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TURBULENCE, CONVECTION AND STABILITY IN ACCRETION DISKS

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ABSTRACT

Allowance for turbulent pressure pulsations, convection and dependence of the turbulence along z in accretion disks results in turbulent scale length much less than the disk (semi) scale height, more effectiveness for the convective energy transport against instability and gas pressure dominance in the flow, even for sonic regimes. Solution for the convective disk exists only for $\beta_0 \geq 0.14$ and forces instability to depend on the turbulence level of the disk. For the viscosity law we have adopted (and for the standard one) the disk will be stable up to α (turbulent Mach number) ~ 1 (sonic regime). When applied to Galatic and Extragalactic X–rays sources, these results suggest that absence (presence) of fluctuations may indicate subsonic (supersonic) turbulent regime as well as confirm (refute) the assumed viscosity law, rather than a clue for externally (internally) produced photons cooled by the thermal plasma.

Subject headings: accretion – convection – turbulence – X–rays: binaries

I. Introduction

Recently, a number of astrophysicists have addressed to the stability problem of accretion disks with electron–positron pairs (Lightman 1982; Svenssson 1982; Moskalik and Sikora 1986; Sikora and Zbyszewska 1986; Kusunose and Takahara 1988; Björnsson and Svensson 1990; White and Lightman 1990). It is well known that for accretion disks cooled by internally produced photons and also by externally produced photon (Kusunose and Takahara 1990; Meirelles 1990) there is a critical accretion rate above which equilibrium pair production–annihilation no longer holds. This implies an annular region centered on $r = 10 \frac{GM}{c^2}$, in which no steady state is possible (Kusunose and Takahara 1988; White and Lightman 1989–1990). Below this critical accretion rate there are two consistent solutions to the disk equations: the low pair density and the high pair density branches. It has been found by White and Lightman 1990, that disks with high density of pairs when perturbed are unstable against pair runaway. They produce pairs very explosively, then cool and collapse toward cool optically thick disks without pairs. They also found that disks with low density of pairs and with accretion rate above the critical one have identical behaviour. As the inner region of the optically thick disks is secularly unstable, these disks would fluctuate between optically thick and optically thin states. These fluctuations should be followed by changes in the spectrum which would serve as model for galatic and extragalactic X–rays sources.

It should be argued, however, that the role of convection has been underestimated in the accretion disks of the 1970's. When convection is properly taken into account fluctuations between optically thick and optically thin phases will only occur if turbulence is highly supersonic.

In this paper we shall be mainly concerned with the evolution in time of optically thick disks without pairs. Our main result is to show that the role of convection will determine the evolution in time of accretion disks even not being the main energy transport in accretion disks.

Another result of this paper is to show that instability sets in the disk only for β_0 (relative radiation pressure) greater than 0.8. However, it is shown that this value of β_0 is only reached for α_0 (turbulent Mach number) ≥ 7. For such a high value of α_0, shock instabilities will be present in the disk long before the secular instability of Lightman and Eardley 1974.

We believe these facts provide us enough motivation to retake the instability problem of the accretion disks of the 1970's.

II. On the convective energy transport in accretion disks: a brief review

Convective energy transport in accretion disks is important, not only for determining its vertical structure, but also by the fact that convection itself may be the source of turbulence. Besides this, for some systems (like Cygnus X–1 and NGC 4151) observational data suggest the existence of a corona, whose inception above the accretion disk may be highly dependent on the amount of convective energy transport (Liang and Thompson 1979; White and Holt 1981; Liang and Nolan 1984).

Though the question of necessary conditions for the onset of convection in accretion disks (with opacity and viscosity laws expressible as products of powers of the density and temperature) had been answered by Tayler (1980) with the application of the Nauer–Osterbrock (Nauer and Osterbrock, 1953) criterion for convection in stars, the question of the amount of energy transported by convection is controversial. Bisnovatyi–Kogan and Blinnikov (1977) have concluded, under the adiabatic vertical structure assumption, that convection is the most relevant energy transport mode for disk

accretion at subcritical luminosity. Shakura et al. (1978) have obtained an approximate energy equation, whose polytropic solution yields, at the most, 30% for the convective energy transport, independently of the mass of the central object, the accretion rate and the turbulence level of the flow. Assuming similar dependence on the scale length of the disk(ℓ) and on the surface density(U) for the heat flux and convective flux Shakura et al. have concluded that convection will only change the increment of the instability growth as compared to the convection free model (Shakura and Sunyaev 1976).

However, it should be argued that such a dependence on ℓ and U only holds in the stationary regime.

Minor restrictions to Shakura et al. (1978) treatment of the convective energy transport in the inner region of accretion disks are: no allowance is given for the dependence of the turbulence along z and the fact that polytropic solution of the approximate energy equation of Shakura et al. (1978) only exists for $\alpha_0 = 0.44$.

Piran (1978) has concluded that Liang's convective disk was both thermal and secularly unstable. However, this conclusion was obtained using an expression for c_p (specific heat at constant pressure) valid only for gas pressure dominated flows, as is the case of Liang's unperturbed disk. In the perturbed disk one has to use the full expression for c_p which takes into account the contribution of matter as well as the contribution of radiation.

In this paper we propose a way of estimating the amount of energy transported by convection in the inner region of accretion disks where opacity is given mainly by electron scattering. Our method of doing this improves previous ones in the literature in the following aspects:

a. we employ the correct energy equation,

b. parametrizing the ratio of radiation and gas pressures (ξ), we use the correct solution of the radiative transfer equation in the plane parallel atmosphere approximation,

c. allowance for the dependence of turbulence along z is taken into account with the variation of the turbulent scale length and velocity with z, the distance to the symmetry plane of the disk.

III. The unperturbed disk equations

In this section we shall obtain the description of the accretion disk in terms of the following variables and parameters: density ρ, temperature T_9 (in units of 10^9°K), total pressure P, gas pressure P_g, ratio of radiation and gas pressure ξ, ratio of radiation and total pressure β, ratio of energies transported by convection and radiation q, disk scale length ℓ, turbulent velocity v_t, turbulent scale length ℓ_t, mass of the central compact object M_{34} (in units of 10^{34}g), accretion rate $\dot{M}_{17}$ (in units of 10^{17}g s^{-1}), keplerian angular velocity Ω, turbulent Mach number α. The disk has azimuthal symmetry and we shall employ cylindrical coordiantes (r,φ,z), where r is the (radial) distance to the compact object (in units of GM/c^2), φ is the azimuth and z is the distance to the symmetry plane of the disk. To simplify our equations we shall take γ (adiabatic index) equal to 1, instead of the correct value of 4/3 (Shakura et al., 1978).

Account of turbulent pressure pulsations will slightly modify the equation of hydrostatic equilibrium, which reads

$$(1+\alpha^2)\frac{\partial}{\partial z}(1+\xi)\,P_g = -\rho\,\Omega^2 z \quad . \tag{1}$$

From the above equation we see that turbulent velocity is defined as

$$v_t = \frac{\alpha}{\sqrt{1+\alpha^2}}\left[\frac{P}{\rho}\right]^{1/2} \tag{2}$$

with α constant along z.

Assuming equality between keplerian and turbulent times, we have for the turbulent scale length ℓ_t,

$$\ell_t = \frac{v_t}{\Omega} \quad , \tag{3}$$

a well known result from homogeneous turbulence (Landau and Lifshitz, 1971).

The turbulent Reynolds stress between adjacent layers of the disk is

$$W_{r\varphi} = -2r\,\frac{d\Omega}{dr}\int_0^{\ell} \eta\,dz \quad , \tag{4}$$

where η is the turbulent viscosity given by

$$\eta = \frac{1}{3}\,\rho\,v_t\,\ell_t \quad , \tag{5}$$

or, using equations (2) and (3),

$$\eta = \frac{1}{3}\,\frac{\alpha^2}{1+\alpha^2}\,\frac{P}{\Omega} \quad . \tag{6}$$

From equations (4) and (6) we may write the expression for the heat generation in the disk,

$$Q^{+} = -\frac{1}{2} r \frac{d\Omega}{dr} W_{r\varphi} = \frac{3}{4} \Omega \frac{\alpha^2}{1+\alpha^2} \int_0^{\ell} P \, dz \quad . \tag{7}$$

In stationary regime this energy is transported to the disk surface by radiative and convective energy fluxes, respectively

$$F_r = -c \, \xi \frac{\partial}{\partial \tau} P_g \tag{8}$$

(Rybicki and Lightman, 1979) and

$$F_t = -\eta T \frac{dS}{dz} = -\eta \left\{ c_p \frac{dT}{dz} - \frac{1}{\rho} \frac{dP}{dz} \right\} \tag{9}$$

(Shakura et al., 1978) where c is the velocity of light, τ is the optical depth and c_p is the specific heat given by (Chandrasekhar, 1950)

$$c_p = \frac{R}{3} (5 + 40\xi + 32\xi^2) \tag{10}$$

R being the gas constant.

We now use equations (7),(8) and (9) to obtain the energy equation

$$\frac{c m_H}{\sigma_T \rho \ell} \frac{\xi}{1+\xi} \frac{1}{(1+\alpha^2)} \frac{\partial}{\partial x} P + \eta \left\{ \frac{c_p}{\ell} \frac{\partial}{\partial x} T - \frac{1}{\rho \ell} \frac{\partial}{\partial x} P \right\} = -\frac{3}{4} \Omega \frac{\alpha^2 \ \ell}{1+\alpha^2} \int_0^{x} P \, dx \quad , \tag{11}$$

with m_H the hydrogen mass, σ_T the Thompson electron scattering cross section and x is distance to the symmetry plane expressed in units of the (semi) disk length ℓ.

To solve equations (1) and (11) we shall express ρ, T and P in terms of central values $(x = 0)$, i.e.,

$$\begin{aligned} \rho &= \rho_c \, j(x) \\ T &= T_c \, t(x) \\ P &= P_c \, p(x) \quad . \end{aligned} \tag{12}$$

For the functions j, t and p we shall impose at $x = 0$

$$j = t = p = 1$$

and null boundary condition at $x = 1$, i.e., we shall search a polytropic solution

$$\begin{aligned} j &= (1 - x^2)^n \\ t &= (1 - x^2)^s \\ p &= (1 - x^2)^m \quad . \end{aligned} \tag{13}$$

The reason for looking for such a kind of solution is due to the fact that for stellar situations, ocurrence of convection takes place with temperature gradients quite close to adiabatic gradients. Although in the disk case, the situation be much more complex owing to not static medium, we believe a temperature gradient much greater than the adiabatic one will give birth to tremendous energy exchange, lowering the temperature gradient to a value close to the adiabatic one.

Using equations (12) and (13) equations (1) and (11) change respectively to

$$\frac{\partial}{\partial x}(1-x^2)^m = -H(1-x^2)^n x \tag{14}$$

$$Ax - B(1-x^2)^m \frac{\partial}{\partial x}(1-x^2)^s - D(1-x^2)^m x = E\int_0^x (1-x'^2)^m dx' \tag{15}$$

with

$$H = \frac{\rho_c\,\Omega^2\,\ell^2}{P_c}$$

$$A = \frac{c m_H\,\Omega^2\,\ell}{\sigma_T(1+\alpha^2)}\frac{\xi}{1+\xi}$$

$$B = \frac{\alpha^2\,P_c\,c_p\,T_c}{3(1+\alpha^2)\ell\,\Omega} \tag{16}$$

$$D = \frac{\alpha^2\,P_c\,\Omega\,\ell}{3(1+\alpha^2)}$$

$$E = \frac{3}{8}\,\frac{\alpha^2\,\Omega\,P_c\,\ell}{1+\alpha^2}\ .$$

The solution of the system of equations (14) and (15) is

$$\begin{aligned} & n = 0\ ,\quad m = s = 1 \\ & H = 2 \\ & 2B - D = E/3 \\ & A = \tfrac{2}{3}E \end{aligned} \tag{17}$$

or

$$P_c = \frac{2\,m_H\,c}{\sigma_T\,\alpha^2}\,\frac{\xi}{1+\xi}\,\Omega \tag{18}$$

$$\rho_c = \frac{m_H\,c}{\sigma_T\,\alpha^2(1+\alpha^2)}\,\frac{\xi}{(1+\xi)^2}\,\frac{\Omega}{R\,T_c} \tag{19}$$

$$\ell = \frac{2}{\Omega}\left\{(1+\alpha^2)(1+\xi)\,R\,T_c\right\}^{1/2} \tag{20–a}$$

$$\ell_t = \frac{\alpha}{\sqrt{6(1+\alpha^2)}}\,\ell\ . \tag{20–b}$$

From equation (1) (assuming complete ionization $P_c = 2(1+\alpha^2)(1+\xi)\rho_c R\,T_c$), together with equations (10) and (17), we obtain

$$-31 + (109 - 51\alpha^2)\,\xi + 128\xi^2 = 51\,\alpha^2\ . \tag{21}$$

An interesting consequence of equation (21) is that solution for the disk equations only exists for $\xi > 0.225$, which corresponds to $\alpha \geq 0$.

It should be remarked from equation (20–b) that the scale length of the turbulence will only be comparable to the disk (semi) scale height for supersonic regimes.

Finally, from the angular momentum conservation equation

$$\rho\,\frac{V_r\,V_k\,r}{2} = -\frac{\partial}{\partial r}\left\{\frac{1}{3}\,\frac{\alpha^2}{1+\alpha^2}\,Pr^2\right\} \tag{22}$$

where V_r and V_k are respectively the radial and the keplerian velocities, we obtain after integration over z and r,

$$T_c = 2.6 \times 10^2 \frac{(1+\xi)(1+\alpha^2)}{\xi^2} \left[\frac{\dot{M}_{17}\, S}{M_{34}\, r^{3/2}} \right]^2 \tag{23}$$

where $S = 1-\delta\, r^{-1/2}$, with δ being the ratio angular momentum of the flow to the keplerian one at $r = 1$. To obtain equation (23) we have used the mass conservation equation

$$\dot{M} = -4\pi r \ell \int_0^1 \rho\, V_r\, dx \quad . \tag{24–a}$$

It should be said that a constant density over z doesn't contradict the null boundary condition at $x=1$. Indeed, if we set $\rho = \rho_c H(1-x)$, where H is the Heaviside function, we will obtain the same energy equation (15), because the extra term (after differentiating)

$$+ B(1-x^2)^m \left[\frac{\partial}{\partial x}(1-x^2)^s\right] \delta(1-x)$$

($\delta \equiv$ Dirac Delta function) is identically zero.

IV. The time dependent disk equations

In this section we shall obtain the behavior of the accretion disk against perturbations in the radial direction. We shall ignore perturbations in the azimuthal direction, i.e., we shall assume that even perturbed the φ component of the velocity in the disk is keplerian. The implication of this assumption is that the disk equations are formally the same, exception made for the continuity and energy equations.

In the following we shall briefly review some relations involving internal energy, pressure and turbulent Mach number. As we are taking into account turbulent pressure pulsations in the hydrostatic equilibrium equation together with allowance for the dependence of the turbulence along z, some modifications will be introduced in the energy equation compared to that usually appearing in the literature. For details we refer the reader to the works of Shakura and Sunyaev, 1976 and Piran 1978.

Integrating the continuity equation

$$\frac{\partial}{\partial t}\rho + \bar{\nabla}\cdot(\rho\,\bar{v}) = 0 \tag{24–b}$$

over z, with $\rho = 0$ at $z = \ell$, we obtain

$$\frac{\partial U}{\partial t} = \frac{1}{2\pi}\frac{\partial}{\partial r}\dot{M} \quad . \tag{25}$$

However, using equation (22) integrated over z, we may write

$$\frac{\partial U}{\partial t} = \frac{2}{9r}\frac{\partial}{\partial r}\frac{1}{\Omega r}\frac{\partial}{\partial r}\frac{\alpha^2}{1+\alpha^2}\Omega^2\, U\, \ell^2\, r^2 \tag{26}$$

where U, the surface density of matter, is given by

$$U = 2\ell \int_0^1 \rho \, dx \quad . \tag{27}$$

We now turn to the energy equation. Assuming uniformity of the expansion or contraction along z, we integrate the thermal equation,

$$\frac{dE}{dt} = \frac{P}{\rho^2}\frac{d\rho}{dt} + q^+ - q^- \tag{28}$$

where E is the internal energy per mass, q^+ and q^- represent respectively frictional heating and cooling, to obtain

$$\frac{\partial}{\partial t}\varepsilon\ell + P\frac{\partial\ell}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(\varepsilon + P)\,V_r\,\ell r + V_r\frac{\partial}{\partial r}\ell P + Q^+ - Q^- \quad . \tag{29}$$

For a turbulent gas composed of matter and radiation, the internal energy density is given by

$$\varepsilon = \frac{3}{2}P_g(1 + 2\xi) = \frac{3}{2}\left[\frac{1 + \beta + \beta\alpha^2}{1 + \alpha^2}\right]P \quad . \tag{30}$$

To obtain equation (30) we have used the equation of state

$$P = (1 + \xi)(1 + \alpha^2)P_g \tag{31}$$

and the relation

$$\xi = \frac{\beta\,(1 + \alpha^2)}{1 - \beta\,(1 + \alpha^2)} \quad . \tag{32}$$

Substitution of expression (30) into equation (29) using (17), yields

$$\frac{1}{4}\frac{\partial}{\partial t}\left[\frac{1 + \beta + \beta\alpha^2}{1 + \alpha^2}\right]U\,\Omega^2\,\ell^2 + \frac{1}{6}U\,\ell\,\Omega^2\frac{\partial\ell}{\partial t} =$$

$$= \frac{2}{9}\frac{\alpha^2}{(1+\alpha^2)^2}\frac{\partial}{\partial r}\left\{\frac{5 + 3\beta + \alpha^2(3\beta + 2)}{12}\right\}\Omega\,\ell^2\frac{\partial}{\partial r}U\,\ell^2\,\Omega^2 +$$

$$+ \frac{V_r}{6}\frac{\partial}{\partial r}U\,\ell^2\,\Omega^2 + \frac{\alpha^2}{4(1+\alpha^2)}U\,\ell^2\,\Omega^3 - \frac{cm_p}{\sigma_T}\beta\,\ell\,\Omega^2\,(1+q) \quad . \tag{33}$$

Compared to the other terms, the one proportional to V_r is of order superior on ℓ/r and will be neglected.

To study the temporal behavior of U and ℓ, we write

$$U = U_0\,(1 + u)$$
$$\ell = \ell_0\,(1 + h) \tag{34}$$

where the subscript $_0$ stands for unperturbed values.

To linearize equation (26) and (33) we shall assume that the intensity of the perturbations is small compared to the unperturbed values. Besides, we shall also assume that the perturbations vary much faster than the unperturbed values, i.e., we shall neglect variations of the unperturbed variables. Therefore, the equation of continuity (26) becomes

$$\frac{\partial}{\partial t} u = \frac{2}{9} \frac{\alpha^2}{1+\alpha^2} \Omega \ell_0^2 \frac{\partial^2}{\partial r^2}(u + 2h) \quad . \tag{35}$$

Writing

$$\begin{aligned} T &= T_0 + T_1 \\ \beta &= \beta_0 + \beta_1 \\ q &= q_0 + q_1 \end{aligned} \tag{36}$$

we obtain from equations (10),(11),(31),(32) and

$$\beta P = a T^4 \tag{37}$$

how T_1, β_1 and q_1 depend on h and u, i.e.,

$$\frac{\beta_1}{\beta_0} = (7h - u) \left[\frac{1 - \beta_0 f}{1 + 3 \beta_0 f} \right] \tag{38}$$

$$\frac{T_1}{T_0} = \frac{1}{4(1+3\beta_0 f)} \left\{ h(8-4\beta_0 f) + 4\beta_0 f u \right\} \tag{39}$$

$$\frac{q_1}{q_0} = \frac{19}{12} u - \frac{25}{12} h + \left\{ \frac{35 + 1295\beta_0 f + 441 \beta_0^2 f^2 + 21 \beta_0^3 f^3}{12(5 + 25\beta_0 f - 33\beta_0^2 f^2 + 3\beta_0^3 f^3)} \right\} \frac{\beta_0}{\beta_1} \quad , \tag{40}$$

where f stands for $1 + \alpha^2$.

We now insert equations (36),(38),(39) and (40) into equation (33) to obtain the linearized energy equation

$$S_0 \frac{\partial h}{\partial t} + S_1 \frac{\partial u}{\partial t} = (S_2 h + S_3 u) \Omega \tag{41}$$

with

$$S_0 = 16 \left[5 + 25 \beta_0 f - 33\beta_0^2 f^2 + 3\beta_0^3 f^3 \right] \left[6 + 2f + (45 + 6f)\beta_0 f - 3\beta_0^2 f^2 \right] \tag{42}$$

$$S_1 = 16 \left[5 + 25 \beta_0 f - 33 \beta_0^2 f^2 + 3 \beta_0^3 f^3 \right] \left[-33 - 24 f - (135 + 72 f)\beta_0 f - 3 \beta_0^2 f^2 \right] \tag{43}$$

$$S_2 = (f-1) \left[85 - 6140 \beta_0 f + 3880 \beta_0^2 f^2 + 1176 \beta_0^3 f^3 + 867 \beta_0^4 f^4 \right] \tag{44}$$

$$S_3 = (f-1) \left[420 + 3380 \beta_0 f - 4856 \beta_0^2 f^2 - 1476 \beta_0^3 f^3 + 96 \beta_0^4 f^4 \right] \quad . \tag{45}$$

In the S coefficients we have used $q_0 = 1/3$, calculated using equations (15), (16) and (17). We now search for equations (35) and (41) a solution of the kind $u = e^{\omega t} u(R)$ and $h = e^{\omega t} h(r)$. Making the substitution $\psi = u + 2h = e^{\omega t} \psi(r)$, we obtain

$$9(S_0 \omega^2 - S_2 \Omega \omega)\psi = \left\{ 2(S_0 - 2S_1)\omega + 2(2S_3 - S_2)\Omega \right\} \frac{\alpha^2}{f} \Omega \ell_0^2 \frac{\partial^2}{\partial r^2} \psi \quad . \tag{46}$$

V. Perturbation types and instability

As we are assuming λ, the length scale of the perturbations, very small compared to the radial distance, coefficients in equation (46) may be treated as constants and setting $\psi \simeq e^{i r/\lambda}$, we obtain

$$\left[\frac{\omega}{\Omega}\right]^2 - \left[\frac{\omega}{\Omega}\right] \left\{ \frac{S_2}{S_0} + \frac{2}{9}\left[2\frac{S_1}{S_0} - 1\right] \left[\frac{f-1}{f}\right] \left[\frac{\ell_0}{\lambda}\right]^2 \right\} + \frac{2}{9\, S_0}(2S_3 - S_2) \left[\frac{f-1}{f}\right] \left[\frac{\ell_0}{\lambda}\right]^2 = 0 \quad (47)$$

The S coefficients satisfy

$$\begin{array}{lll} S_0 > 0 & , & \text{always} \\ -S_1/S_0 > 0 & , & \text{always} \\ S_2 > 0 & , & \beta_0 f \leq 0.014 \\ 2S_3 - S_2 > 0 & , & \beta_0 f \leq 0.795 \ . \end{array} \quad (48)$$

As we don't take into account shock waves, our analysis is valid only for $f \leq 2$. Therefore, for $\beta_0 f < 0.795$, the perturbations take the form of damped concentric waves along the disk. The behavior of ω, in the limits $\lambda \to 0$ and $\lambda \to \infty$ is respectively

$$\frac{\omega_\pm}{\alpha^2\,\Omega} = \begin{cases} -\dfrac{2(2S_3-S_2)}{9S_0 f} \\[2ex] -\dfrac{2(S_0-2S_1)}{9S_0 f}\left[\dfrac{\ell_0}{\lambda}\right]^2 \end{cases} \quad (49)$$

$$\frac{\omega_\pm}{\alpha^2\,\Omega} = \begin{cases} 0 \\[2ex] \dfrac{S_2}{S_0} \end{cases} \quad (50)$$

For $\beta_0 f > 0.795$, only the ω_- mode has a similar behaviour. For the ω_+ mode, the intensity of the perturbations grows with time. This instability corresponds to the secular instability discovered by Lightman and Eardley (1974). As discussed by Lightman and Eardley (1974), in that case the time evolution of the disk is governed by a nonlinear diffusion equation with a negative effective diffusion coefficient. Therefore, matter in the disk will clump rather than smooth, and high density zones get higher in density, low density zones get lower.

We now inquire ourselves if these instabilities are attainable in the disk. For that we set $f = 2$, i.e., $\alpha = 1$, in equation (21) to obtain

$$-82 + 58\xi + 128\xi^2 = 0 \quad (51)$$

which gives $\xi = 0.6$ or $\beta \approx 0.375$. For $\beta_0 f = 0.795$ we get $\alpha \simeq 3.1$.

Therefore, the disk will be unstable only if turbulence is supersonic.

VI. Conclusions

Allowance for dependence of the turbulence along z results in a turbulent scale length ℓ_t comparable to the (semi) disk height scale only for supersonic regimes, contrarily to Shakura, Sunayev and Zilitinkevich (1978) (hereafter SSZ) which obtain equality between these scales for sonic regimes. This implies that the degrees of freedom associated to the turbulence will increase by a factor of about $(\ell/\ell_t)^3$. Therefore, turbulence will be highly developed in the disk, as compared to SSZ's results.

Another point that should be mentioned concerns temperature gradients which essentially go like $\sim 1/\beta$ and they are much greater in our case.

It should also be mentioned that the value for the ratio convective to radiative energy transport (q_0) we have obtained ($q_0 = 1/3$) is equal to SSZ's value. A non–constant value for the turbulent conductivity that goes to zero at the surface of the disk, where the flow is laminar, implies a lesser value for the convective energy transport. It should be stressed that, owing to the dependence of the radiative flux on x (proportional to x) the polytropic solution we have obtained is unique. This also should happen to SSZ's treatment.

Although with equal convective energy transport, our disk is thicker and hotter than theirs by a factor of, respectively, $8 \times \frac{10^{-2} f^2 (1+\xi)}{\xi^2}$ and $\frac{14^4 (1+\xi) f (M_{17}\ S)^2}{\xi^2}$.

It should be kept in mind, however, that there is solution for the convective disk equations only for $\beta_0 \geq 0.184$. It also yields for the ratio of convective and radiative energies, q_0, a value independent of the Mach number.

This, together with the inclusion of turbulent pressure pulsations in the equation of state for the pressure makes the ratio of radiative and gas pressures highly dependent on the turbulent Mach number. As a consequence, the flow in that region, where electron

scattering is the main source of opacity, will only be radiative pressure dominated for supersonic regimes.

Concerning stability we should distinguish between the effects of convection and turbulent pressure pulsations separately.

Compared to the convection free model, convection will stabilize the flow against perturbations in the radial direction. For negligible Mach number, instability sets in the disk for $\beta_0 \approx 0.795$ and $\beta_0 = 0.6$ for the α standard α accretion disk model. Turbulent pressure pulsations affect stability in the reverse way, the greater the turbulent Mach number the less the value of β_0 for the onset of instability. It should be remarked, however, that the secular instability will only be present in the disk if the flow is supersonic. In that case shock instability will occur long before.

With a few modifications (specially $\ell = \ell_t$) the results we have obtained do apply to the α standard accretion disk model.

Finally, we conclude that fluctuations between optically thick and optically thin states followed by changes in the spectrum as a model for galactic and extragalactic X–rays sources will only work for viscosity laws quite different from ours and the standard one.

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