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ABSTRACT

With the allowance for a density dependent Coulomb logarithm the solution, for the two temperature soft photon comptonized disc with pairs becomes double–valued. Keeping the electronic temperature constant and varying α, we obtain a critical electronic temperature, below which pair equilibrium is possible. For temperature greater than the critical value, even allowance for pairs to escape the disc with the velocity of light cannot restore equilibrium. Treating α as a free parameter, allowing for temperature variation along the disc, we obtain a critical accretion rate, above which pair equilibrium is no longer possible. Below, the solution keeps its double–valueness. The effects of advection are seen to be negligible. It is also shown that the assumptions of ionic temperature lesser than virial temperature and thin disc are compatible only for supersonic turbulent regimes.

I. Introduction

Recently, there has been a renewed interest in accretion discs, owing to their possibility as a scenario for pair production. Though the first work on relativistic thermal plasmas dates from the beginning of the seventies (Bisnovatyi–Kogan, Zel'dovich and Sunyaev, 1971), it was only after the pioneering papers of Stoeger (1977), Pozdnyakov, Sobol and Sunyaev (1977), and Liang (1979) that astrophysicists became aware of this fact. However, both Stoeger's and Liang's works focuse a small region of parameter space, for which the pairs may outnumber the ionizing electrons. Recent works (White and Lightman, 1989; Tritz and Tsuruta; Takahara and Kusunose, 1985) based on up–to–date microphysics of pair production (Lightman and Band, 1981; Lightman, 1982; Svensson, 1982) give much more accurate results. However, the available results are only numerical. Our paper differs from others in the literature in the sense that we obtain analytic solutions for the ratio pairs–ions much greater and less than 1. Besides, we treat the turbulent Mach number both as variable along the disc in order to have a consist power law solution for the spectrum of the source, as a free parameter. Considering only two–photon pair production, we don't resort to the usual approximation of constant Coulomb logarithm and use, instead, a density dependent one. We solve the disc equations subject to null inflow rate at $r = r_0$, the outer edge of the two–temperature region. These equations are solved in the case n_+(number of positrons) $\gg N_i$(number of protons) and in the case $n_+ \ll N_i$, for the regime pair production–annihilation plus wind dominance as well as for the regime pair production balanced by inflow rate. We then show that the general solution, a multivalued one, encompasses asymptotically both solutions we have obtained. We obtain the consistency of our solution in parameter space. One of the main results of our work is to show that pair equilibrium in discs cooled by externally produecd photons only holds below a maximum electronic temperature, when α varies, or a maximum accretion rate when α is kept constant and T_e varies. These results are pratically independent of the inflow rate.

II. The disc equations

Besides the standard assumptions from the theory of the accretion discs (Shakura and Sunyaev, 1973; Novikov and Thorne, 1973), i.e.,

1. the azimuthal velocity v_k is keplerian,
2. thin disc, i.e., ℓ (disc semi–scale height) $\ll$ r (radial distance to the central compact object),
3. the stress tensor, essentially given by $t_{r\varphi}$, is related to the pressure through $t_{r\varphi} = \alpha P$ where α is the turbulent Mach number,
4. every kind of particle in the disc has thermal distribution,
5. energy is tranferred only in the z direction, normal to the symmetry plan of the disc, and the specific assumptions from the " two–temperature accretion disc" (Shapiro et al., 1976), i.e.,
6. protons and electrons (pairs) out of thermal equilibrium,
7. pressure given by the protons,
8. the cooling of the gas is due to unsaturated inverse comptonization of soft photons, from a very copious external source. This is equivalent to set the Kompaneetz y parameter equal to 1, throughout the inner region,
9. ions and pairs are coupled through collisional energy exchange only,
10. turbulence is sub–sonic,
11. the inner region is the result from an instability developed in the outer cool disc.

We shall also assume:

12. the variation of the turbulent Mach number along the disc in such a way as to keep the electronic temperature constant in the inner region,
13. inflow and formation of a pair wind.

Assumption (12) is intended to describe systems endowed with power law spectra, the spectral index n in the range 0,3–1. A small variation of the electronic temperature may cast n completely outside this range. Assumption (13) takes into account some non–thermal processes that may acelerate the pairs making them able to escape the disc through a wind. Besides, pairs may be drifted by the flow (inflow).

Throughout this article we shall adopt the following set of units

a) radial distance r in units of the inner radius $R_i = \frac{GM}{C^2}$,

b) T_i, T_e respectively ionic and electronic temperatures in units of 10^9 °K,

c) M_{34}, the mass of the central object, in units of 10^{34}g,

d) $\dot{M}_{17}$, the accretion rate in units of 10^{17}gs^{-1},

e) unless otherwise stated, any physical variable will be expressed in the C.G.S. system,

f) particle number densities will be expressed in units of 10^{16}cm^{-3}.

The hydrostatic equilibrium equation together with the thin disc assumption read for the pressure

$$P = \frac{\rho\, \Omega^2\, \ell^2}{3} \ , \qquad (1)$$

ρ is the density and Ω is the angular keplerian velocity.

The semi–scale height of the disc follows straightforward from equation (1) and assumption (7)

$$\ell = 1.22 \times 10^4\, M_{34}\, r^{3/2}\, T_i^{1/2} \ . \qquad (2)$$

The scattering optical depth, with the inclusion of pairs, becomes

$$\tau = N_i \sigma_T \ell \left[1 + 2 \frac{n_+}{N_i} \right] \quad , \tag{3}$$

where N_i, n_+ are respectively the proton and the pairs number densities and σ_T is the Thomson cross section for electron scattering. Using assumption (8), we obtain

$$I = \frac{4 \times 10^9 k T_e}{m_e c^2} \left[1 + \frac{4 \times 10^9 k T_e}{m_e c^2} \right] \left[\tau + \tau^2 \right]$$

$$\simeq \frac{4 \times 10^9 k T_e}{m_e c^2} \tau \quad , \tag{4}$$

with k,m_e,c respectively Boltzmann constant, electronic mass and velocity of light.

The effect of the wind is to make the accretion rate variable along the disc. However, under the assumption of unsaturated inverse comptonization, it can be verified that this variation is negligible, and we may treat the accretion rate $\dot{M}$ as a constant and use for the heat generation in the disc, the well known expression from the accretion disc theory,

$$Q^+ = \frac{3}{8\pi} \dot{M} \Omega^2 S$$

$$= 1.94 \times 10^{25} \frac{\dot{M}_{17} S}{M_{34}^2 r^3} \quad , \tag{7}$$

with $S = 1 - \delta r^{-1/2}$, δ being the ratio actual angular momentum of the flow to the keplerian one at $r = 1$.

From the angular momentum conservation equation together with the definition of the stress tensor $t_{r\varphi}$, we get for the density

$$\rho = \frac{3}{4\pi} \frac{\dot{M} S}{\alpha \ell^3 \Omega}$$

$$= 0.33 \frac{\dot{M}_{17} S}{\alpha M_{34}^2 r^3 T_i^{3/2}} \text{ g cm}^{-3} \quad . \tag{8}$$

The collisional energy exchange term will be given by (Spitzer, 1962)

$$F_{ee\pm} = 2.67 \times 10^{-23} N_i (N_i + 2n_+) \frac{\ell T_i \ell n \Lambda}{T_e^{3/2}} \quad , \tag{9}$$

where $\ell n \Lambda$ is the Coulomb logarithm.

Below we shall give the solutions of the system of equations (1) to (9). We shall consider the cases $n_+ \ll N_i$ and $n_+ \gg N_i$. For the former the solutions are given as a function of r,$\ell n \Lambda$ and T_e, for the latter, as a function of r,$\ell n \Lambda$,α and T_e.

a) $N_i \gg n_e$

$$T_i = 4 \times 10^6 \left[\frac{\dot{M}_{17} S}{M_{34} r \ell n \Lambda} \right]^2 T_e^7 \Delta^4 \tag{10}$$

$$\ell = 1.4 \times 10^7 \frac{r^{1/2} \dot{M}_{17} S}{\ell n \Lambda} T_e^{7/2} \Delta^2 \text{ cm} \tag{11}$$

$$\alpha = 1.5 \times 10^{-3} \frac{M_{34}\, r^{3/2}}{T_e^6\, \dot{M}_{17}\, S\, \Delta^3} (\ln \Lambda)^2 \tag{12}$$

$$N_i = \frac{16.4 \ln \Lambda}{r^{3/2}\, \dot{M}_{17}\, S\, T_e^{9/2}\, \Delta^3} \tag{13}$$

$$P = 2.92 \times 10^{16} \frac{\dot{M}_{17}\, S\, \Delta}{M_{34}^2\, r^{7/2} \ln \Lambda} T_e^{5/2} \text{ dyn cm}^{-2} \tag{14}$$

where $\Delta = 1+0.67\, T_e$ takes into account relativistic effects

b) $N_i \ll n_+$

$$T_i = \frac{0.27\, (\ln \Lambda)^2}{\alpha^2\, T_e^5\, (1 + 0.67\, T_e)} \tag{15}$$

$$\ell = \frac{6.36 \times 10^3\, M_{34}\, r^{3/2} \ln \Lambda}{\alpha\, T_e^{5/2}(1 + 0.67\, T_e)} \text{ cm} \tag{16}$$

$$n_+ = \frac{1.47 \times 10^4\, \alpha\, T_e^{3/2}}{M_{34}\, r^{3/2} \ln \Lambda} \tag{17}$$

$$N_i = 1.35 \times 10^8 \frac{\dot{M}_{17}\, S\, \alpha^2\, T_e^{15/2} (1 + 0.67\, T_e)^3}{M_{34}^2\, r^3 (\ln \Lambda)^3} \tag{18}$$

$$P = 5.03 \times \frac{10^{16}\, \dot{M}_{17}\, S}{M_{34}^2\, r^3 \ln \Lambda} T_e^{5/2} (1 + 0.67\, T_e) \text{ dyn cm}^{-2} \tag{19}$$

III. The solution of the balance equation for the pairs

From the definition of the Debye number, for electronic temperature $T_e \geq 1$ (Golant, Zhilinsky and Sakharov, 1980)

$$\Lambda = 2.5 \times 10^{15} \frac{T_e}{(N_i + n_+)^{1/2}} \tag{20}$$

we obtain for $N_i \gg n_+$

$$e^{2\ln\Lambda} = 3.8 \times 10^{13}\, \dot{M}_{17}\, S\, r^{3/2}\, T_e^{13/2} \tag{21}$$

with the approximate solution

$$\ln \Lambda \simeq 14.25 + \frac{1}{2} \ln T_e^{13/2}\, r^{3/2}\, \dot{M}_{17}\, S \tag{22}$$

and for $N_i \ll n_+$

$$e^{2\ln\Lambda} = 4.25 \times 10^{10} \frac{M_{34}\, r^{3/2}\, T_e^{1/2}}{\alpha} \ln \Lambda \tag{23}$$

We now shall stress the following assumptions we have made on deriving the balance equation for the pairs:

1. for the creation of pairs, we shall only consider the following reactions

   a) $\gamma + \gamma \rightarrow e^+ + e^-$,

2. the cross section for this reaction will be in the ultrarelativistic limit given by (Jauch and Rohrlich, 1976)

$$\sigma = \frac{3}{8}\sigma_T \frac{(m_e c^2)^2}{h^2 \nu_1 \nu_2} \ell n \left[\frac{4 h^2 \nu_1 \nu_2}{(m_e c^2)^2}\right] \tag{24}$$

where h is the Planck constant, ν_1 and ν_2 are the frequencies of the interacting photons.

The balance equation for pairs in the stationary case, may be written (Lightman, 1982; Svensson, 1982; Zdziarski, 1985)

$$\frac{1.34 \times 10^{-6}}{M_{34}\ r} \frac{\partial}{\partial r}\left\{r\, n_+ V_r\right\} = c<\sigma>n_\gamma^2 - \frac{3}{16} c\, \sigma_T T_*^{-2} n_- n_+ g - \beta \frac{c}{\ell} n_+ \tag{25}$$

with

$$T_* = \frac{10^9\ k}{m_e c^2} T_e$$

$$g = \left[\frac{1}{2} + \frac{T_*^2}{\ell n(T_* + 1.3)}\right]^{-1}$$

$$n_- = N_i + n_+ \tag{26}$$

$$V_r = 1.34 \times 10^{-6}\, \alpha \left(\frac{\ell}{r}\right)^2 r\, \Omega\, S^{-1}\ .$$

β is the wind velocity and $<\sigma>$ is the cross section (24) averaged over the interacting photon distributions.

In the average, considering only contributions from the leading terms in the exponential integrals, we obtain,

$$\frac{\partial}{\partial r}\left\{\Delta\, r\, \dot{M}_{17} T_e n_+\right\} = M_{34}^2 \left\{\frac{0.4\ g}{r^{1/2} T_e^{9/2}\ \Delta^3} + \frac{1.46\ \beta\ r^{1/2}}{\Delta^2\ T_e^{7/2}}\right\} \frac{n_+}{\dot{M}_{17}\ S}$$

$$- 6.67 \times 10^4 \frac{(\dot{M}_{17}S)^2}{M_{34}^2\ r^5} \tag{27}$$

for $N_i \gg n_+$ and

$$\frac{\partial}{\partial r}\left[T_e \Delta \dot{M}_{17} \frac{n_+^2}{N_i}\right] = \left\{4.8 \times 10^{-2} \frac{g}{T_e^2} + 10^{-2} \beta\, T_e \Delta\right\} M_{34}^2\, r\, n_+^2 - 5.26 \times 10^4 \frac{(\dot{M}_{17}S)^2}{M_{34}^2\ r^5}\ , \tag{28}$$

for $N_i \ll n_+$. It is interesting to remark that the balance equation for pairs reduces to a linear differential equation for $n_+(N_i \gg n_+)$ and a non linear differential equation for $\frac{n_+^2}{N_i}(N_i \ll n_+)$.

For $N_i \gg n_+$, the wind term is negligible as compared to the annihilation one and an approximate solution of equation (27) may be given by

$$n_+ \simeq (\delta_1 + \delta_2)\, N_i \tag{29}$$

where N_i is the ion number density, equation (13), and

$$\delta_1 = 20\,\frac{(\dot{M}_{17}\;S)^4\,T_e^9\,\Delta^6}{M_{34}^4\,r^3\,g}$$

$$\delta_2 = + \left[\frac{\partial}{\partial r} \ln (r^{1/2} S) + \frac{0.4\;g\;M_{34}^2}{T_e^{11/2}\,\Delta^4\,\dot{M}_{17}^2\,S\,r^{3/2}} \right]^{-1} \tag{30}$$

$$\left[\frac{\partial}{\partial r}\delta_1 - \delta_1 \frac{\partial}{\partial r} \ln (r^{1/2} S) + C \right]$$

C being an integration constant.

For $N_i \ll n_+$, the situation is more complex. However, approximate solutions may be obtained depending on the relative strength of the different rates involved in equation (28). If the annihilation plus wind rate is the dominant one, we will have

$$n_+ = \frac{1.1 \times 10^3\;\dot{M}_{17}\;S\;T_e}{M_{34}^2\,r^3(4.8\;g + \beta\;T_e^3\;\Delta)^{1/2}} \tag{31}$$

$$N_i \simeq \frac{5.5 \times 10^4(\dot{M}_{17}\;S)^3\;T_e^{13/2}}{M_{34}^4\;r^6(4.8\;g + \beta\,T_e^3\;\Delta)} \tag{32}$$

and for a inflow rate dominated regime,

$$n_+ = 6.25 \times 10^{14}\,T_e^2\,e^{-2T_e^{7/2}S\phi} \tag{33}$$

and

$$N_i = 2.4 \times 10^{29}\,\frac{\dot{M}_{17}\;T_e^5\;\Delta}{\phi}\,e^{-4T_e^{7/2}S\phi} \tag{34}$$

where ϕ is given by

$$\phi = C + 8.22 \times 10^3\,\frac{\dot{M}_{17}^2}{M_{34}^2}\left[\frac{1}{r^4} - \frac{1.6}{r^{9/2}} + \frac{0.65}{\gamma^5}\right] \tag{35}$$

and C is another integration constant.

To the solutions we have obtained we shall impose that the value of α at $r = r_0$, the external radius of the inner region is given by the condition of instability (Shakura and Sunyaev, 1976; Shapiro et al., 1976; Meirelles and Marques, 1989) i.e.,

$$3\,P_g = 2\,P_r \tag{36}$$

which yields

$$\alpha_0 = 8 \times 10^{-20}\,\frac{M_{34}^7\,r_0^{21/2}}{(\dot{M}_{17}\;S_0)^8} \tag{37}$$

where P_g and P_r are respectively the gas and radiation pressures in the external region. The subscript **0** stands for values calculated at $r = r_0$. We shall also impose null inflow rate at $r = r_0$. It should be said that equation (37), for given turbulent Mach number, mass of the central object and accretion rate determines the point (r_0) where the instability occurs. However, as we have assumed α variable in the inner region, its value at r_0 should be consistent with equation (37). Or, equating the expression of $\alpha = \alpha\,(r, T_e, M_{39}, \dot{M}_{17})$ for the inner region with α given by eq.(37) we obtain $r_0 = r_0\,(T_e, M_{39}, \dot{M}_{17})$. Such a procedure ensures $r_0 > 1$, i.e., the existence of the inner region.

To calculate C in equation (33) one needs the value of α at $r \neq r_0$.

IV. The general solution for the production of pairs in the unsaturated comptonization two temperature disc model: the maximum electronic temperature for equilibrium

We have considered up to now the solutions $N_i \gg n_+$ and $n_+ \gg N_i$, separately. We shall show now that the abandon of the assumption of constant Coulomb logarithm makes the general solution for pair production a double–valued one, that depending on a special relation among temperature, mass of the central compact object, accretion rate and radial distance may yield both solutions we have obtained. We shall also show the existence of a maximum electronic temperature, above which equilibrium pair production–annihilation no longer holds.

If we define a new variable f as

$$f = 1 + 2\frac{n_+}{N_i} \tag{38}$$

the physical variables in the inner region will change to

$$T_i = 4 \times 10^6 \left[\frac{\dot{M}_{17}^2}{M_{34}\ r\ \ln\ \Lambda}\right]^2 T_e^7\, \Delta^4\, f^2 \tag{39}$$

$$\ell = 1.4 \times 10^7 \frac{r^{1/2}\ \dot{M}_{17}\ S}{\ln\ \Lambda} T_e^{7/2}\, \Delta^2\, f \tag{40}$$

$$N_i = 16.4 \frac{\ln\ \Lambda}{r^{3/2}\ \dot{M}_{17}\ S\ T_e^{9/2}\ \Delta^3\ f^2} \tag{41}$$

$$\alpha = 1.5 \times 10^{-3} \frac{M_{34}\ \ r^{3/2}(\ell n\ \Lambda)^2}{T_e^6\ \dot{M}_{17}\ S\, \Delta^3\ \ f^2}\ . \tag{42}$$

From equation (41) we express $\ln \Lambda$ as a function of N_i and f. Substitution in equation (20) yields

$$e^{0.12\, r^{3/2}\, \dot{M}_{17}\, S\, T_e^{9/2}\, \Delta^3\, N_i\, f^2} = 1.5 \times 10^{15} \frac{T_e^2}{N_i(1+f)}\ . \tag{43}$$

The pair balance equation becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{ r\ \dot{M}_{17}\, T_e\, \Delta\ f(f-1)N_i \right\} = \frac{2.9\ \times\ 10^{-2}}{T_e^2}\, g\ M_{34}^2(f^2-1)N_i^2$$

$$+\ 6 \times 10^{-3}\, M_{34}^2\, T_e\, f(f-1)N_i^2\, \beta - 6.35 \times 10^4 \frac{(\dot{M}_{17}\ \ S)^2}{M_{34}^2\ \ r^6}\ . \tag{44}$$

Equation (43) has an approximate solution given by

$$N_i \approx \frac{2.44\ \times\ 10^2}{\dot{M}_{17}\ S\ r^{3/2}\ T_e^{9/2}\ \Delta^3\ f^2}\ . \tag{45}$$

At $r = r_0$, the external radius of the inner region, the inflow rate vanishes, therefore, using equation (45) we obtain

$$\frac{1.73 \times 10^3}{(\dot{M}_{17}\ S)^2}\ \frac{M_{34}^2\, g\,(f^2-1)}{T_e^{11}\ \Delta^6\ f^4} + \frac{35.65\ \ M_{34}^2\ \ f(f-1)\beta}{(\dot{M}_{17}\ \ S)^2\ T_e^8\, \Delta^6\ f^4} = 1.27 \times 10^5 \frac{(\dot{M}_{17}\ \ S)^2}{M_{35}^2\ \ r^3}\ . \tag{46}$$

For systems with negligible wind

$$f^2 = \frac{a_1 \pm \sqrt{a_1^2 - 4a_1}}{2}$$

$$a_1 = \frac{1.36 \times 10^{-2} \, M_{34}^4 \, r^3 \, g}{(\dot{M}_{17} S)^4 \, T_e^{11} \, \Delta^6} . \qquad (47)$$

From eq.(47) we see that equilibrium only occurs if

$$T_e^{11}(1 + 0.67 \, T_e)^6 \leq 6.8 \times 10^{-3} \frac{M_{34}^4 \, r^3 \, g}{(\dot{M}_{17} S)^4} \qquad (48)$$

which defines the maximum electronic temperature, in the absence of wind, for equilibrium pair production–annihilation. For T_e lesser than the solution of equation (48), the solution has two branches: the high and the low f.

It is interesting to remark that the multivalueness of the solution persists even for systems with a strong wind. In that case, neglecting annihilation, we obtain

$$a^2 f(f-1) = f^4$$

$$a^2 = \frac{5.6 \times 10^{-4} \, \beta \, M_{34}^4 \, r^3}{(\dot{M}_{17} S)^4 \, T_e^8 \, \Delta^6} . \qquad (49)$$

Equation (49) yields a maximum electronic temperature given by the solution of

$$T_e^8 \, \Delta^6 = 8.3 \times 10^{-5} \frac{\beta \, M_{34}^4 \, r^3}{(\dot{M}_{17} S)^4} . \qquad (50)$$

For $a_1 \gg 4$, we obtain the solutions $f \gg 1$ and $f \simeq 1$ which we have already studied, i.e.,

$$f \simeq \begin{cases} 0.12 \dfrac{M_{34}^2 \, r^{3/2} \, g^{1/2}}{(M_{17} \, S)^2 \, T_e^{11/2} \, \Delta^3} \gg 1 \\[2ex] 1 + \dfrac{20 \, (\dot{M}_{17} \, S)^4 \, T_e^{11} \, \Delta^6}{M_{39}^4 \, r^3 \, g} \sim 1 \end{cases} . \qquad (51)$$

Besides this, we see from eq.(45) that in the upper branch $(n_+ > N_i)$ $n_+ < \dfrac{244}{\dot{M}_{17} \, S \, T_e^{9/2} \, \Delta^3}$, and in the lower branch $(n_+ < N_i)$ with $n_+ > \dfrac{244}{\dot{M}_{17} \, S \, T_e^{9/2} \, \Delta^3}$.

It should be stressed that from equations (46) to (51) we have used r instead of r_0 to do not burden notation.

V. Consistency of the model

In a previous paper we have analysed for the pair–free model, the constraints imposed on the turbulent Mach number, accretion rate and mass of the compact object in order to have a consistent solution for the two–temperature disc model.

Analysing the assumptions ionic pressure dominance, thin disc, ionic temperature lesser than the virial temperature, unsaturated inverse comptonization and subsonic turbulence, we have concluded that consistency is met whenever the following inequality holds;

$$7.55 \times 10^3 \frac{\alpha \dot{M}_{17} S}{M_{34} r^{3/2}} \leq \ell n \Lambda \leq 36 \alpha^7 \left[\frac{M_{34}}{\dot{M}_{17} S} \right]^5 r^{3/2} \quad . \tag{52}$$

However, it has been shown (Meirelles, 1990) that even for the pair free model one obtains a multivalued solution: one ionic pressure dominated and the other a radiation pressure dominated gas.

Therefore, the consistency condition expressed by inequality (52) holds only in the upper branch of the pair free model solution.

As we have imposed a null inflow rate boundary condition, we shall start our analysis by finding out under which conditions this will occur.

If we had not the condition at $r = r_0$, $\alpha = \alpha_0$, equation (37) we would have simply a maximum electronic temperature expressed by inequality (48) in the case of negligible wind and equation (50) in the case of a dominant wind. However, we have to couple equation (37) to equation (42), which yields, at $r = r_0$, assuming that most of the luminosity comes from the region $r < r_0$,

$$r_0 = 16.85 \frac{\mathcal{L}_{37}^{16/21} \alpha_0^{2/21}}{M_{34}^{2/3}}$$

$$f = 9.1 \times 10^{-3} \frac{M_{34}^{59/168} \alpha_0^{158/147}}{\mathcal{L}_{37}^{59/147} T_e^{27/28} \Delta} q \tag{53}$$

$$\dot{M}_{17} S_0 = 0.17 \mathcal{L}_{37} \quad .$$

Where $\mathcal{L}_{37}$ is the luminosity in units of 10^{37} erg s^{-1} and q is the ratio ionic to electronic temperature at $r = r_0$.

Using equation (53) we obtain in the absence of wind

$$T_e^M \leq 2.15 \left[\frac{M_{34}^2 \alpha_0^{2/7}}{\mathcal{L}_{37}^{12/7}} \right]^{1/17} , \tag{54}$$

and for systems with a dominant wind $(T_e > 3.4 \beta^{-0.2})$,

$$T_e^M \leq 1.9 \left[\beta^{1/2} \frac{M_{34} \alpha_0^{1/7}}{\mathcal{L}_{37}^{6/7}} \right]^{1/7} . \tag{55}$$

From equations (54) and (55) we see that the wind will never be strong enough to dominate annihilation at $r = r_0$, with null inflow rate.

From now on we shall consider only systems with

$$T_e < 3.4 \beta^{-0.2} \quad .$$

We then obtain at $r = r_0$,

$$f \pm \begin{cases} 10^3 \dfrac{M_{34} \alpha_0^{1/7}}{\mathcal{L}_{37}^{6/7} T_e^{17/2}} \\ \\ 1 + 5.6 \times 10^{-7} \dfrac{\mathcal{L}_{37}^{12/7} T_e^{17}}{M_{34}^2 \alpha_0^{2/7}} \end{cases} . \tag{56}$$

Therefore, the ratio of the ionic and electronic temperature is given by

$$q_{\pm} = \begin{cases} \dfrac{1.1 \times 10^{5}\, M_{34}^{109/168}}{\mathcal{L}_{37}^{85/168}\, T_e^{211/28}\, \alpha_0^{137/147}} \\[2ex] 1.1 \times 10^{2}\, \dfrac{\mathcal{L}_{37}^{59/147}\, T_e^{27/28}\, \Delta}{M_{34}^{59/168}\, \alpha_0^{158/147}} \end{cases} \tag{57}$$

Using equations (53), (56) and (57) we may calculate the ratios radiation pressure to ionic pressure, ionic temperature to virial temperature, and disc scale length to radial distance, given respectively by

$$\left[\frac{P_r}{P_g}\right]_{\pm} = \begin{cases} 1.6\, \dfrac{\alpha_0^{157/294}\, M_{34}^{109/336}}{T_e^{294/56}\, \mathcal{L}_{37}^{85/336}} \\[2ex] 4.2 \times 10^{-2}\, \dfrac{\mathcal{L}_{37}^{59/294}\, \alpha_0^{68/147}}{T_e^{29/56}\, M_{34}^{59/336}} \end{cases} \tag{58}$$

$$\left[\frac{T_i}{T_v}\right]_{\pm} = \begin{cases} 1.67 \times 10^{2} \left[M_{34}^{1/56}\, T_e^{183/28}\, \alpha_0^{41/49}\, \mathcal{L}_{37}^{43/168} \right]^{-1} \\[2ex] 0.11\, \dfrac{T_e^{83/28}\, \mathcal{L}_{37}^{57/49}}{M_{34}^{57/56}\, \alpha_0^{48/49}} \end{cases} \tag{59}$$

$$\left[\frac{\ell}{r}\right]_{\pm} = \begin{cases} 22.3\, \dfrac{M_{34}^{111/112}\, \mathcal{L}_{37}^{43/336}}{T_e^{183/56}\, \alpha_0^{41/98}} \\[2ex] 0.57\, \dfrac{M_{34}^{165/336}\, \mathcal{L}_{37}^{57/98}\, T_e^{83/56}}{\alpha_0^{72/147}} \end{cases} \tag{60}$$

Consistency will be satisfied as long as we can find values in parameter space that make these three ratios less than 1.

It can be seen that ionic pressure dominated flows are realizable. However, a necessary condition for ionic temperature and disc semi–scale length respectively lesser than virial temperature and radial distance is

$$\alpha_0 > 8.16 \times 10^{2}\, M_{34}^{127/100}\, T_e^{-77/10}\,. \tag{61}$$

Using equation (54) we obtain the minimum turbulent Mach number at $r = r_0$, i.e.,

$$\alpha_0 \geq 2.1\, M_{34}^{32/100}\, \mathcal{L}_{37}^{7/10}\,. \tag{62}$$

Therefore, the flow will be supersonic and consistency will fail in the two–temperature accretion disc with pair production balanced by annihilation plus wind. A similar conclusion was given by Kusunose (1988) and Kusunose and Takahara (1987). White and Lightman (1989) have also found that for $\dot{m}\left[\frac{\dot{M}\, c^2}{L_{Edd}}\right]$ great, the solution is not consistent.

VI. Pair production with electronic variable temperature and α constant

Kepping the electronic temperature as a free parameter is due to the fact that for unsaturated comptonization the solution of the Kompaneetz's equation yields for the specific intensity I_ν,

$$I_\nu = \frac{C}{\nu}\, e^{-h\nu/kT} \tag{63}$$

where C should be determined by the equality between heating and cooling, i.e.,

$$C = 1.98 \times 10^{25} \frac{\dot{M}_{17}\, S}{M_{34}^2\, r^3\, E_1\left[\frac{h\nu_0}{kT}\right]} \tag{64}$$

where E_1 is the exponential integral of the first order and ν_0 is the characteristic photon frequency of the soft source.

From these last equations the disc spectral luminosity will be

$$\mathcal{L}_\nu = 1.35 \times 10^{38} \frac{\dot{M}_{17}}{\nu} \int_1^{\nu_0} \frac{S}{r^2} \frac{e^{-h\nu/kT}}{E_1\left[\frac{h\nu_0}{kT}\right]} dr \tag{65}$$

from where we see that if $T = T(r)$, only for $h\nu \ll kT$ (the flat part of the spectrum) $\mathcal{L}_\nu \sim 1/\nu$. For a very astrophysically interesting class of objects (X–rays binaries, Quasars, etc.), with significant emission in the X–ray portion of the spectrum, $h\nu \sim kT$. This implies that a small variation of T, with radial distance may lead to a spectral index quite different from the observed. However, if T is constant along r, $\mathcal{L}_\nu \sim 1/\nu$ for any ν.

Besides, we have seen that the assumption of a constant electronic temperature

makes the system of equations less "non–linear", allowing for analytic results. On the other hand, a constant electronic temperature would require such an efficient physical mechanism to redistribute energy, equalizing temperature along the disc, which plausibility is very hard to believe.

Therefore, let us see how the solutions we have obtained change when we considere the turbulent Mach number a free parameter, allowing T_e to vary along the disc.

From equation (41) we obtain

$$\ln \Lambda = 6.1 \times 10^{-2}\, r^{3/2}\, \dot{M}_{17}\, S\, T_e^{9/2}\, \Delta^3\, f^2\, N_i \tag{66}$$

and substitution into equation (42) results in

$$T_e^3\, \Delta^3 = \frac{1.8 \times 10^5\, \alpha}{M_{34}\, r^{9/2}\, \dot{M}_{17}\, S\, f^3\, N_i^2} . \tag{67}$$

We now use the approximate solution of equation (43), i.e.,

$$N_i = \frac{2.44 \times 10^2}{\dot{M}_{17}\, S\, r^{3/2}\, T_e^{9/2}\, \Delta^3\, f^2} \tag{43}$$

in equation (67) to obtain an approximate solution to the electronic temperature,

$$T_e \simeq \left[\frac{M_{34}\, r^{3/2}}{\alpha\, \dot{M}_{17}\, S\, f}\right]^{1/9} . \tag{68}$$

Finally, substituting equations (68) and (45) into equation (44) results in the continuity equation for the pairs

$$\frac{1.07\times 10^3}{r}\left[\frac{\alpha\ \dot{M}_{17}}{M_{34}}\right]^{11/18}\frac{\partial}{\partial r}\left\{\frac{S^{11/18}}{r^{51/36}}\frac{(f-1)}{f^{7/18}}\right\} =$$

$$= 1.55\times 10^5\left[\frac{M_{34}}{\dot{M}_{17}S}\right]^{1/9}\frac{\alpha^{17/9}}{r^{17/6}}\frac{g(f^2-1)}{f^{19/9}} + \tag{69}$$

$$+ 2.2\times 10^3\left[\frac{M_{34}}{\dot{M}_{17}S}\right]^{4/9}\frac{\alpha^{14/3}\ (f-1)\beta}{f^{19/9}\ r^{21/9}} - 1.27\times 10^5\frac{(\dot{M}_{17}\ S)^2}{M_{34}^2\ r^3} .$$

Let us analyse the behavior of the solution of equation (69), near r_0, the outer radius of the inner region, given by the solution of equation (37). We shall take as boundary condition that the ratio of the inflow rate to the annihilation rate is p, i.e.,

$$\frac{1.07\times 10^3}{r}\left[\frac{\alpha\ \dot{M}_{17}}{M_{34}}\right]^{11/18}\frac{\partial}{\partial r}\left\{\frac{S^{11/18}}{r^{51/36}}\frac{(f-1)}{f^{7/18}}\right\}\Bigg|_{r=r_0} =$$

$$= -1.55\times 10^5\left[\frac{M_{34}}{\dot{M}_{17}S}\right]^{1/9}\left[\frac{\alpha^{17/9}}{r^{17/6}}\frac{g(f^2-1)}{f^{19/9}}\right]\Bigg|_{r=r_0} . \tag{70}$$

We the obtain, assuming negligible wind,

$$f^2 - 1 = a\, f^{19/9} \tag{71}$$

with

$$a = \frac{0.82}{(1+p)\ r_0^{1/6}}\left[\frac{\dot{M}_{17}\ S_0}{M_{34}}\right]^{19/9}\frac{1}{\alpha^{17/9}} .$$

Equation (71) will have 2 possitive solutions for $a < 0.8$, 1 positive solution for $a = 0.8$, and no solution for $a > 0.8$. Therefore, $a = 0.8$ defines the (maximum) critical accretion rate given by

$$\dot{M}_{17}^c = \frac{\alpha^{17/19}\ M_{34}\ r_0^{3/38}\ (1+p)^{9/19}}{S_0} . \tag{72}$$

For $a < 0.8$, the approximate solutions of equation (71) are

$$f \simeq \begin{cases} 1 + a/2 \\ a^{-9} \end{cases} . \tag{73}$$

For systems with strong pair wind, such that the wind rate dominates over the annihilation rate, we obtain

$$f = 1 + b\, f^{13/9} \tag{74}$$

with

$$b = \frac{57.7}{\beta\ r_0^{2/9}}\left[\frac{\dot{M}_{17}\ S_0}{M_{34}}\right]^{22/9}\frac{\alpha^{-14/9}}{(1+p)} .$$

Again, we will have 2 positive solutions for $b < 0.4$, 1 positive solution for $b = 0.4$, no solution for $b > 0.4$. The critical accretion rate defined for $b = 0.4$, is given by

$$\dot{M}^c_{17} = 0.13 \frac{\beta^{9/22}\ \alpha^{7/11}\ M_{34}\ r_0^{3/11}}{S_0} (1+p) \quad . \tag{75}$$

For $\dot{M}_{17} < \dot{M}^c_{17}$, given above, the approximate solutions of eq.(75) are

$$f = \begin{cases} 1 + b \\ b^{-9/4} \end{cases} . \tag{76}$$

After a brief comparison between the critical rates given by equations (72) and (75) we conclude that a system endowed with a strong wind has a critical accretion rate lesser than a system without wind.

Now, we turn the effect of inflow rate in the critical accretion rate. From the definition of p, given by equation (70), we have, approximately,

$$p \simeq 1.7 \times 10^{-2} \left[\frac{\dot{M}_{17}\ S_0}{M_{34}}\right]^{13/18} \alpha^{-23/18}\ r_0^{-7/12} \quad . \tag{77}$$

Using equations (37) and (72) we obtain

$$p = 3.2 \times 10^{-4} \frac{r_0^{36/100}}{M_{34}^{1/11}\ \alpha^{3/2}} \tag{78}$$

and

$$282 p^{18/13}\ \alpha^{216/437}\ r_0^{180/247} = 1+p \quad . \tag{79}$$

Now we solve equations (78) and (79) for Cygnus x–1. Setting $M_{34} = 3$, we obtain for $\alpha = 0.1$

$$\begin{aligned} r_0 &\simeq 4.9 \\ p &\simeq 8.9 \times 10^{-2} \\ \dot{M}^c_{17} &\simeq 0.8 \end{aligned} \tag{80}$$

and for $\alpha = 0.5$

$$\begin{aligned} r_0 &\simeq 39.9 \\ p &\simeq 3.1 \times 10^{-3} \\ \dot{M}^c_{17} &\simeq 2.5 \quad . \end{aligned} \tag{81}$$

We see from (80) and (81) that the assumption of null inflow rate at $r = r_0$, used in the disc with constant electronic temperature, is a reasonable one. With the values given by (80) and (81) we obtain, respectively,

$$\begin{aligned} T_e &\simeq 1.75 \\ N_i &\simeq 0.12 \\ n_+ &\simeq 0.20 \\ T_i &\simeq 3.4 \times 10^4 \\ \ell &\simeq 4.2 \times 10^7 \\ T_v &\simeq 1.6 \times 10^3 \end{aligned} \tag{82}$$

and

$$\begin{aligned} T_e &\simeq 1.76 \\ N_i &\simeq 1.17 \times 10^{-3} \\ n_+ &\simeq 1.96 \times 10^{-3} \\ T_i &\simeq 1.14 \times 10^4 \\ \ell &\simeq 5.5 \times 10^8 \text{cm} \\ T_v &\simeq 2 \times 10^2 \quad . \end{aligned} \tag{83}$$

From the above results we see that for accretion rate equal to the critical one, the thin disc approximation fails and the ionic temperature will be greater than the virial temperature.

VII. Conclusions:

We have shown that non linear effects in the two temperature soft photon comptonized disc are exhibited when we allow for a variable coupling ($\ln \Lambda$) for ions, electrons and positrons. The physical meaning is that non linearity comes into play when the number of particles interacting with a test particle varies along the disc. Such effects will make the solution a double–valued one, a behavior identical to the solution for discs cooled by internally provided photons.

The variation of $\ln \Lambda$ will occur as long as we do use the definition of the Debye number, a given function of the electronic temperature and density of particles. In the upper branch pairs outnumber protons; also in this branch, positron number density never exceeds $244/(\dot{M}_{17}\, S\, T_e^{9/2}\, \Delta^3)$. In the lower branch positrons are less abundant than protons and its number density exceeds the above value everywhere.

The solution will be multi–valued as long as T_e doesn't exceed a maximum value. This value, above which pair production no longer balances pair annihilation, is about one order of magnitude lesser than the value previously found by Bisnovatyi–Kogan, Zel'dovich and Sunyaev, 1971. Even for systems with a strong wind there is a maximum electronic temperature below which pair production may be balanced by annihilation plus wind, with null inflow rate. This temperature is lesser than the maximum temperature for equilibrium pair production annihilation. As this temperature is below the threshold for wind

dominance, the wind will never dominate near the outer edge of the two temperature region of accretion discs.

Intuitively one would expect that the ratio positrons to protons would grow very fast with temperature. However, in the upper branch, where positrons outnumber protons, this ratio grows like $T_e^{-11/2}\,\Delta^{-3}$, for $T_e \ll T_e^{Max}$, eq.(51). In the low f branch this ratio goes like T_e^{11}, for $T_e \ll T_e^{Max}$. As T_e approaches T_e^{Max} the ratio has identical behavior for both branches. For $T_e > T_e^{Max}$, the disc is no longer stationary and we should solve time dependent equations.

It may be seen, from equations (37), (42) and (48) that

$$T_e^{Max} \leq 2.2\, M_{34}^{4/63}\, r_0^{-1/21} \tag{84}$$

a result pratically idenpendent of the mass of the central object and location of the outer radius of the inner region. We have neglected the inflow rate (not the accretion rate) using null boundary condition at $r = r_0$. However, taking into account its effects at $r = r_0$, we would obtain for the ratio inflow rate to annihilation rate

$$p \simeq 1.36 \times 10^{-2} \frac{T_e}{S_0\ r_0\ \Delta^{7/2}} \ . \tag{85}$$

From eq.(84) we conclude that its contribution is negligible. It should be said that such a boundary condition has also been used by Triz and Tsuruta (1989).

In the pair free model one obtains a ionic pressure dominated solution and radiation pressure dominated solution. Full consistency for the ionic pressure dominated solution is only achieved for mildly subsonic turbulence (Meirelles and Marques, 1989). When production of pairs is taken into account, both solutions become ionic pressure dominated.

However, consistency will not be possible because the solutions are supersonic.

It should be remarked that our results contradict previous results of Tritz and Tsuruta (1988) and White and Lightman, 1989, that discs cooled by external photons have only one solution to the disc equations at all radii and for all accretion rates. Indeed, if we had considered not a flow with constant electronic temperature, but instead, a flow with constant turbulent Mach number α, we would obtain a maximum accretion rate for pair equilibrium given by

$$\dot{M}_{17} \leq \frac{\alpha^{17/19}\ M_{34}\ r_0^{3/38}}{S_0}(1+p)^{9/19} . \tag{86}$$

The effects of the inflow rate in the critical accretion rate are provided by p, given by

$$p = 3.2 \times 10^{-4} \frac{r_0^{36/100}}{M_{34}^{1/11}\ \alpha^{3/2}} . \tag{87}$$

We see that the contribution of the inflow rate to the critical accretion rate will hardly exceed 1%, a result similar to the disc with constant electronic temperature. This result is reasonably below White and Lightman (1989) results and in good agreement with the results of Tritz and Tsuruta (1989). It should be said that the results obtained by Tritz and Tsuruta (1989) are fairly well reproduced by our low f branch. If we want to compare the critical accretion rate we obtained for discs with internally produced photons, it should be kept in mind that our results are calculated at $r = r_0$ and their results at $r = r_{crit} = 32/3$. Surprisingly, we obtain a critical accretion rate in good agreement with White and Lightman (1989) results.

Finally it should be said that even allowing pairs to escape the disc at the speed of light, pair equilibria cannot be restored because the critical accretion rate is increased by only a factor of a few.

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In a future paper we shall study the modifications introduced by identical procedure in the two--temperature bremsstrahlung discs with pairs.

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Note added in proof: after this work was ready, it came to our knowledge that M. Kusunose and F. Takahara have recently found that hot, thermal accretion discs with an external source of photons are also subject to a pair critical accretion rate under certain conditions.