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ON THE CONDUCTIVE ENERGY TRANSPORT IN SOFT COMPTONIZED ACCRETION DISCS

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ABSTRACT

We have solved the soft comptonized two temperature accretion disc with the inclusion of the conductive energy transport, which we show to be the main cooling mechanism in the hot inner two temperature region. We also show that the size of the inner region grows as compared to the conduction free model and, if most of the radiation comes from that region, the flow is supersonic. The main mechanisms usually invoked for turbulence generation, shear and convective instabilities are shown to be not operative, which justifies the assumption of not developed turbulence. It is shown that the most probable source of viscosity are magnetic fields.

Key words: accretion discs – hydrodynamics – turbulence – convection – conduction – radiative transfer

I. INTRODUCTION

The observed spectrum of Cygnus X–1 is very similar to a power law with spectral index $n \simeq 1$ ($\mathcal{L}_E \propto E^{-1}$) for energies up to 100 KeV, above which there is an exponential cut–off ($\mathcal{L}_E \sim E^{-n} \exp -\frac{E_{100}}{100}$) (Liang and Nolan, 1984).

Two known radiation mechanisms could produce a power law spectrum: Synchrotron radiation from a relativistic power law distribution of electrons and unsatured inverse Comptonization. In the specific case of Cygnus X–1 there is a strong argument contrary to the adoption of the synchrotron radiation model: a Lorentz averaged factor of about 10^3, which would imply low efficiency for the conversion of the gravitational energy into radiation, requiring a very high accretion rate to supply the luminosity $\mathcal{L}_\chi \sim 10^{37-38}$. So, we are left with only a mechanism, unsatured inverse Comptonizaton.

However, to obtain a power law integrated spectrum it is necessary to keep the electronic temperature constant along the region where the radiation comes from.

Though the slightly radial distance dependent temperature obtained in the two temperature accretion disc (Shapiro et al., 1976) be small, the temperature variation along the region where the X–radiation is supposed to come from, may cast the spectral index n very far from the values required by observations. This would imply a softer X–ray spectrum, even for states of high luminosity.

To conciliate the model with observations we must face 2 problems: the first is related to the viscosity dependence of the temperature and the problem here is long standing one in Astrophysics because the microscopic viscosity in the disc fails to provide significant transport by many orders of magnitude (Pringle, 1981). It was originally assumed (von Weizsäcker, 1948) that since the flow within the disc is strongly shearing and often highly supersonic, it would be turbulent on scales comparable to the disc thickness. However, it has been pointed out by Safranov (1972), that since a typical disc easily satisfies the Rayleigh criterion for stability,

$$\frac{d}{dr}(r^2\Omega) > 0 \quad , \tag{1}$$

such an assumption is unjustified. Shakura and Sunyaev (1973) proposed parametrizing the unknown viscosity, ν, by a constant α defined by

$$\nu = \alpha \frac{C_s^2}{\Omega} \; . \tag{2}$$

Since then, modeling of discs has usually proceeded on the assumption that α is an unknown constant whose value must be adjusted to fit observations.

In spite of the evident difficulty in deriving a value of α from first principles, there have been a number of efforts in that direction since Shakura and Sunyaev's paper. Among the possibilities that have been discussed are the ideas of: effective viscosity derived from turbulence induced by convection, which is due to the heating caused by viscosity (Lin and Papaloizou, 1980; Cabot et al., 1987a,b); viscosity due to tangled magnetic fields in the disc sheared by differential rotation (Shakura and Sunyaev, 1973; Eardley and Lightman, 1975; Coroniti, 1981); viscosity generated by the instability due to the constructive interference after reflection from a radial boundary (Papaloizou and Pringle, 1984–85).

The second difficulty resides in the fact that a two temperature model yields high temperature gradients, which makes the ionic conductive (or convective) radial energy transport a very efficient one

$$\frac{Q_c}{Q_r} \sim 10^4 \tag{3}$$

(Q_c and Q_r respectively conductive and radiative fluxes) by no means negligible.

In this paper we shall speculate with the idea of an unknown process responsible for the generation of viscosity, which will be parametrized in the usual way, though not necessarily assigning to α the meaning of turbulent Mach number. Besides, we shall consider the ionic conductive radial energy transport. As we do not assume developed turbulence, we shall take a classical conductive constant. Regarding the problem of the spectral index, we shall impose constant electronic temperature along the disc, which implies to find out the law of variation of α with the radial distance.

II. THE EQUATIONS OF THE DISC

We shall consider a gas of fully ionized hydrogen, with pressure essentially given by the protons. As usual the φ component of velocity is keplerian and the disc is geometrically thin. Electrons and ions are out of thermal equilibrium and the ionic temperature is much greater than the electronic temperature. Electrons and protons are coupled only through collisional energy exchange, the cooling is dominated by unsaturated inverse Comptonization.

Throughout this paper we shall employ the following set of dimensionless variables:

a) r, the radial distance, in units of the inner radius $R_i = R_g$, where R_g is the gravitational radius;

b) T_i, T_e respectively ionic and electronic temperature in units of 10^9°K;

c) M_{34}, mass of the central compact object in units of 10^{34} g;

d) $\dot{M}_{17}$, accretion rate in units of 10^{17} g s^{-1}.

The introduction of the conductive energy transport and the assumption of constant electronic temperature with $\alpha = \alpha(r)$ do not make any structural modification on the disc equations, but the energy equation in which we shall also consider radial transport.

Following Shapiro et al. (1976), we obtain the following solution for the thin disc structure variables: ρ (density, g cm^{-1}), P (pressure, dyn cm^{-2}), ℓ (semi–scale height, cm), T_i

$$\ell = 1.2 \times 10^4 \dot{M}_{34} r^{3/2} T_i^{1/2} \tag{4}$$

$$\rho = 0.33 \frac{\dot{M}_{17} S}{\alpha M_{34}^2 T_i^{3/2} r^3} \tag{5}$$

$$P = 7.94 \times 10^{16} \frac{\dot{M}_{17} S}{\alpha M_{34}^2 r^3 T_i^{1/2}} \tag{6}$$

$$T_i = \frac{1.06 \times 10^3 \dot{M}_{17} S}{\alpha M_{34} r^{3/2}} T_e \tag{7}$$

where $S = 1 - 0.9 r^{-1/2}$.

With the assumption of constant electronic temperature we would obtain from the conduction free disc

$$\alpha = 2.22 \times 10^{-2} \frac{M_{34}}{T_e^6 \dot{M}_{17} S} r^{3/2} (\ln \Lambda)^2 \tag{8}$$

However, with the inclusion of the conductive energy transport together with the assumption of isothermal atmosphere (in z direction), the energy equation changes to

$$A \frac{S}{r^3} = -B \frac{\partial}{\partial r} \left\{ \frac{(ST_e)^{7/2}}{\alpha^{7/2} r^{21/4}} \right\} + \frac{C}{r^{9/4} \alpha^{1/2} T_e^3} \tag{9}$$

with

$$A = 1.93 \times 10^{25} \, \frac{\dot{M}_{17}}{M_{34}^2}$$

$$B = \frac{3.96 \times 10^{29}}{\ln \Lambda} \left[\frac{\dot{M}_{17}}{M_{34}^2} \right]^{7/2} \tag{10}$$

$$C = 3.54 \times 10^{23} \, \frac{\dot{M}_{17}}{M_{34}^{3/2}} \; .$$

One can easily check in equation (9) that $A/B \gg C/B$, a fact that will permit us to treat the radiative flux as a perturbation. With such an approximation together with the boundary condition

$$\left. \frac{dT_i}{dr} \right|_{r=r_0} = 0 \tag{11}$$

where r_0 is the outer radius of the inner region, we obtain the solution of equation (9),

$$\alpha = E^{-2/7} \, \frac{S}{r^{3/2}} \tag{12}$$

$$T_i = 1.03 \times 10^3 \, \frac{\dot{M}_{17} \, T_e}{M_{34}} \left\{ \frac{A}{2T_e^{7/2} \, B} \, r^{-2} - \left[\frac{0.36A}{2T_e^{7/2} \, B} + \frac{CA^{1/7}}{5B^{8/7} \, T_e^7} \right] r^{-5/2} + \right.$$

$$\left. + \frac{0.13 \, CA^{1/7}}{B^{8/7} \, T_e^7} \, r^{-3} + E \right\}^{2/7} \tag{13}$$

with

$$E \approx 1.43 \times 10^{12} \left\{ \frac{\dot{M}_{17} \, S_0^2 \, T_e^6}{M_{34} \, (\ln \Lambda)^2 \, r_0^3} \right\}^{7/2} . \tag{14}$$

To obtain equation (9), we have used the expression for collisional energy exchange F_{ee^+} (Spitzer, 1962)

$$F_{ee^+} = 9.23 \times 10^{24} \rho^2 \, \frac{\ell \, T_i \, \ln \Lambda}{T_e^{3/2}} \tag{15}$$

and for the (classical) conductive constant (Golant, Zhilinky and Sakharov, 1980)

$$K_c = 2.29 \times \frac{10^{19} \, T_i^{5/2}}{\ln \Lambda} \; . \tag{16}$$

It is interesting to remark from equation (9), that the radiative flux is given by

$$F = 1.92 \times 10^{25} \, \frac{\dot{M}_{17}^{3/2} \, S_0}{M_{34}^2 \, \ln \Lambda \, r_0^{3/2}} \, S^{-1/2} \, r^{3/4} \; . \tag{17}$$

To obtain equation (9) we have set constant $\ln \Lambda$.

III. CONSISTENCY OF THE MODEL

In a previous paper (Meirelles and Marques, 1989), we have analysed the constraints imposed on the turbulent Mach number, accretion rate and mass of the compact object in order to have a consistent solution for the two–temperature disc model.

Analysing the assumptions ionic pressure dominated gas, thin disc, ionic temperature lesser than the free fall temperature, unsaturated inverse Comptonization we have concluded that consistency is met whenever the following inequality holds,

$$7.55 \times 10^3 \frac{\alpha \dot{M}_{17} S}{M_{34} \gamma^{3/2}} \leq \ln \Lambda \leq 36\alpha^7 \left[\frac{M_{34}}{\dot{M}_{17} S} \right]^5 r^{3/2} . \tag{18}$$

However, with the new assumptions and with the inclusion of conductive energy transport clearly the consistency condition will change.

We now obtain the validity condition for each assumption of the model.

a) Ionic pressure dominated gas

$$\alpha \leq 29.24 \frac{M_{34} (\ln \Lambda) r_0^{3/2} S^{3/2} T_e}{\dot{M}_{17}^{1/2} S_0 r^{9/4}} . \tag{19}$$

b) Thin disc

$$\alpha \geq 0.27 \frac{\dot{M}_{17} S T_e}{M_{34} r^{1/2}} . \tag{20}$$

c) Unsaturated inverse Comptonization and scattering optical depth < 1

$$T_e > 1.5 . \tag{21}$$

d) Ionic temperature less than the free–fall temperature

$$\alpha \geq 0.29 \frac{\dot{M}_{17} S r^{1/2}}{M_{34}} T_e . \tag{22}$$

Clearly inequality (22) encompasses (20), therefore we may write

$$0.29 \frac{\dot{M}_{17} S r^{1/2} T_e}{M_{34}} \leq \alpha \leq \frac{29.24 M_{34} (\ln \Lambda) r_0^{3/2} S^{3/2} T_e}{M_{17}^{1/2} S_0 r^{9/4}} . \tag{23}$$

A necessary condition for the existence of a solution of inequality (23) is

$$\dot{M}_{17} \leq \frac{21.83 M_{34}^{4/3} (\ln \Lambda)^{2/3} r_0 S^{1/3}}{S_0^{2/3} r^{11/6}} . \tag{24}$$

The value of α at $r = r_0$, is given by the condition of instability, i.e.,

$$2 P_r = 3 P_g \tag{25}$$

which yields

$$\alpha_0 = 8 \times 10^{-20} \frac{M_{34}^7 r_0^{21/2}}{(\dot{M}_{17} S_0)^8} . \tag{26}$$

If we assume that most of the luminosity comes from the inner region, we get

$$\mathcal{L} \approx 2.7 \times 10^{38} \frac{\dot{M}_{17}^{3/2} S_0}{\ln \Lambda} \left\{ \frac{1}{5.5} r_0^{1.25} + r_0^{0.75} + 2.1 r_0^{0.25} \right\} \qquad (r_0 \gg 1) . \tag{27}$$

Substituting $\dot{M}_{17}$ given by eq. (27) in the left hand side of inequality

(23), we get

$$\frac{T_e^{7/2}}{M_{34}\ \mathcal{L}_{37}^{2/3}} \leq \frac{0.34\ r_0}{\left[\ \frac{1}{5.5}\ r_0^{1.25} + r_0^{0.75} + 2.1\ r_0^{0.25}\ \right]^{2/3} S_0^{5/3}} \qquad (28)$$

with $\mathcal{L}_{37}$, the luminosity in units of 10^{37} erg s^{-1}. To obtain (28) we have set $\ln \Lambda = 15$.

The right hand side of inequality (23) is easily satisfied, so we should say that inequality. (28) together with the condition

$$\alpha_0 \leq 1 \qquad (29)$$

are the consistency conditions we are looking for.

Let us now apply them for Cygnus X–1. Using the values $M_{34} = 3$, $T_e \simeq 2$, $\mathcal{L}_{37} = 3$ (Liang and Nolan, 1984), we see that inequality (28) changes to

$$5.32 \leq \frac{r_0}{\left[\ \frac{1}{5.5}\ r_0^{1.25} + r_0^{0.7} + 2.1\ r_0^{1/2}\ \right]^{2/3}} \qquad (29)$$

satisfied by any $r_0 \geq 1$.

However, from equations (12) and (26) we obtain

$$r_0^9 = 6.57 \times 10^{14}\ S_0^{19/3} \qquad (30)$$

which yields $r_0 \approx 40$. Substituting back into equation (26) gives $\alpha_0 = 1.7$, and the flow fails to be subsonic.

If we relax the assumption that most of the radiation comes from the

inner region and use instead

$$\dot{M}_{17} = \frac{0.33}{\varepsilon} \qquad (31)$$

where ε is the efficiency for the conversion gravitational energy into radiation, we obtain

$$\varepsilon = 26.3\ \frac{S_0}{r_0^{9/7}} \qquad (32)$$

and

$$\alpha_0 = 0.29\ r_0^{-3/14} \qquad (33)$$

which shows that the flow is subsonic everywhere.

As we have assumed a black hole for the central compact object, we must have

$$0.07 \leq \varepsilon \leq 0.4 \qquad (34)$$

which implies

$$22.35 \leq r_0 \leq 10^2 \qquad (35.a)$$

IV. CONCLUSIONS

With the inclusion of the (classical) conductive energy transport and the assumption of constant electronic temperature our model will make a better fit to the spectrum of Cygnus X–1, because our solution gives exactly a spectral index $n = 1$. With these new assumptions the ionic temperature gradients will be softened. The radiative flux will no longer be given for the usual expression from the accretion disc theory, because the radiative transport along the direction normal to the surface of the disc will be drastically altered owing to the conductive transport along the radial direction. The size of the inner region will be increased as compared to the size of the conduction free model. When these results are applied to Cygnus X–1 we can no longer find $\alpha_0 < 1$ if the radiation comes mainly from the inner region. When this assumption is relax we obtain $\alpha_0 < 1$ and

$$22.35 \le r_0 \le 10^2 \ . \tag{35.b}$$

If we assume a description of the turbulence of Landau, we must have

$$R_c \approx \frac{L\, v_k}{\nu} \tag{36}$$

where R_c is the critical Reynolds number, v_k is the keplerian velocity, L a caracteristic length in the disc. Setting L equal to the radial distance, we obtain

$$R_c \approx 1.48 \times 10^3\, r_0^{-11/14} \tag{37}$$

or

$$R_c \le 1.29 \times 10^2 \ .$$

However, from experimental results it is currently accepted that for turbulence generated by shear

$$R_c > 1.3 \times 10^3$$

(Monin and Yaglom 1971, page 77).

For turbulence generated by convenction, a necessary condition is (Tayler 1980)

$$\frac{PdT}{TdP} > 1/4 \ . \tag{38}$$

Using equations (6), (7) and (13) we obtain

$$\frac{PdT}{TdP} \simeq \frac{4}{21}\frac{A}{BE}\, r^{-2} \tag{39}$$

which is much less than 1/4. Therefore, the disc is not convectively unstable, and the assumption of not developed turbulence is justified.

Finally, we must inquire ourselves about the source of viscosity in the disc. Besides the possibilities we have already mentioned, as the ionic temperature is sufficiently high to produce neutrons by the dissociation of the $^4He\ (T_i > 30)$ or even by the reaction $pp \to pn\pi^+$ $(T_i > 300)$, neutron viscosity (Guessoum and Kazanas, 1990) could be operative in the inner region (r < 100) in accretion discs. However, we shall not consider it here because nuclear abundance curves in Guessoum and Kazanas (1990) paper focuse a very narrow range in $(r,\dot{M})$ space, we are not allowed to infer general conclusions.

The Papaloizou and Pringle process for viscosity generation is, to a certain extent, similar to the mechanism proposed by Vishniac and Diamond (1989), who have showed that the resulting viscosity is, approximately, given by

$$\nu \simeq \left[\frac{\ell}{r}\right]^2 \ell\, v_s \ , \tag{40}$$

where v_s is the sound speed.

It is easily seen from equation (40), that the resulting α parameter is much less the one obtained in the text. We are, therefore, left only with magnetic fields as a probable source of viscosity.

From equations (33), assuming $r_0 \approx 40$ (eq. 35.b), yields $\alpha_0 \simeq 0.13$, and we obtaind from the α prescription

$$\frac{B^2}{4\pi} \simeq 1.36 \times 10^{10}\ G^2 \quad . \tag{41}$$

As the magnetic pressure is about one tenth of the gas pressure, the magnetic field may be considered a weak field, with its effects being neglected in the conductive constant, as we did. A better argument is that the Larmor frequency is much less than the ion–ion frequency.

We now compare the cyclotron flux

$$\mathcal{F}_{cy} = 3.17 \times 10^{10}\, \rho\, T_e\, B^2\, \ell \ \mathrm{erg\ cm^{-1}\ s^{-1}} \tag{42}$$

to the total radiative flux, eq.(17).

It is easily seen that for $r = 40$ and $r = 2$, the ratio of the cyclotron emission to the total emission is, respectively, 2×10^{-3} and 5×10^{-3}, i.e., cyclotron emission has negligible effect on the emission. Even, if comptonization of this radiation were taken into account, the results would change, at the most, by a factor of 10.

We, then, conclude that magnetic fields are the most likely source of viscosity.

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