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FIELD LINE DYNAMICS IN THE MARTIN-TAYLOR MODEL FOR A TOKAMAK WITH ERGODIC LIMITER

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Abstract

We consider a model (T.J.Martin, J.B.Taylor:Plas. Phys. 26(1984),321) for magnetic field ergodization in the outer portion of a Tokamak chamber, by means of current ring limiters. It describes an analytical mapping for the field lines through a Poincaré-type surface of section. We analyse some analytical issues of this mapping concerning local stability. The characterization of some interesting dynamical regimes in this mapping is shown by using deviation measures as well as their corresponding power spectra.

The dynamics of magnetic field lines in plasma confinement systems, like Tokamaks, Stellarators, RFP's, etc., is an issue of overwhelming importance in the modern Plasma Physics research. It is particularly convenient in situations when irregular behaviour is present, giving some insights on the particle diffusion problem [1]. From a theorist point of view, there is another spot on the field line dynamics: its equations, after a judicious choice of variables, can be taken as canonical equations, derived from a Hamiltonian function [2]. This fact enables us to use the powerful machinery developed in the past decades to characterize regular and stochastic motion of an integrable system submitted to perturbations [3].

A common technique to reduce the actual problem to a more tractable one is the use of a Poincaré surface of section. The successive piercings of a field line on a surface like this produce a map, in which the coordinates of each point are uniquely determined by the coordinates of the preceding point and so on. Analytical relations between these coordinates are seldom found in the literature. A recent example is the so-called Martin-Taylor model [4] for a large aspect-ratio Tokamak with Ergodic Magnetic Limiters. These are external current configurations that produce resonant fields which, combined with the equilibrium structure, generate magnetic islands. The resultant magnetic structure presents very interesting features, the most important being chaotic-like behaviour. We have analysed the magnetic "ergodization" by Ergodic Magnetic Limiters by means of a theoretical procedure using invariant flux functions and impulsive perturbations, obtaining an improved form for the field line map, and results will be published elsewhere [5].

The basic geometry to be used in this note is depicted in fig. 1. The original toroidal Tokamak chamber is wrapped successively into a cyllinder (large aspect ratio approximation) and a rectangular box (in the analysis of the chamber region close to the inner wall). The coordinates on the x-axis correspond to the (rectified) poloidal circumference in the

Tokamak. Hence, the period for these coordinates is $2\pi b$ (where b is the Tokamak minor radius). On the y-axis we depict the radial distance from the torus wall (the zx-plane), and coordinates on the z-axis stand for positions in the toroidal direction. Points with $y > 0$ are located inside the Tokamak vessel, whereas $y < 0$ defines the external medium.

Embracing the Tokamak torus, the Ergodic Magnetic Limiter consists of a grid with m pairs of wires of length L (see figure 1), conducting a current I in alternate directions. The magnetic field generated by this kind of configuration falls off rapidly as y increases (for a detailed derivation see ref. [6]), and is superposed to a uniform toroidal field B_0 in the z-direction.

The magnetic field line equations are:

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} \tag{1}$$

and is convenient to make a Poincaré map by cutting the torus with a section plane at $z = 0$. Integrating (1) inside and outside the limiter region, Martin and Taylor have found the following two-dimensional map [4]:

$$X_{n+1} = \mathcal{F}(X_n, Y_n) + \beta\mathcal{G}(X_n, Y_n) \tag{2}$$

$$Y_{n+1} = \mathcal{G}(X_n.Y_n) \tag{3}$$

where:

$$\mathcal{F}(X,Y) = X - Pe^{-Y}\cos X \tag{4}$$

$$\mathcal{G}(X,Y) = Y + \ln\left(\frac{\cos\mathcal{F}(X,Y)}{\cos X}\right) \tag{5}$$

The map variables are related with geometrical coordinates as follows:

$$X = \frac{mx}{b} \tag{6}$$

$$Y = \frac{m}{b}\left(y + \frac{b}{2}\right) \tag{7}$$

The map (2,3) has two control parameters:

$$P = \frac{m^2\mu_0 IL}{b^2\pi B_0} \tag{8}$$

which measures the limiter strength in terms of the field line diffusion, and the "magnetic shear":

$$\beta = \frac{4\pi}{q_b} \tag{9}$$

where q_b is the safety factor at the wall position. Phase portraits for typical values of P and β are found in the ref. [4].

The Jacobian matrix **J** of the map (2,3) has determinant equal to the unity, so the map is symplectic. Its approximate fixed points are [4]:

$$\bar{X} = \begin{Bmatrix}\pi\\0\end{Bmatrix} + P\exp\left(\frac{-2\pi m}{\beta}\right) \tag{10}$$

$$\bar{Y} = \frac{2\pi m}{\beta} + \frac{P}{\beta}\exp\left(\frac{-2\pi m}{\beta}\right) \tag{11}$$

In order to study their local stability one can look for the eigenvalues of **J**, i.e., the roots of the secular equation:

$$\det(\mathbf{J} - \lambda\mathbf{I}) = 0 \tag{12}$$

which can be written as:

$$\lambda = 1 - 2R \pm 2\sqrt{R(R-1)} \tag{13}$$

after defining the "Greene Residue" [7]:

$$R = \frac{1}{4}(2 - \mathrm{Tr}\mathbf{J}) \tag{14}$$

If $0 < R < 1$ the fixed point $(\bar{X}, \bar{Y})$ will be elliptic, its eigenvalues being complex conjugates:

$$\lambda = e^{\pm 2\pi i\rho} \tag{15}$$

where:

$$\rho = \frac{1}{2\pi}\arccos(1-2R) \tag{16}$$

is an average rotation angle of the libration orbits around $(\bar{X},\bar{Y})$. Otherwise $(\bar{X},\bar{Y})$ will be hyperbolic.

The Greene residue (14) for the map (2,3) reads:

$$R = \frac{1}{4}\left\{\left[\beta + Pe^{-Y_n}\left(\cos X_n + \beta\sin X_n\right)\right]\tan\left[X_n - Pe^{-Y_n}\cos X_n\right] - \beta\tan X_n - Pe^{-Y_n}\sin X_n\right\} \tag{17}$$

For small values of the limiter current I, we can expand (17) in powers of P to obtain:

$$R = -\frac{\beta Pe^{-Y_n}\cos X_n}{4} \tag{18}$$

which is exactly the same as the Chirikov-Taylor standard map.

Using the first couple of fixed points in (10,11), namely $(\bar{X}\approx\pi)$, the residue, for typical values of m (around 10) and β (around π), belongs to the interval $(0,1)$, since both β and P are strictly positive numbers. So this point is elliptic, whereas the other couple, namely $(\bar{Y}\approx 0)$ is hyperbolic. This is illustrated in the phase portraits of ref. [4].

Fixing values for P and β, it is possible to analyse different dynamical regimes in the Martin-Taylor map by selecting a given initial condition and computing its "radial":

$$\Delta Y_n = Y_n - Y_0 \tag{20}$$

as well as "poloidal" deviation:

$$\Delta X_n = X_n - X_0 \tag{21}$$

In figures 2 and 4 we show typical oscillations, arising from single initial conditions picked up from regions where presumably there is regular and irregular behaviour, respectively.

The power spectrum related to fig. 4 suggests a quasi-broadband spectra in a limited frequency interval, although a more precise characterization of chaotic motion (e.g., Lyapounov exponents or K-entropy) is needed for a rigorous statement.

Other valuable measurement (mean quadratic deviation) can be made by taking N initial conditions, equally spaced along the Y = const. direction. It is defined as:

$$<\Delta Y_n^2> = \frac{1}{N}\sum_{i=1}^{N}\left[Y_n(i) - Y_0(i)\right]^2 \tag{22}$$

In figure 6 the peculiar behaviour would suggest the existence of accelerator modes in this mapping, but they are forbidden due to the absence of two- dimensional periodicity characteristic of the standard map [3]. Fig. 7 shows a power spectrum which diverges for small frequencies (the "1/f noise"). This phenomenon, well known in dissipative systems [8], has been observed also in a conservative one [9], related to the cantori structure of the phase space.

Finally, fig. 4 shows a curious example of intermittent transition between regular and irregular behaviour for simple poloidal deviations. Intermittency , like 1/f noise, is widely studied for dissipative systems [8], but it has been observed in the Chirikov-Taylor standard map as well [10]. At the best of our knowledge there is no complete theory for this kind of situation in conservative maps.

The Martin-Taylor map, in spite of its simplicity, provides a tool for studies of magnetic stochasticity in situations where an external field acts upon an integrable equilibrium structure. This is the case in Tokamaks with Ergodic Magnetic Limiters, but applications in other areas, like astrophysics , have not been fully explored up to the moment. Other maps with similar characteristics (see ref. [5]) are expected to present the same qualitative behaviour. It is worthnoting that a kind of map analysis had been guided the design of a machine implementation of an Ergodic Magnetic Limiter in the TEXT Tokamak. The area-preserving maps technology might be applied in these problems, our local stability analysis being just an example.

While stochasticity is commonly recognized in nonintegrable maps like the Martin-Taylor's one, novel features need a deeper understanding, as pointed by the unexpected results of figures 5 and 6. For example, one might speculate the relationship between intermittent oscillations and the behaviour of map points close enough a separatrix layer, where rigorous estimates (like Nekhoroshev bounds) exists for travelling times [12].

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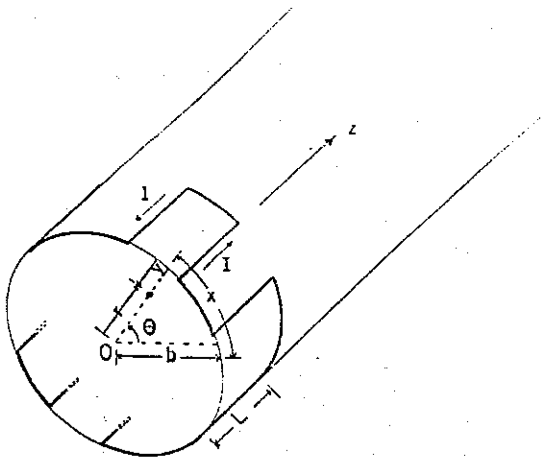

Fig. 1: Basic geometry used in this note ($x=b\theta$; $y=b-r$)

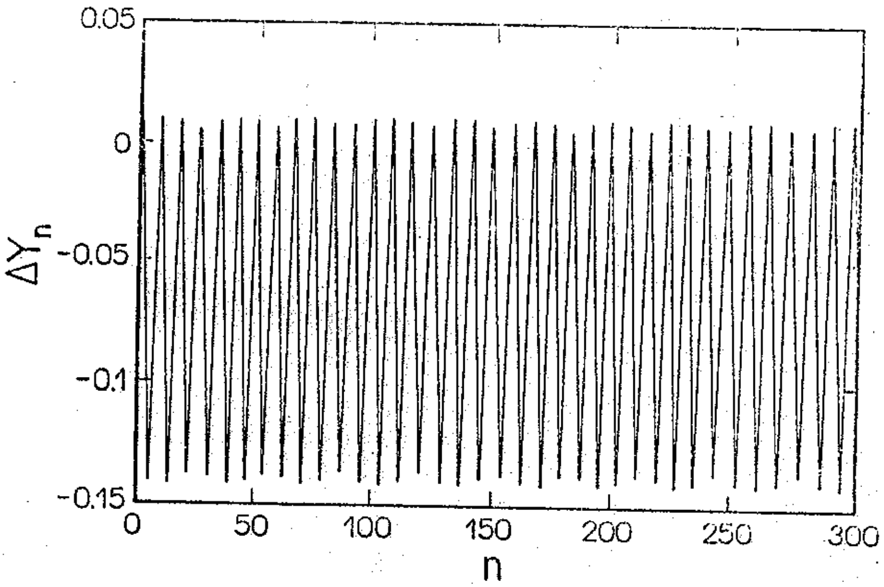

Fig. 2: Radial deviations with $P=0.25$; $\beta=6.28$ and the initial condition $X_0=3.00$, $Y_0=1.05$.

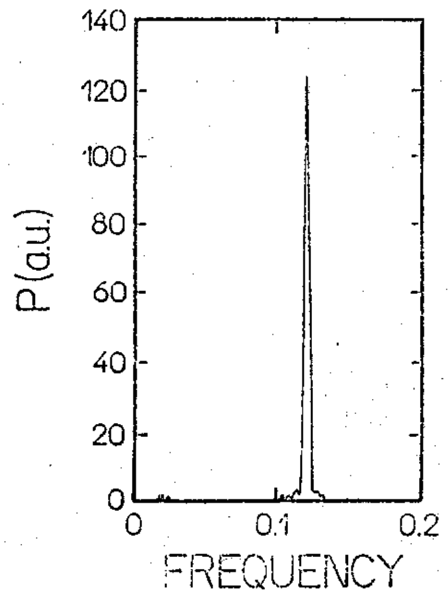

Fig. 3: Power spectrum related to Fig. 2. The sampling frequency is taken to be the unity.

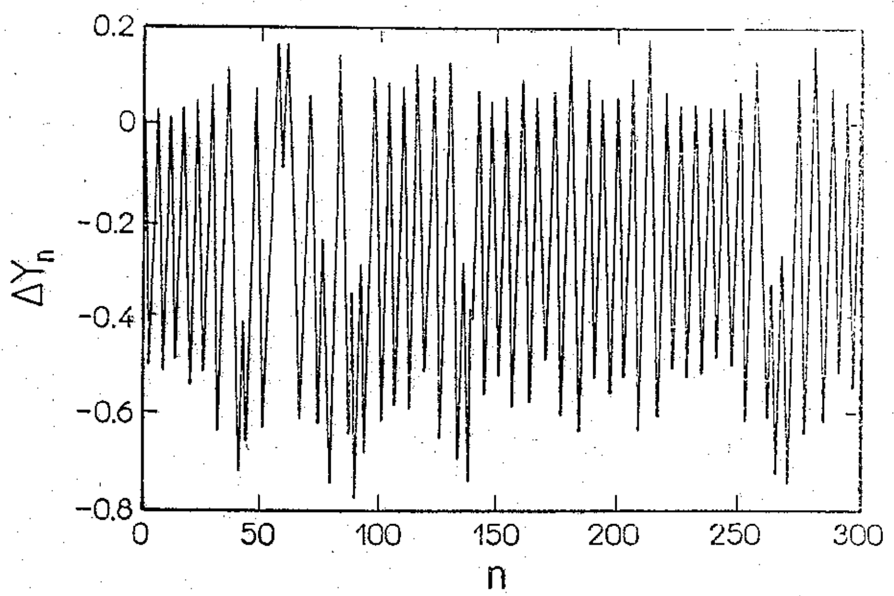

Fig. 4: Radial deviations with P=0.25; β=6.28 and the initial condition X_0=3.00, Y_0=0.20.

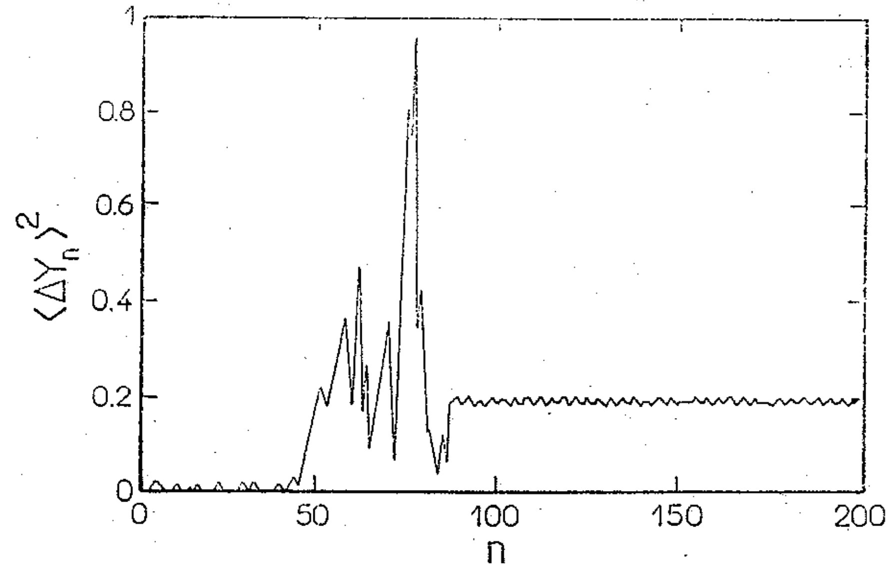

Fig. 6: Radial average quadratic deviations with P=0.25; β=6.28 and N=30 initial conditions equally spaced along the line Y_0=0.30.

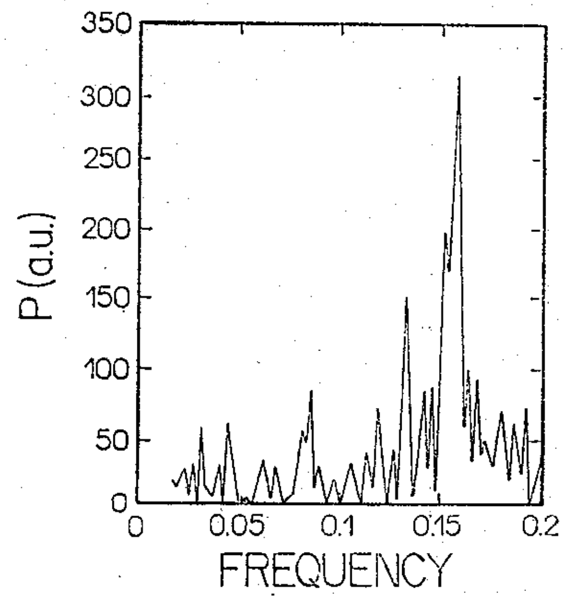

Fig. 5: Power spectrum related to Fig.4.

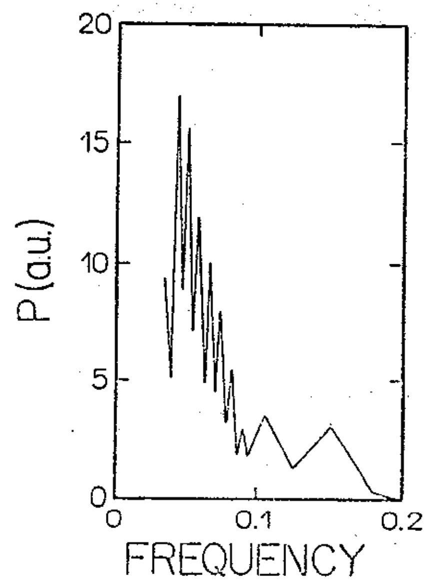

Fig. 7: Power spectrum related to Fig. 6.

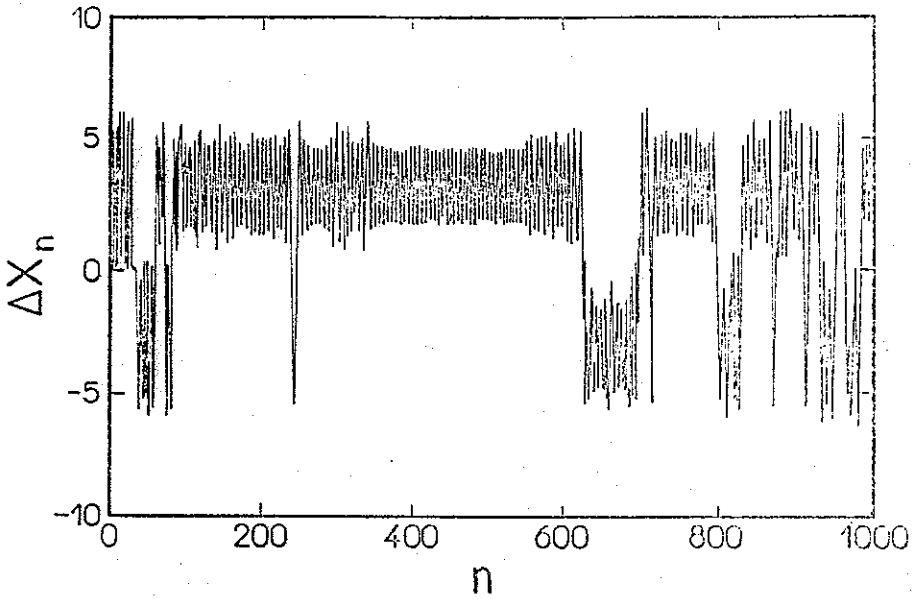

Fig. 8: Poloidal deviations with P=0.25; β=6.28 and the initial condition X_0=0.00, Y_0=0.19.