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IFUSP/P-921

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ACCRETION DISKS

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Junho/1991

# NEUTRON VISCOSITY IN TWO-TEMPERATURE ACCRETION DISKS

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## Abstract

We consider disks with viscosity given by neutron collisions with accreting ions. By making a fit to Guessoum and Kazanas's (1990) results for neutron abundance we obtain the (neutron) viscosity and then solve the disk structure equations taking into account electron radiative cooling by inverse unsaturated Comptonization, Comptonized bremsstrahlung and pure bremsstrahlung. Applying these results to Cygnus X-1, we show that the most consistent models are those cooled by inverse unsaturated Comptonization and pure bremsstrahlung. It is also shown that neutrino emission by neutron decay may be a relevant source of cooling, mainly for the pure bremsstrahlung model.

Subject headings: stars: accretion—X-rays: binaries—radiative transport.

## 1. Introduction

Certainly, one of the main drawbacks in accretion disk theory is the lack of knowledge about physical processes that may answer for the viscosity needed to transport angular momentum outwards, making accretion possible.

Though, at least for practical purposes, this difficulty has been circumvented by the seminal Shakura and Sunyaev's (1973) paper, by parametrizing the unknown viscosity in terms of a single parameter, namely  $\alpha$ , the question about the origin of the viscosity still remains.

Shear turbulence, a widely accepted physical mechanism for viscosity generation in astrophysical context, should be excluded because accretion disks easily satisfy the Rayleigh criterion for stability, i.e.,

$$\frac{d}{dr}(\Omega^2 r) > 0 \quad ,$$

(Safranov, 1972). Other mechanisms, such as turbulence driven by convection, are highly dependent on the mass of the central object, accretion rate and on the temperature regime for protons and electrons (Bisnovatyi-Kogan and Blinnikov 1977). For a two temperature accretion flow, with masses and accretion rates peculiar to X-ray binaries, it has been shown by Meirelles (1990) that the conditions required for the onset of the convection (Tayler 1980) are not met. It is also shown that relatively moderate fields, not strong enough to dominate over other transport properties of the plasma (Paczynski, 1978), may prevail over some viscosity generation mechanism like the Papaloizou-Pringle (1984) one.

It should be argued, however, that these fields, besides being invoked "ad hoc", have no dynamics associated to them.

Recently, a step towards self-consistency and understanding of the Physics associated to viscosity generation, though restricted to a special class of astrophysical

objects, has been given by Guessoum and Kazanas (1990), by proposing neutrons as a source of viscosity in the inner region ( $R < 100 R_g$ ) of accretion disks around black holes.

According to them, viscosity could be provided by neutron collisions with the accreting ions. The neutrons, in turn, would be produced by the dissociation of  ${}^4\text{He}$  (inherent in the accreting material, with assumed cosmic abundance) when the ion temperature  $T_i$  exceeds 3 MeV and also by the reaction  $pp \rightarrow pn\pi^+$ , for  $T_i \geq 30$  MeV.

In their approach, however, the main concern is to determine conditions under which a particular viscosity can account for a steady state accretion of matter onto a compact object at sufficiently high rates to provide the luminosities observed in Galactic X ray sources.

In doing so, they treat the electronic temperature as a free parameter and obtain the ion temperature by equating the viscous dissipation to the energy transferred by the ions to the electrons through Coulomb collisions. No subsequent cooling of the electrons, by radiation or by any other way, nor the structure of the accretion disks are considered.

In this paper we shall extend Guessoum and Kazanas's (1990) results by taking into account both the structure of the accretion disks as well as electron cooling by radiation, which we shall assume, separately, unsaturated inverse comptonization of soft photons produced externally, comptonization of bremsstrahlung photons and pure bremsstrahlung.

Differently from Guessoum and Kazanas's (1990) paper, we shall assume the disk as a thin one.

## II. Disk equations

Before obtaining the disk equations let us first explain the notation we shall employ to describe the disk:  $P$  (pressure),  $\ell$  (disk semi-height),  $T_i$  (ion temperature in units of  $10^9$  °K),  $T_e$  (electronic temperature in units of  $10^9$  °K),  $M_{34}$  (mass of the black hole in

units of  $10^{34}$  g),  $\dot{M}_{17}$  (accretion rate in units of  $10^{17}$  g s $^{-1}$ ),  $r$  (radial distance in units of  $GM/c^2$ ),  $z$  (distance from the symmetry plane),  $\alpha$  (adimensional viscosity parameter),  $Y$  (neutron abundance),  $y$  (Kompaneetz comptonization parameter),  $\rho$  (matter density),  $N_n$  (neutron number density),  $\lambda_n$  (neutron mean free-path),  $\Omega$  (keplerian angular velocity),  $W_{r\varphi}$  (integrated over  $z$  viscous stress). Unless otherwise stated, all the variables will be expressed in the C.G.S. system.

Exception made for the viscosity parameter  $\alpha$ , which will be calculated self-consistently, we shall make the usual assumptions from the standard  $\alpha$  model, i.e.,

1. keplerian velocity,
2. thin disk,
3.  $W_{r\varphi} = 2\alpha P$ ,
4. radiative energy transport only in the  $z$ -direction,
5. hydrostatic equilibrium in  $z$  direction,
6. equality between heating and cooling,
7. pressure essentially given by the gas.

From the assumptions of hydrostatic equilibrium, thin disk and gas pressure dominance we obtain for the disk semi scale height

$$\ell = 7.1 \times 10^3 M_{34} r^{3/2} T_i^{-1/2} \text{cm}, \quad (1)$$

and from the angular momentum conservation equation,

$$\rho = \frac{1.66 \dot{M}_{17} S}{\alpha M_{34}^2 r^3 T_i^{3/2}} \text{g cm}^{-3} \quad (2)$$

where  $S = 1 - \delta r^{-1/2}$ ,  $\delta$  being the ratio angular momentum of the flow to the keplerian one, at  $r=1$ .

Now, from the collisional energy exchange term (Spitzer, 1962)

$$F_{ei} = 9.24 \times 10^{24} \rho^2 \ell \frac{T_i}{T_e^{3/2}} \ln \Lambda \text{ erg cm}^{-2} \text{ s}^{-1}, \quad (3)$$

where  $\ln \Lambda$  is the Coulomb logarithm, and from the heat generation function

$$Q^+ = 1.94 \times 10^{25} \frac{\dot{M}_{17} S}{M_{34}^2 r^3} \text{ erg cm}^{-2} \text{ s}^{-1} \quad (4)$$

we obtain using equations (1) and (2), setting  $\ln \Lambda \approx 15$ ,

$$T_i = \frac{5.6 \times 10^2}{\alpha^{4/3}} \left[ \frac{\dot{M}_{17} S}{M_{34}^2 r^{3/2}} \right]^{2/3} T_e^{-1}. \quad (5)$$

To make further progress we should specify now the radiative cooling for the electrons. Before deducing the temperature expression for the unsaturated comptonized model it would be interesting to stress some points to justify the approximations we shall employ. In that model one needs the density of external soft photons to be up comptonized in the hot inner region. If we assume the outer disk as the source of these soft photons, we must face the following problem:

- if that part of the disk emits modified black body-like radiation, the soft flux is highly dependent on the viscosity ( $\alpha$  parameter), that can not be due to the neutrons because temperature is too low,
- if that part emits black body radiation, temperature will be even lower. Placing the inner edge of the outer disk at  $r \approx 100$ , we obtain  $T \sim 10^{-4}$ . We then should need a too high enhancement factor to obtain the observed spectral temperature ( $T_e \sim 1$ ).

Besides, it should be remarked that knowledge of the viscosity is not enough to determine the electron temperature that should be determined by the electron cooling (spectral flux integrated over frequency).

To treat the unsaturated inverse comptonization we then shall adopt the usual procedure found in the literature (Shapiro, Lightman and Eardley 1976; White and Lightman 1989).

Setting the Kompaneetz  $y$  parameter equal to 1, we obtain

$$T_i = \frac{3.18 \times 10^3}{\alpha} \frac{\dot{M}_{17} S}{M_{34}^2 r^{3/2}} T_e. \quad (6)$$

Comparison with equation (5) gives

$$T_e = \frac{0.42}{\alpha^{1/6}} \left[ \frac{M_{34} r^{3/2}}{\dot{M}_{17} S} \right]^{1/6}. \quad (7)$$

For comptonized bremsstrahlung, we should equate the electron cooling (Rybicki and Lightman, 1979)

$$F_{bc} = 1.15 \times 10^{25} \rho^2 T_e^{1/2} \ell \left\{ \ln \frac{2.25}{x_{\text{coh}}} \right\}^2 \text{ erg cm}^{-2} \text{ s}^{-1}, \quad (8)$$

where  $x_{\text{coh}}$  is the coherent photon energy in units of  $k T_e$ , to the heat generation function, eq.(4), to obtain

$$T_i \approx 8.3 \times 10^{-2} T_e^2 \left\{ \ln \frac{2.25}{x_{\text{coh}}} \right\}^2. \quad (9)$$

From the definition of  $x_{\text{coh}}$  (Rybicki and Lightman 1979), using eq.(5), we obtain

$$x_{\text{coh}} \approx 6.8 \times 10^{-6} \alpha T_e^{-3/4} \quad (10)$$

Inserting this last result into eq.(9) and using eq.(15) we get

$$2.87 \times 10^3 e^{-3.47 T_i^{1/2}/T_e} = \left[ \frac{M_{17} S}{M_{34} r^{3/2}} \right]^{1/2} T_e^{-3/2} T_i^{-3/4} \quad (11)$$

Finally, from the expression for pure bremsstrahlung cooling,

$$F_b = 2.04 \times 10^{25} \rho^2 T_e^{1/2} \ell \text{ erg cm}^{-2} \text{ s}^{-1}, \quad (12)$$

we obtain

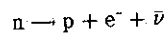
$$T_e^2 = 6.8 T_i \quad (13)$$

### III. On the neutron viscosity

In this section we shall briefly obtain the neutron viscosity. For a more detailed treatment one is referred to Guessoum and Kazanas's (1990) paper.

The reactions that, theoretically, supply or absorb neutrons in the inner region of accretion disks, may be found in Guessoum and Kazanas's (1990) paper.

As pointed out by Guessoum and Kazanas (1990), the last reaction



is only interesting for systems with transit time comparable or greater than the neutron  $\beta$ -decay time (e.g. Quasars).

If  $\rho_n$ ,  $v_n$  and  $\ell_n$  are respectively the matter density associated to the neutrons, neutron thermal velocity and mean free path, then the viscosity will be (Weaver 1976)

$$\eta = \frac{1}{3} \rho_n v_n \ell_n \quad (14)$$

If we assume further that the distribution function for the neutrons is a Maxwellian of temperature equal to the ion temperature, we obtain for the kinematic viscosity, after replacing  $v_n$  and  $\ell_n$  by their thermal averages, (Bond, Watson and Welch 1965)

$$\nu = \frac{5\sqrt{\pi}}{8} y_n \left[ \frac{k T_i}{m_n} \right] \lambda_n(k T_i) \quad (15)$$

where  $y_n$  is the neutron abundance,  $m_n$  the neutron mass and

$$\lambda_n = \frac{1}{N \bar{\sigma}_n(k T_i)}, \quad (16)$$

with  $N$  being the total number particle density. For  $\bar{\sigma}_n(k T_i)$  we shall use simply (Guessoum and Kazanas 1990)

$$\bar{\sigma}_n \approx 2.74 \times 10^{-22} T_i^{-0.85} \text{ cm}^2 \quad (17)$$

Substitution of  $\sigma_n$  given by eq.(15) in equations (14) and (13) using (2) results in

$$\nu = 5.9 \times 10^6 Y_n \frac{M_{34}^2 r^3}{M_{17} S} T_i^{2.85} \alpha \quad (18)$$

However, from the definition of the  $\alpha$  parameter,

$$\nu = \frac{1}{3} \alpha v_s \ell, \quad (19)$$

we obtain

$$2.74 \times 10^5 \frac{\dot{M}_{17} S}{M_{3.4} r^{3/2}} = Y_n T_i^{1.85} \quad (20)$$

A fit of Guessoum and Kazanas's (1990) data yields for the neutron abundance

$$Y_n \approx 4.1 \times 10^{-2} T_i^{-1/4} T_e^{1.3} \left[ 1 - \frac{34.8}{T_i} \right]^{2.63} \quad (21)$$

Equations (18) and (19) give the relation between electronic and ionic temperatures for disks with viscosity given by collisions of neutrons with ions, i.e.,

$$T_e = 1.6 \times 10^5 \left[ \frac{\dot{M}_{17} S}{M_{3.4} r^{3/2}} \right]^{0.77} T_i^{-1.23} \left[ 1 - \frac{34.8}{T_i} \right]^{-2} \quad (22)$$

For two temperature soft photon comptonized disk the ion temperature is the solution of

$$T_i \left[ 1 - \frac{34.8}{T_i} \right]^{1.46} = 1.98 \times 10^4 \left[ \frac{\dot{M}_{17} S}{M_{3.4} r^{3/2}} \right]^{0.77} \quad (23)$$

Now we specialize for Cygnus X-1 and use data from Liang and Nolan's (1984) paper, i.e.,

$$\dot{M}_{39} \approx 3, \quad \mathcal{L}_x \approx 3 \times 10^{37} \text{ erg s}^{-1}, \quad \dot{M}_{17} \approx 0.6$$

Below we give the values of  $T_i$ ,  $T_e$ ,  $\alpha$ ,  $\ell$ ,  $Y_n$  for  $r$  varying from 1 to 100.

$r$	$T_i$	$T_e$	$\alpha$	$\ell$	$Y_n$	$\tau$
1	965	1.7	$9.7 \times 10^{-2}$	$1.14 \times 10^6$	$1.53 \times 10^{-2}$	0.87
2	1200	1.55	0.13	$3.59 \times 10^6$	$1.3 \times 10^{-2}$	0.96
4	730	1.83	$9.1 \times 10^{-2}$	$7.9 \times 10^6$	$1.7 \times 10^{-2}$	0.81
10	340	2.24	$8.2 \times 10^{-2}$	$2.13 \times 10^7$	$2.35 \times 10^{-2}$	0.66
20	190	2.7	$6.8 \times 10^{-2}$	$4.5 \times 10^7$	$2.7 \times 10^{-2}$	0.55
100	69	4.46	$3.2 \times 10^{-2}$	$3 \times 10^8$	$1.8 \times 10^{-2}$	0.33

Table 1: the run of the physical variables for the inverse unsaturated comptonization model.

We now turn to the disks cooled by comptonized bremsstrahlung. In that case the ion temperature is the solution of the equation

$$1.84 \times 10^{11} \left[ \frac{M_{17} S}{M_{34} r^{3/2}} \right]^{0.66} T_i^{2.6} \left[ 1 - \frac{34.8}{T_i} \right]^3 = e^{2.17 \times 10^{-5}} \left[ \frac{M_{34}^2 r^{3/2}}{M_{17} S} \right]^{0.77} T_i^{1.73} \left[ 1 - \frac{34.8}{T_i} \right]^2 \quad (24)$$

Again, we specialize for Cygnus X-1, and give below values of  $T_i$ ,  $T_e$ ,  $\ell$ ,  $\alpha$ ,  $Y_n$  for  $r$  varying from 1 to 100.

$r$	$T_i$	$T_e$	$\ell$	$\alpha$	$Y_n$	$\tau$
1	755	2.23	$1.02 \times 10^6$	$5.8 \times 10^{-2}$	$2.23 \times 10^{-2}$	0.62
2	843	2.45	$3 \times 10^6$	$5.76 \times 10^{-2}$	$2.50 \times 10^{-2}$	0.74
4	638	2.19	$7.43 \times 10^6$	$5.65 \times 10^{-2}$	$2.23 \times 10^{-2}$	0.53
10	395	1.8	$2.3 \times 10^7$	$5.38 \times 10^{-2}$	$1.78 \times 10^{-2}$	0.30
20	268	1.56	$5.38 \times 10^7$	$5.03 \times 10^{-2}$	$1.44 \times 10^{-2}$	0.18
100	1181	1.1	$4 \times 10^8$	$3.86 \times 10^{-2}$	$6.44 \times 10^{-3}$	$5.6 \times 10^{-2}$

Table 2. the run of the physical variables for the comptonized bremsstrahlung model.

Finally, we consider disks cooled by pure bremsstrahlung. Using equation (13), we obtain the equation satisfied by the ion temperature,

$$T_i^{1.73} \left[ 1 - \frac{34.8}{T_i} \right]^2 = 6.1 \times 10^4 \left[ \frac{M_{17} S}{M_{34} r^{3/2}} \right]^{0.77} \quad (25)$$

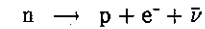
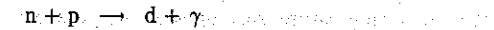
Below, we give the values of  $T_i$ ,  $T_e$ ,  $\ell$ ,  $\alpha$ ,  $Y_n$  for  $r$  varying from 1 to 100.

$r$	$T_i$	$T_e$	$\ell$	$\alpha$	$Y_n$	$\tau$
1	138	30.6	$4.3 \times 10^5$	$2.98 \times 10^{-2}$	0.55	6.79
2	150	31.94	$1.27 \times 10^6$	$3.07 \times 10^{-2}$	0.61	7.79
4	123	28.92	$3.25 \times 10^6$	$2.81 \times 10^{-2}$	0.47	5.55
10	90	24.74	$1.1 \times 10^7$	$2.29 \times 10^{-2}$	0.27	3.06
20	72	22.13	$2.78 \times 10^7$	$1.85 \times 10^{-2}$	0.16	1.87
100	50	18.44	$2.59 \times 10^8$	$8.89 \times 10^{-3}$	$1.8 \times 10^{-2}$	0.57

Table 3. the run of the physical variables for the pure bremsstrahlung model.

#### IV. On the $\bar{\nu}$ cooling

We have only considered cooling on the disk by electron radiation. However, the last two reactions considered by Guessoum and Kazanas (1990)



may constitute a source of cooling, producing high energy photons and neutrinos that escape the disk as soon as they are produced. The first reaction constitutes a problem in itself and will not be considered here.

To consider to cooling due to neutrino emission we shall adopt the following

reasoning. Neutrons are produced at  $r_1$  and decay at  $r$ , related to  $r_1$  by the neutron decay time, i.e.,

$$\int_{r_1}^r \frac{dr}{Vr} = 720 \text{ s} \quad (26)$$

The amount of neutrons (per second) produced at  $r_1$  is, simply,

$$N_1 = \frac{Y_n^{(1)}}{m_n} \dot{M} \quad (27)$$

where  $Y_n^{(1)}$  is the neutron abundance at  $r_1$ . Therefore, at  $r$ , this will result in a neutron flux given by

$$q_n = \frac{Y_n^{(1)} \dot{M}}{m_n 4\pi r^2} \quad (28)$$

Assuming that the neutrino carries, on the average, an energy of about 1 MeV, we obtain the neutrino energy flux at  $r$ ,

$$\mathcal{F}_\nu \approx 4.25 \times 10^{-2} Q^+(r) \left[ \frac{Y_n^{(1)} r^{1/2}}{S T_i^{1/2}} \right] \quad (29)$$

We see that, close to the inner radius, neutrino cooling may become very important. Furthermore, if we assume null boundary condition to the angular momentum, at  $r=1$ , it becomes the dominant source of cooling close to the inner radius.

For the disks we have considered, the pure bremsstrahlung cooled disk is the one most affected by neutrino cooling.

## V. Conclusions

High ion temperature to make neutron collision an efficient source of viscosity will thicken the inner region of accretion disks, making the semi scale height comparable or even greater than the radial distance. Rigorously speaking, for all the models we have considered, the thin disk approximation only holds for  $r \leq 20$ . For  $r$  greater than this value one would have to include the effects of  $z$ -structure, a task beyond our aim in this paper. This effect is more sensitive to unsaturated inverse comptonization or comptonized bremsstrahlung cooling. However, electrons are much hotter in that case. The combined effects of lowering  $T_i$  and raising  $T_e$  will slightly increase neutron abundance in pure bremsstrahlung disks. It is worth while to remark that neutron viscosity is greater in pure bremsstrahlung disks, due to the fact that the increase in the density by  $r$  exceeds the decrease in the neutron mean free path and temperature.

It was shown that for some systems, with infall time greater than the neutron decay time, neutrino emission may be an important source of cooling close to the inner radius. This will be relevant for pure bremsstrahlung disks.

Exception made for the thickening of the disk, common to the three models, making the semi scale height comparable to the radius, the unsaturated inverse comptonization and the pure bremsstrahlung are the most consistent models. For the comptonized bremsstrahlung disk, the frequency for which the medium becomes optically thin is less than the coherent frequency.

It should be said that accounting of the structure of the disk and the electron cooling have allowed for solutions to the disk equations for a wider temperature range than the previously found by Guessoum and Kazanas (1990). The mass we have used for Cygnus X-1 is equal to their value, however for the accretion rate we have made an option for a lesser value ( $\dot{M} \sim 10^{-9} M_\odot$ ) because a higher value ( $\dot{M} \sim 10^{-8} M_\odot$ ) would imply a too low efficiency for gravitational energy conversion into radiation.



Finally, concerning the high electron temperatures found, specially for the pure bremsstrahlung disk, it becomes clear that electron-positron production will play a fundamental role in the physics of accretion disks with neutron collisions as the main viscosity. It is our intention to consider it in a next paper.

We thank the Referee for raising some pertinent points concerning self-consistency of our results.

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