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ON THE ROLE OF SUPERSONIC TURBULENCE AND HOMOGENEITY IN THE STRUCTURE OF ACCRETION DISCS IN ACTIVE GALACTIC NUCLEI

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ON THE ROLE OF SUPERSONIC TURBULENCE AND HOMOGENEITY IN THE STRUCTURE OF ACCRETION DISCS IN ACTIVE GALACTIC NUCLEI

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SUMMARY

In this paper is pointed out that turbulent pressure pulsations should be taken into account in the equation of state for the pressure, in supersonic regimes. At the light of the celebrated α standard accretion disc model, it is shown that the structure and emission of optically thin region of steady accretion discs in Active Galactic Nuclei (AGN) may be drastically altered. It is shown that z–homogeneity is a very strong assumption that may lead to solution with matter flowing back from the compact object. The solutions obtained are everywhere thermally unstable. It is also shown that temperatures in the range $(1\text{–}5) \times 10^3$ °K yield masses quite different from the values usually assumed for Quasars, unless a given relation between M and $\dot{M}$ is used.

Key words: quasars – accretion – turbulence – lines: formation.

1. INTRODUCTION

The purpose of this paper is to point out some aspects concerning the role of turbulent pressure and z–non–homogeneity, whose account in the hydrostatic equilibrium and angular momentum conservation equations may withdraw some criticism concerning the applicability of the α standard accretion disc model to Active Galactic Nuclei (AGN). In a recent paper by Collin–Souffrin (1987) the structure and emission of the optically thin region of steady accretion discs in Active Galactic Nuclei (AGN) is obtained at the light of the celebrated α accretion disc model (Shakura and Sunyaev, 1973, 1976). Some of the main results obtained by the author, such as how far this region extends from the center, temperature, density and scale height of the disc as well as the region of gas pressure dominance and the effects of self gravity are highly dependent on the value of the turbulent Mach number. From the assumption of z–homogeneity one should expect the constancy of the scale–height of the disc, a result that if not confirmed may lead to matter flowing from the compact object in the case of ℓ growing with the radial distance (Urpkin, 1983).

Though not questioning the idea of accretion discs to explain some features (or all of them) of the emission spectrum, it should be emphasized, the applicability of the model to these objects, the way it stands in the paper, should be seen with reticence.

2. THE INCLUSION OF THE TURBULENT PRESSURE PULSATIONS IN THE HYDROSTATIC EQUILIBRIUM EQUATION

It is usual do separate the velocity field V in the disk into an average velocity field, here denoted U and a stochastic one represented by u. Clearly,

$$V_i = U_i(\bar{x}) + u_i(\bar{x},t) \tag{1}$$

For the continuity equation we have, in the stationary case,

$$\frac{\partial}{\partial x_i}(\rho U_i) = 0 \qquad (2)$$

and

$$\frac{\partial}{\partial x_i} u_i = 0 \quad . \qquad (3)$$

We have neglected pulsations in the density ρ and assumed homogeneous turbulence.

Assuming stationarity for the turbulence and taking the time average of the pressure P, u_i and U_i the force equation reads

$$U_j \frac{\partial}{\partial x_j} U_i + \frac{1}{\rho} \frac{\partial}{\partial x_i} \langle P \rangle + \frac{\partial}{\partial x_j} \langle u_i u_j \rangle - \nu \frac{\partial^2}{\partial x_j^2} U_i = \frac{\partial}{\partial x_i} \psi \quad . \qquad (4)$$

Linear terms in u_i have disappeared with the averaging. ψ stands for the gravitacional potential and ν for the molecular viscosity. Compared with the other terms, the term, due to the molecular viscosity is of the order of $1/Re$ ($Re \equiv$ Reynolds number $\gg 1$) and should be neglected.

Following Favre et al. (1976) we make the assumption of proportionality between the Reynolds stresses and the turbulent kinetic energy

$$u_i u_j = a \sum_{k=1}^{3} u_k^2 = a u^2 \qquad (5)$$

a is for order unity, and will be set equal to 1.

Then, the z component of equation (4) is, assuming thin disk and $U_z = 0$,

$$\frac{\partial}{\partial z} P + \rho \frac{\partial}{\partial z} \langle u^2 \rangle = -\rho \Omega^2 z \qquad (6)$$

with P standing for the sum of gas and radiation pressures and Ω for the keplerian angular velocity.

If matter in the disk is constant along z and falls abruptly to zero at $z = \ell$ (ℓ = semi–scale height of the disk) (Shakura, Sunyaev, 1976),

$$-\frac{\partial}{\partial z}\left\{\left[1 + \frac{m_t^2}{\gamma}\right] P\right\} = \rho \Omega^2 z \qquad (7)$$

m_t is the turbulent Mach number and γ is the adiabatic index. We shall set γ equal to 1 instead of the correct value $4/3$.

We now obtain the relation between the viscosity parameter α and the turbulent Mach number m_t. According to Shakura and Sunyaev (1973,1976) setting magnetic fields to zero,

$$\alpha = \frac{\ell_t \, u}{\ell \, v_s} \qquad (8)$$

where

$\ell_t \equiv$ turbulent scale length

$v_s \equiv$ sound velocity.

However, $m_t = u/v_s$ so

$$m_t = \alpha \frac{\ell}{\ell_t} \qquad (9)$$

with $\ell/\ell_t \geq 1$.

Using eq. (9), we may write

$$P_t = m_t^2 \rho v_s^2 = \alpha^2 \left[\frac{\ell}{\ell_t}\right]^2 \rho v_s^2 \quad . \qquad (10)$$

The prescription of ℓ_t is very difficult and usually one sets $\ell_t = \ell$. This implies $\alpha = m_t$.

It should be noticed that (7) is quite general and is similar to the expression used by Shakura et al. (1978), White and Holt (1981, preprint) and Kriz and Hubeny (1986).

3. THE STRUCTURE OF THE ACCRETION DISK

In the following we shall obtain the structure of accretion disk in the optically thin and gas pressure dominated region. As in Collin–Souffin's (1987) paper, we shall assume homogeneity in z for the density and the temperature and, exception made for the inclusion of the turbulent pressure pulsations in the hydrostatic equilibrium equation, our equations are identical to hers. Consequences of z homogeneity will be analysed later on this paper. Let us briefly review the equations.

The continuity equation (2) is easily integrated to give

$$\dot{M} = -4\,\pi\, r\, \ell\, V_r\, \rho \qquad (11)$$

$\dot{M}$ is the accretion rate, r the radial distance and V_r the radial velocity. In an inhomogeneous media V_r and ρ in equation (11) should be interpreted as averaged values over z.

From the φ component of equation (4), we obtain through the α prescription

$$V_r = -\alpha\, v_s \frac{\ell}{r} \,. \qquad (12)$$

Neglecting pressure gradients along the r direction, the heat generation function is given by

$$D(r) = \frac{3}{8\pi}\,\Omega^2\,\dot{M} \qquad (13)$$

and the radiative flux is

$$F(r) = \pi \int_0^\infty B_\nu(t)\left[1 - e^{-\tau_\nu}\right] d\nu \qquad (14)$$

B_ν is the Planck function and τ_ν, the optical depth, is given by

$$\tau_\nu = 2\,\rho\,\ell\, x_\nu \qquad (15)$$

x_ν is the exponential mass absorption coefficient ($cm^2 g^{-1}$).

Integrating equation (7) over z, with the boundary condition $P(z=\ell) = 0$, taking the average over z and equating this to the gas pressure, we obtain

$$\ell = 1.56 \times 10^4\, \frac{(1+\alpha)^{1/2}}{\Omega}\, T^{1/2}\ \text{cm} \qquad (16)$$

T is the temperature. Using (11), (12) and (16) we obtain for the density

$$\rho = \frac{6.29 \times 10^{-14}\ \dot{M}\Omega^2}{\alpha(1+\alpha^2)^{3/2}\, T^{3/2}}\ \text{g cm}^{-3} \,. \qquad (17)$$

In the stationary regime heat generation equals radiative cooling so, using equations (13), (14), (15) and (16) we obtain

$$x_\nu = 6.8 \times 10^{11}\ \frac{\alpha(1+\alpha^2)\Omega}{T^3} \,. \qquad (18)$$

For x_ν we shall adopt the values by Alexander et al. (1983), as given by Collin–Souffrin (1987). From now on we shall express the radial distance in units of parsec (r), the mass in units of 10^9 solar masses (M_9), the scale height in units of 10^{15} cm (ℓ), the accretion rate in units of 1 solar mass per year ($\dot{M}_1$) and the density in g cm^{-3}.

The solutions for the physical variables will be labelled by a subscript i, running from 1 to 5, corresponding to different expressions of the absorption coefficient.

From table 1 in Collin–Souffrin's paper, the temperature at r_L, the radius at which the disk becomes optically thin, never exceeds 2400°K. So, we shall confine our attention to temperatures lesser than 3200°K and shall neglect bound–free, free–free, H, H^- opacities and electron scattering.

In the following we shall give the expressions for T, ℓ and ρ for different expressions of the absorption coefficient. We shall also give expressions for r_L and T_L, the temperature at which the disk becomes optically thin.

1) $\log T < 3.2$

$$x_1 = 63\,\rho^{0.625}$$

$$T_1 = 2.24 \times 10^2 \, \frac{\alpha^{26/33} \, (1+\alpha^2)^{31/33} \, r^{6/33}}{\dot{M}_1^{10/33} \, M_9^{2/33}}$$

$$\ell_1 = 6.2 \, \frac{\alpha^{13/33} \, (1+\alpha^2)^{32/33} \, r^{35/22}}{\dot{M}_1^{5/33} \, M_9^{35/66}}$$

$$\rho_1 = \frac{5.42 \times 10^{-12} \, \dot{M}_1^{16/11} \, M_9^{12/11}}{\alpha^{24/11} \, (1+\alpha^2)^{32/11} \, r^{36/11}} \qquad (19)$$

$$T_{1L} = 1.97 \times 10^2 \, \dot{M}_1^{-0.195} \, \alpha^{26/41} \, (1+\alpha^2)^{3/4}$$

$$r_{1L} = \frac{0.5 \, \dot{M}_1^{73/123} \, M_9^{41/123}}{\alpha^{104/123} \, (1+\alpha^2)}$$

2) $3.2 < \log T < 3.35$

$$x_2 = 10^{-27} \, T^9 \, \rho^{0.625}$$

$$T_2 = 1.2 \times 10^3 \, \frac{\alpha^{26/177} \, (1+\alpha^2)^{31/177} \, r^{2/59}}{\dot{M}_1^{10/177} \, M_9^{2/177}}$$

$$\ell_2 = 11.3 \, \frac{\alpha^{13/177} \, (1+\alpha^2)^{104/177} \, r^{89/59}}{\dot{M}_1^{5/177}}$$

$$\rho_2 = \frac{4.37 \times 10^{-13} \, \dot{M}_1^{64/59} \, M_9^{60/59}}{\alpha^{72/59} \, (1+\alpha^2)^{52/27} \, r^{180/59}} \qquad (21)$$

$$T_{2L} = 1.09 \times 10^3 \, \frac{\alpha^{13/118} \, (1+\alpha^2)^{31/236}}{\dot{M}_1^{0.043}}$$

$$r_{2L} = 5.19 \times 10^{-2} \, \frac{\dot{M}_1^{217/555} \, M_9^{1/3}}{\alpha^{26/177} \, (1+\alpha^2)^{31/177}} \, .$$

For $3.35 < \log T < 3.5$, the expression for x depends on the value of ρ in the following manner:

3) $\log \rho > -7$

$$x_3 = 7.41 \times 10^{-7}\, T^{1.47}$$

$$T_3 = 64.72\, \frac{\alpha^{0.22}\,(1+\alpha^2)^{0.22}\, M_9^{0.11}}{r^{0.34}}$$

$$\ell_3 = 2.62\, \frac{\alpha^{0.11}\,(1+\alpha^2)^{0.61}}{\dot{M}_9^{0.44}}\, r^{1.33}$$

$$\rho_3 = \frac{3.5 \times 10^{-11}\, \dot{M}_1\, M_9^{0.83}}{\alpha^{1.34}\,(1+\alpha^2)^{1.84}\, r^{2.49}} \qquad (22)$$

$$T_{3L} = 39.94\, \frac{\alpha^{0.41}\,(1+\alpha^2)^{0.41}}{\dot{M}^{0.223}}$$

$$r_{3L} = 4.24\, \frac{\dot{M}^{0.631}\, M^{1/3}}{\alpha^{0.55}\,(1+\alpha^2)^{0.55}} .$$

4) $\log \rho < -10$

$$x_4 = 10^{17}\, T^{-6}$$

$$T_4 = \frac{1.03 \times 10^5\, r^{1/2}}{M_9^{1/6}\, \alpha^{1/3}\,(1+\alpha^2)^{1/3}}$$

$$\ell_4 = 1.05 \times 10^2\, \frac{(1+\alpha^2)^{1/3}\, r^{7/4}}{M_9^{0.5}\, \alpha^{1/6}}$$

$$\rho_4 = \frac{5.5 \times 10^{-16}\, \dot{M}_1\, M_9^{5/4}}{\alpha^{1/2}\,(1+\alpha^2)\, r^{9/4}} \qquad (23)$$

$$T_{4L} = \frac{6.86 \times 10^3\, \dot{M}^{1/10}}{\alpha^{1/5}\,(1+\alpha^2)^{1/5}}$$

$$r_{4L} = 4.43 \times 10^{-3}\, \dot{M}^{1/5}\, M^{1/3}\, \alpha^{4/15}\,(1+\alpha^2)^{4/15}$$

5) $-10 < \log \rho < -7$

$$\log x = (2.49 \log \rho + 18.9) \log T - 7.71 \log \rho - 60.1 .$$

Now T and T_L are given, respectively, as the solutions of

$$3.74 (\log T)^2 - (\log T) \log \left\{ \frac{1.2 \times 10^{13} (M_9 \dot{M}_1)^{2.49}}{\alpha^{2.49}\,(1+\alpha^2)^{3.74}\, r^{7.47}} \right\}$$

$$+ \log \left\{ 8.35 \times 10^{-2}\, \frac{\dot{M}_1^{7.71}\, M_9^{8.21}}{\alpha^{7.71}\,(1+\alpha^2)^{11.57}\, r^{26.13}} \right\} = 0 \qquad (24)$$

and

$$6.22 (\log T_L)^2 + (\log T_L) \log \left\{ \frac{7.23 \times 10^{-44}}{\alpha^{2.49}\,(1+\alpha^2)^{3.74}} \right\}$$

$$- \log \left\{ \frac{1.36 \times 10^{-76}}{\alpha^{7.71}\,(1+\alpha^2)^{11.57}\, \dot{M}_1\, M_9^{1/2}} \right\} = 0 . \qquad (25)$$

The correct solutions of equations (24) and (25) are those that yield T in the appropriate range.

4. CAN THIS MODEL BE APPLIED TO QUASARS?

Clearly, the answer to that question is highly dependent upon the picture we assume for a Quasar, i.e., the mass, the accretion rate and how they are related to each other. For that we shall assume the evolutionary scheme described by Luminet (1981), according to which the Quasar phase is fuelled by stellar collisions which begin to dominate over other supplying gas processes at a core (nucleus) mass of about $M_9 \simeq 0.035$ and lasts till the mass of nucleus reaches $M_9 \simeq 1$. In that phase the mass and the accretion rate are related by

$$\dot{M}_1 = 10^2 M_9^3 \quad . \tag{26}$$

A glance at the results we just obtained shows that temperatures lesser than 3200°K are only produced in the disk, for masses and accretion rates in the Quasar range, if turbulence is supersonic. To clear this point, we shall find out what relation M_9 and α should satisfy in order to have T_L and ρ_L in the prescribed value for each region (x_i).

1) $1.19 \times 10^{-2}\, \alpha^{1.08} (1+\alpha^2)^{1.28} < M_9 < 2.6 \times 10^{-2}\, \alpha^{1.08} (1+\alpha^2)^{1.28}$ (27)

2) $8.34 \times 10^{-4}\, \alpha^{0.85} (1+\alpha^2)^{1.01} < M_9 < 1.23 \times 10^{-2}\, \alpha^{0.85} (1+\alpha^2)^{1.01}$ (28)

3) $3.13 \times 10^{-4}\, \alpha^{0.62} (1+\alpha^2)^{0.62} < M_9 < 5.24 \times 10^{-4}\, \alpha^{0.62} (1+\alpha^2)^{0.62}$ (29)
$M_9 < 2.5 \times 10^{-4}\, \alpha^{0.01} (1+\alpha^2)^{-0.28}$

4) $5.17 \times 10^{-3}\, \alpha^{2/3} (1+\alpha^2)^{2/3} < M_9 < 1.63 \times 10^{-2}\, \alpha^{2/3} (1+\alpha^2)^{2/3}$ (30)
$\alpha < 4.8$

5) $0.13\, \alpha^{0.18} (1+\alpha^2)^{0.28} < M_9 < 0.13\, \alpha^{0.29} (1+\alpha^2)^{0.43}$ (31)
$3.88 \times 10^2\, \alpha^{2/5} (1+\alpha^2)^{3/5} < T_L < 6.16 \times 10^3\, \alpha^{2/5} (1+\alpha^2)^{3/5}$.

It is readily seen that only relations (27), (28) and (31) are compatible with masses usually assumed for quasars. It is also seen that this compatibility is only achieved for $\alpha > 1$. If turbulence pressure pulsations were not included α would be much greater.

It is worth to remark that only for region 4 temperature is a decreasing function of the radius. As the density is everywhere decreasing with radius, region 4 will only occur in Quasars for r_L belonging to it. However, as seen from relation (29) it only happens for masses below 2.5×10^{-4}, which is definitively outside the quasar mass range. For relations (27), (28), (30) and (31) temperature is a monotonically increasing function of the radius, while the flux behaves differently. Therefore, the disk is thermally unstable everywhere.

Finally, it should be noted that with the inclusion of turbulent pressure pulsations density will decrease in the disk (for Quasars), therefore, the effects of self–gravity will be highly attenuated.

5. IS DISK ACCRETION COMPATIBLE WITH z–HOMOGENEITY?

According to the α standard disk model of Shakura and Sunyaev (1976) the density, the kinematic viscosity coefficient K and the (not integrated over z) stress, assuming Keplerian velocity, are given respectively by

$$\rho = \rho_c(r)\, H(\ell - z)$$

$$K = \frac{\alpha\Omega\ell^2}{3} \tag{32}$$

$$\omega_{r\varphi} = \frac{3}{2} K\rho\Omega$$

where H is the Heaviside function and ρ_c is the constant (over z) value of the density given by

$$\rho_c = \frac{3}{4\pi}\,\frac{\dot{M}\, S}{\ell^3\Omega} \tag{33}$$

with $S = 1 - \delta\left[\frac{r_*}{r}\right]^{1/2}$. r_* is the inner radius of the disk and δ is the ratio of the actual angular momentum of the flow to the Keplerian one at $r = r_*$.

It should be remarked from equation (11) that accretion onto the compact object means $\dot{M} > 0$, which implies $V_r < 0$, i.e., the radial velocity is directed toward the compact object.

Using equation (32) we may write the angular momentum conservation equation as

$$\frac{\rho V_r V_k r}{2} = -\frac{\partial}{\partial r}\left\{\frac{\alpha\rho\ell^2\,\Omega^2 r^2}{3}\right\} \tag{34}$$

V_k being the keplerian velocity. Using now equations (11) and (33) we obtain

$$\frac{r\Omega}{2\ell} = \frac{\partial}{\partial r}\left\{\frac{S\Omega r^2}{\ell}\right\} \tag{35}$$

Making $y = \frac{S\Omega r^2}{\ell}$, equation (35) changes to

$$\frac{\partial}{\partial r} y = \frac{1}{2}\,\frac{y}{Sr} \tag{36}$$

which solution is

$$y = C\, r^{1/2}\, S \tag{37}$$

C is an integration constant.

Comparing the definition of y with equation (37) we conclude that for the α standard accretion disk model the disk scale height ℓ is constant.

Expressing V_r by means of (34) and using (33) gives

$$V_r = -\frac{1}{2\pi}\,\frac{\dot{M}}{\rho\, r^2\Omega}\,\frac{\partial}{\partial r}\left[\frac{r\Omega S}{\ell}\right] \tag{38}$$

in the region we are concerned $S \simeq 1$. We see from (38) that accretion means $\frac{\partial}{\partial r}\left(\frac{r\Omega}{\ell}\right) > 0$. However, in every solution we got ℓ varies faster than $r^{1.3}$, which clearly implies $V_r > 0$.

If now we abandon the assumption of equality between the scale disk height and the scale length of the turbulence and use the correct hydrostatic equilibrium equation

$$P = \frac{\rho_c\Omega^2\,\ell^2}{2(1+m^2)}(1-x^2) \tag{39}$$

with $x = z/\ell$, we obtain

$$V_r \simeq -\frac{\alpha\dot{M}}{\Omega\rho r^2(1+m^2)}\left\{\left[\frac{1}{2}\frac{r\Omega}{\ell} - \frac{r^2\Omega}{\ell^2}\frac{\partial\ell}{\partial r}\right](1-x^2)^{1/2} + \frac{r^2\Omega x^2}{\ell(1-x^2)^{1/2}}\frac{\partial\ell}{\partial r}\right\} . \tag{40}$$

If $\frac{\partial\ell}{\partial r} \neq 0$, as $x \to 1$, $|V_r| \to \infty$.

Therefore, the only solution compatible with accretion, if the disk is homogeneous over z, is ℓ constant.

4. CONCLUSIONS

Restricting our results to Quasars, we may say that inclusion of turbulent pressure pulsations will make the optically thin region more extended, i.e., closer to the compact object. We have shown that the Quasar mass regime is only achieved for supersonic turbulence. For this regime the structure of the optically thin region will be affected because the temperature and the scale height of the disk will increase, while the density will decrease in a very pronounced way. This will make the disk less sensitive to the effects of self–gravity. In this regime, the optically thin region will be everywhere thermally unstable. It is worth to remark that temperatures in the optically thin region in range (1–5) 10^3 °K will imply masses completely outside the quasar mass range, unless we use equation (26).

Finally, it should be remarked that homogeneity is compatible with accretion only if ℓ is constant. Otherwise, we may find regions in the disk where matter is flowing back to infinity.

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