$	0.0122	$\frac{25s}{429}$	4.81	0.011	0.005
6 1 7	$\frac{875}{143}$	-0.0988	$\frac{175t}{429}$	4.28	-0.023	0.009
M_{OV}					0.166	-0.522

Figure Caption

Fig. 1. Form factors $Rq^2v(q;\omega_j) \left[R_{pn}^L(q) \right]^2$ of the $0f_{7/2}$ state as a function of q .

Fig. 2. The squares of the energies $\omega_{J\pi}$ in ^{48}Sc and the moments $m(\text{LSJ}^\pi)$, M_{0v}^A and M_{0v}^V for the decay $^{48}\text{Ca}(0_1^+) \rightarrow ^{48}\text{Ti}(0_1^+)$ as a function of the particle-particle couplings s and t .

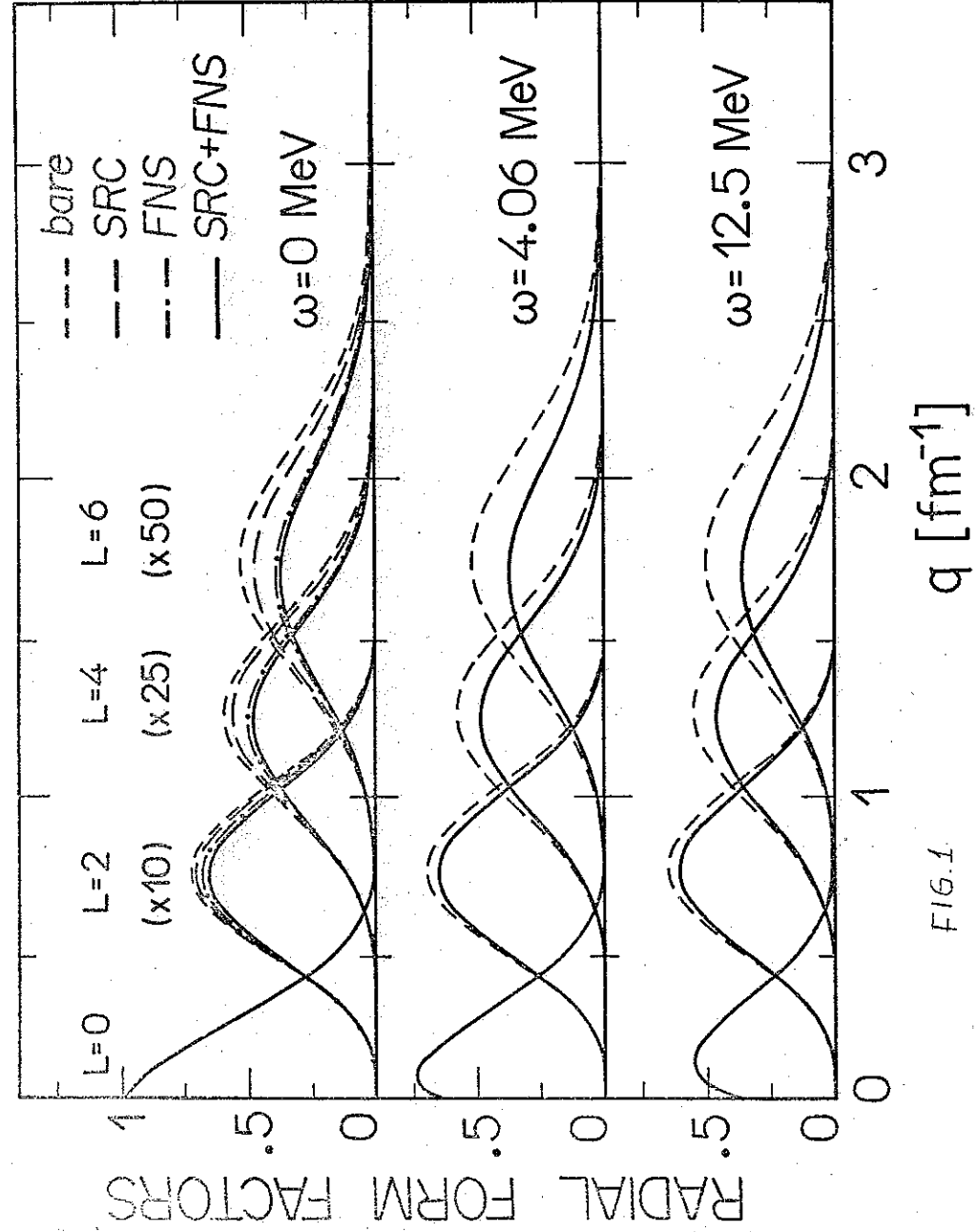


FIG.1

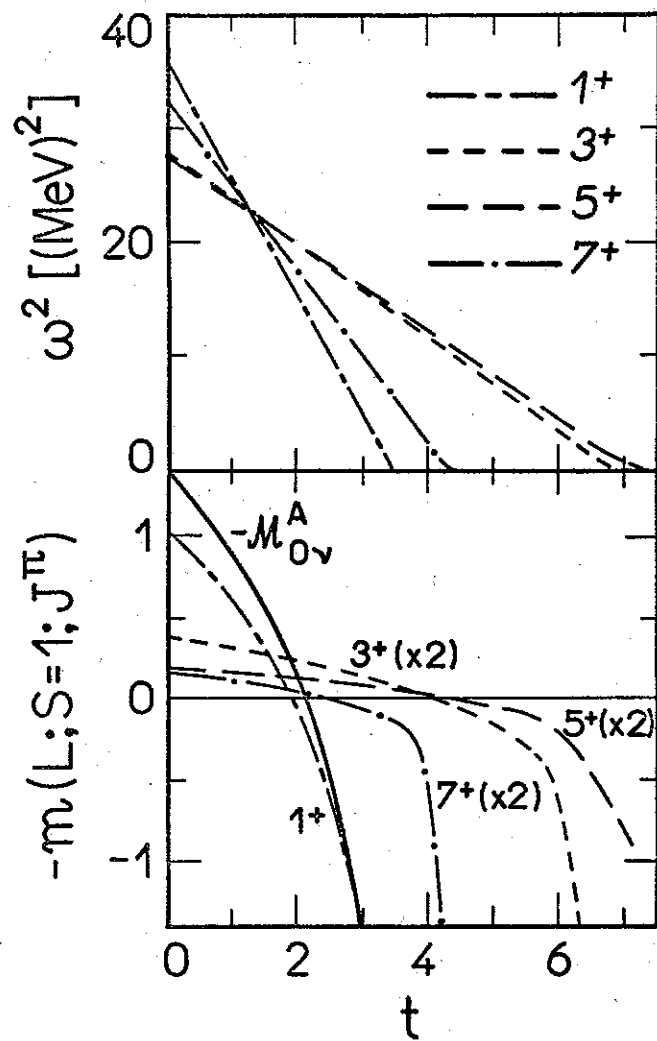
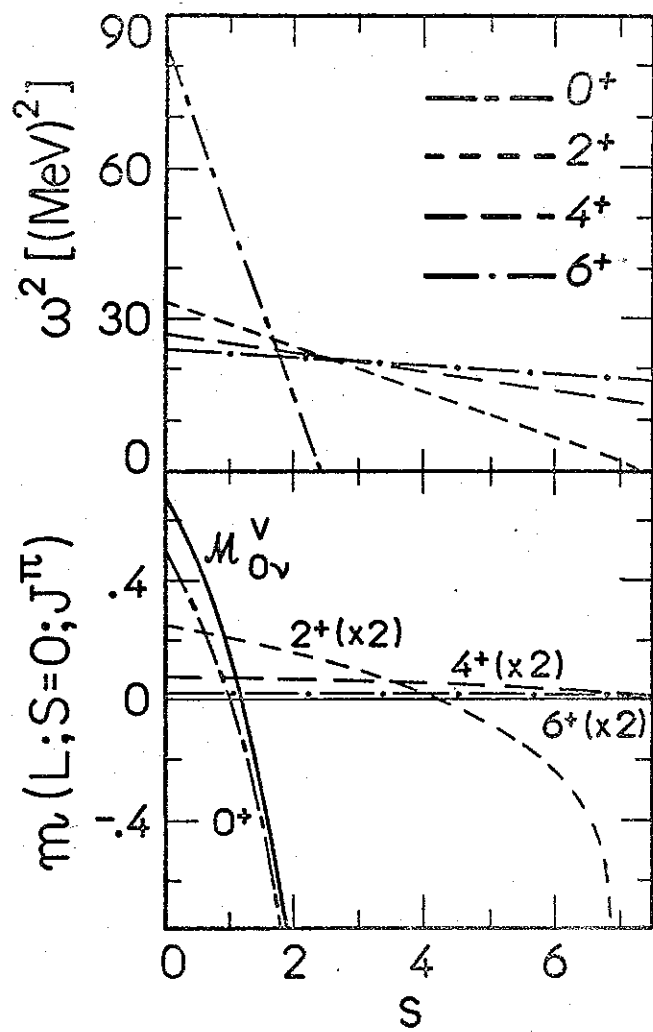


FIG. 2