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ON THE CONDUCTIVE ENERGY TRANSPORT IN TWO-TEMPERATURE ACCRETION DISKS: THE NON (z) ISOTHERMAL ATMOSPHERE CASE

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Abstract:

We here improve previous results we have obtained for conductive energy transport in two–temperature accretion disks. For both inverse unsaturated comptonization and pure bremsstrahlung we show the existence of a critical accretion rate, if the gas dominates the pressure. This critical accretion rate is much less than the values assumed for X–ray binaries. In the limit of great radiation pressures, we obtain a double valued solution for the disk equations, with different temperatures and different conductive to radiative transport ratios. Depending on the values of the accretion rate, mass and viscosity parameter, there will be a radius inside of which the disk will be thermally unstable. These results eventually may explain the states of high and low luminosity of Cygnus X–I.

Key words: accretion disks – hydrodynamics – conduction – radiative transfer – X–ray binaries.

I. Introduction

Since the mid of the seventies, two temperature accretion flows have found a large amount of applicability in Astrophysical Systems, from X–Ray Binaries to Active Galactic Nuclei and Quasars (Rees et al. 1982, Band and Malkan 1989).

As far as we are aware, all this interest started with the successful paper of Shapiro et al. (1976) on theoretically explaining the data of Cygnus X–1. This model is, however, very simplified in the sense that it does not take into account some physical processes that may occur within a plasma in similar conditions as those assumed for the disk around Cygnus X–1.

Recently, a large amount of papers in the literature has been dedicated to the inclusion of some effects not considered by Shapiro et al. 1976. Among them it should be mentioned some applications related to pair production in two temperature accretion disks (Sikora and Zbyszewska 1985; Kusunose 1987; Tritz and Tsuruta 1989; White and Lightman 1989; Meirelles 1991(a)) and spherical accretion (Kusunose and Takahara 1985; Takahara and Kusunose 1985; Guilbert and Stepney 1985; Begelman et al. 1987; Lightman et al. 1987; Parker and Ostriker 1989) and to thermal conduction related to the viscosity generation problem treated by Meirelles 1991(b)), who considered an isothermal (z) atmosphere.

It should be argued, however, that the isothermal (z) atmosphere assumption is a drastic assumption that would explain the high value for the conductive to radiative energy transport ratio found ($\sim 10^4$).

We here improve Meirelles 1991(b) results on the conductive energy transport in two temperature soft photon comptonized accretion disk in the sense that we consider a non–isothermal atmosphere.

To a certain extent, the solution we obtain for the conductive transport is quite general and is also applied for the pure bremsstrahlung disk. Besides, we consider the solution for ξ (ratio radiation to gas pressure) $\gg 1$ and $\xi \ll 1$.

We also analyse here the stability of the disk against perturbations in the radial direction, taking conduction into account. Finally, we apply the results to Cygnus X–1.

II. The stationary disk equations

The main standard assumptions from the accretion disk theory (e.g. Shakura and Sunyaev 1973; Novikov and Thorne 1973) we shall retain for the description of the accretion disk are: thin disk, approximately circular gas orbits about the central black hole and thermal distribution for the particles. We briefly explain the notation we shall employ: ρ (density), T_e (electron temperature in units of 10^9 °K), T_i (proton temperature in units of 10^9 °K), P (total pressure), P_g (gas pressure), ξ (ratio of radiation and gas pressures), β (ratio of radiation and total pressures), q (ratio of energies transported by conduction and radiation), ℓ (disk semi height scale), v_t (velocity associated to dissipative energy proceses), ℓ_t (scale lenght associated to dissipative energy processes), M_{34} (black hole mass in units of 10^{34} g), $\dot{M}_{17}$ (accretion rate in units of 10^{17} gs^{-1}), Ω (Keplerian angular velocity), α (Mach number associated to v_t), r (radial distance in units of GM/c^2), x (distance to the symmetry plane of the disk in units of ℓ), K_t (classical conductivily constant), V_r (radial velocity), Q^+ (heat generation function), F_{ei} (electron–proton energy exchange), y (Kompaneetz parameter), K_v (Kinematic viscosity).

In the accretion disk, we propose, we consider, for every physical variable, the z–dependence up to the eighth power on **x**. Differently from other works in the literature, the kinematic viscosity will not be assumed constant over x.

a) **Hydrostatic equilibrium and pressure**

Hydrostatic equilibrium in the z–direction requires that the total pressure P and the z–component of the gravitational force be related through (Shakura and Sunyaev 1973)

$$\frac{\partial}{\partial x} P = -\rho \Omega^2 \ell^2 x \quad . \tag{1}$$

For the pressure we shall take the contributions from the particles, radiation and magnetic fields (Shakura et al. 1978; Meirelles 1990), i.e.,

$$P = (1+\alpha^2)(1+\xi) P_g \quad . \tag{2}$$

α will be assumed constant over x .

b) **Mass and angular momentum conservation equations**

Radial velocity, density, height scale of the disk and accretion rate are related through the mass conservation equation, i.e.,

$$\dot{M} = -4\pi r \ell \int_0^1 \rho V_r \, dx \quad . \tag{3}$$

Angular momentum conservation equation, in turn, relates radial velocity, density and kinematic viscosity through

$$\rho \frac{V_r \, \Omega \, r^2}{2} = -\frac{\partial}{\partial r} \left\{ \rho K_v r^2 \Omega \right\} \quad . \tag{4}$$

c) **Transport coefficients in the medium**

From equation (1) we define the velocity associated to energy dissipation processes as

$$V_t = \frac{\alpha}{\sqrt{1+\alpha^2}} \left[\frac{P}{\rho} \right]^{1/2} \quad . \tag{5}$$

Assuming equality between Keplerian and energy dissipation times, the length scale associated to V_t is defined as

$$\ell_t = \frac{V_t}{\Omega} \quad . \tag{6}$$

From equations (5) and (6) we obtain for the kinematic viscosity

$$K_v = \frac{\alpha^2}{3(1+\alpha^2)\Omega} \frac{P}{\rho} \quad . \tag{7}$$

Assuming that protons dominate electrons in thermal energy conduction, we write the classical expression for the conductivily constant (Golant, Zhilinsky and Sakharov 1980)

$$K_t = 2.29 \times 10^{19} \frac{T_i^{5/2}}{\ell n \Lambda} \quad , \tag{8}$$

where $\ell n \Lambda$ is the Coulomb logarithm.

If, on the contrary, the electrons dominate the conduction process, we must multiply expression (8) by the square root of the ratio proton to electron mass.

d) Heat generation in the disk

The heat flux generated in the disk is, at a distance x from the symmetry plane (Shakura and Sunyaev 1973; Shakura et al. 1978; Meirelles 1990)

$$Q^{+} = \frac{r^2}{2}\left[\frac{d\Omega}{dr}\right]^2 \ell \int_0^x \rho K_\nu \, dx' \quad . \tag{9}$$

e) Collisional energy exchange between ions and electrons

If the time for the protons cool down to the electron temperature (cooling time) is much longer than the transit time across the disk, the energy transferred from the protons to the electrons is, for $K T_e < m_e c^2$ (Spitzer 1962)

$$F_{ei} = 9.24 \times 10^{24} (\ell n \Lambda) \ell \int_0^x \rho^2 \frac{T_i}{T_e^{3/2}} \, dx' \quad . \tag{10}$$

The Coulomb logarithm in equations (8) and (10) will be set equal to 15.

f) Radiative transport

If there is a source of soft photons impiging the disk, at the inner region, the energy received by the electrons from the protons is rapidly lost through unsaturated inverse comptonization of these external soft photons. Owing to the variation of the electron temperature along x, we shall impose the unsaturated inverse comptonization condition (y=1) as

$$1 = \frac{4k \, \ell \, \sigma_T}{m_e \, c^2 \, m_H} \int_0^1 \rho T_e \, dx \quad , \tag{11}$$

where k is the Boltzmann constant, m_e the electron mass, c the velocity of light, σ_T the Thomson cross section for electron scattering and m_H is the hydrogen mass.

In case of absence of the soft photon source and when the transit time is longer than the cooling time, we will be interested in the bremsstrahlung cooling, given by (Rybicki and Lightman 1979)

$$F_b = 1.6 \times 10^{25} \rho^2 \, \ell \, T_e^{1/2} \quad . \tag{12}$$

In the plane parallel approximation, the radiative flux is related to the radiation pressure through (Rybicki and Lightman 1979)

$$F = -\frac{c m_H}{\rho} \frac{\xi}{\sigma_T \, \ell} \frac{\partial}{\partial x} P_g \quad . \tag{13}$$

g) Conductive energy transport

For the conductive energy transport we shall employ (Golant, Shilinsky and Sakharov 1980)

$$F_c = -1.52 \times 10^{27} \frac{e \, T_i^{7/2}}{\ell} \frac{\partial}{\partial x} T_i \quad , \tag{14}$$

where $e=1$ when the protons dominate the electrons in the conduction process and $e = (m_p/m_e)^{1/2}$ otherwise.

Neglecting pressure gradients in r–direction, we write formally the energy equation

$$F_r + F_c = Q^{+} \quad . \tag{15}$$

Gathering results from (1) to (15), we may write

$$-(1+q^{-1})\,10^{9}\,\frac{k_t}{\ell}\,\frac{\partial}{\partial x}\,T_i = \frac{3}{8}\,\frac{\alpha^2}{1+\alpha^2}\,\Omega\,\ell\int_0^x P\,dx' \quad , \tag{16}$$

where q, the ratio conductive to radiative flux, assumed constant, may be calculated by means of equation (13) together with (10) (or (12)).

To solve equations (1) and (16) we shall express the functions P, ρ, T_i and T_e in terms of central values,

$$\begin{aligned} P &= P_c\,p(x) \\ \rho &= \rho_c\,j(x) \\ T_i &= T_{ic}\,t(x) \\ T_e &= T_{ec}\,i(x)\ . \end{aligned} \tag{17}$$

From equation (2), we see that $p=jt$. For the functions t, j and i we shall impose at $x=0$,

$$j = t = i = 1 \quad . \tag{18}$$

Inserting equation (17) into equations (1) and (16) gives, after differentiating (16),

$$\frac{\partial}{\partial x}\,jt = -H\,j\,x \tag{19}$$

$$-B\,\frac{\partial^2}{\partial x^2}\,t^{7/2} = D\,j\,t \tag{20}$$

where

$$H = \frac{10^{-9}\ \Omega^2\ \ell^2}{(1+\xi)(1+\alpha^2)\ R\ T_{ic}} \ ,$$

$$B = 4.34\times10^{26}\,\frac{T_{ic}^{7/2}}{\ell}\,(1+q^{-1}) \quad . \tag{21}$$

$$D = \frac{3}{8}\,\frac{\alpha^2}{1+\alpha^2}\,\Omega\,\ell\,P_c \quad .$$

III. The detailed solution for the energy and hydrostatic equilibrium equations

We must solve equations (19) and (20) subject to the boundary conditions (18) and

$$\begin{aligned} t &= 0 \quad , \quad x = 1 \\ j &= 0 \quad , \quad x = 1\ . \end{aligned} \tag{22}$$

For that we shall make the following substitution

$$\begin{aligned} t &= 1-f \ , \\ j &= 1-g \ , \end{aligned} \tag{23}$$

where $0 \le f \le 1$, $0 \le g \le 1$. Then, to the lowest order in f (and g) and its derivatives, we obtain from equations (19) and (20), respectively,

$$\frac{\partial f}{\partial x} + \frac{\partial g}{\partial x} = H\,x - H\,x\,g \ , \tag{24}$$

$$\frac{7}{2}\frac{B}{D}\frac{\partial^2}{\partial x^2}f = 1 - f - g \quad . \tag{25}$$

Calculating g from equation (25) and inserting into (24), yields

$$\phi\frac{\partial^3}{\partial x^3}f + \phi x\frac{\partial^2}{\partial x^2}f + H x f = 0 \quad , \tag{26}$$

where $\phi = \frac{7}{2}\frac{B}{D}$.

Expanding f in powers of x^2, i.e.,

$$f = \sum_{j=1} b_{2j}\, x^{2j} \quad , \tag{27}$$

($b_0 = 0$ owing to boundary condition) and inserting into (26), we obtain

$$b_4 = -\frac{H\, b_2}{12} \quad ,$$

$$b_6 = \frac{H(\phi H - 1)b_2}{120\phi} \quad , \tag{28}$$

$$b_8 = -\frac{1}{360\phi}\left[\frac{1}{4}H^3\phi - \frac{H^2}{3}\right] b_2$$

and so on. As the b's coefficients are rapidly decreasing function of j, we may write

$$f \approx b_2 x^2 - \frac{H\, b_2}{12}x^4 + \frac{H}{120\phi}(\phi H - 1)\, b_2 x^6 - \frac{1}{360\phi}\left[\frac{1}{4}H^3\phi - \frac{H^2}{3}\right] b_2 x^8 \quad . \tag{29}$$

From the conditions $g=0$, at $x=0$, using eq.(25) we obtain

$$b_2 = \frac{1}{2\phi} \quad . \tag{30}$$

From the boundary conditions $f = g = 1$ at $x = 1$, we obtain, respectively (up to the third order in x^2),

$$480\phi = 120 + 10H - H^2 \pm \{(120 + 10H - H^2)^2 - 960H\}^{1/2} \tag{31}$$

and

$$b_4 = -\frac{1}{6\phi} \quad . \tag{32}$$

Using eqs. (28), (30) and (32) gives $H = 4$, which inserted into (31) results in

$$\phi_\pm \approx \begin{cases} 0.58 \\ 2.9 \times 10^{-2} \end{cases} \quad . \tag{33}$$

Using now equations (13), (21) and (30) we obtain

$$q^{-1} = \frac{21}{4}(1+q^{-1})\frac{A}{B} \quad , \tag{34}$$

where

$$A = \frac{c m_H\, \Omega^2\, \ell\, \xi}{(1+\alpha^2)(1+\xi)} \quad . \tag{35}$$

Specializing for the upper (+) branch in eq.(33) we obtain

$$D = 6B + 5.25A \quad , \tag{36}$$

which is the equation that gives the temperature along r. We shall consider now the two–temperature regime for the unsaturated inverse comptonization in the limits $\xi \ll 1$ and $\xi \gg 1$.

In the $\xi \ll 1$ limit, we obtain using eqs. (2), (3), (4), (10), (11) and (21)

$$B = \frac{3.1 \times 10^{22}\, T_{ic}^{3}}{M_{34}\, r^{3/2}} , \tag{37}$$

$$T_e = 3.2 \times 10^{-4}\, \alpha^2 \frac{M_{34}\, r^{3/2}}{\dot{M}_{17}\, S}\, T_{ic} , \tag{38}$$

$$\xi = 0.32\, T_{ec}^{-7/2} , \tag{39}$$

$$A = \frac{9.8 \times 10^{35} (\dot{M}_{17}\, S)^{7/2}}{\alpha^7 (M_{34}\, r^{3/2})^{9/2}}\, T_{ic}^{-3} , \tag{40}$$

$$D = 2.9 \times 10^{25} \frac{\dot{M}_{17}\, S}{M_{34}^{2}\, r^{3}} . \tag{41}$$

It should be remarked that D, given by (41), is quite general and independent of any particular cooling process.

Inserting equations (37), (38), (39), (40) and (41) into equation (36) gives

$$T_{ic}^{6} - 157 \frac{\dot{M}_{17}\, S}{M_{34}\, r^{3/2}}\, T_{ic}^{3} + 1.4 \times 10^{13} \left[\frac{\dot{M}_{17}\, S}{\alpha^2\, M_{34}\, r^{3/2}} \right]^{7/2} = 0 \tag{42}$$

A rapid inspection in equation (42) reveals the existence of a critical accretion rate given by

$$\dot{M}_{17}^{c} = 3.6 \times 10^{-7}\, \alpha^2 \frac{M_{34}\, r^{3/2}}{S} . \tag{43}$$

For $\xi \ll 1$ and $\dot{M}_{17} \leq \dot{M}_{17}^{c}$ the disk is stationary. For $\dot{M}_{17} \geq \dot{M}_{17}^{c}$, we can not find a solution to equation (42) and the disk is no longer stationary.

For $\xi \gg 1$, we obtain

$$T_e = 0.13 \left[\frac{\alpha^2\, M_{34}\, r^{3/2}\, T_{ci}}{\dot{M}_{17}\, S} \right]^{2/9} , \tag{44}$$

$$\xi = 404 \left[\frac{\dot{M}_{17}\, S}{\alpha^2\, M_{34}\, r^{3/2}\, T_{ci}} \right]^{7/9} , \tag{45}$$

$$A = \frac{3.6 \times 10^{25}}{\alpha^{7/9}} \frac{(\dot{M}_{17}\, S)^{7/18}}{(M_{34}\, r^{3/2})^{25/18}}\, T_{ic}^{1/9} , \tag{46}$$

$$B = \frac{1.5 \times 10^{21}\, \alpha^{7/9}\, T_{ic}^{61/18}}{(M_{34}\, r^{3/2})^{11/18}\, (\dot{M}_{17}\, S)^{7/18}} \tag{47}$$

and

$$3.2 \times 10^{3} \frac{\dot{M}_{17}\, S}{M_{34}^{2}\, r^{3}} = \frac{\alpha^{7/9}\, T_{ic}^{61/18}}{(M_{34}\, r^{3/2})^{11/18}\, (\dot{M}_{17}\, S)^{7/18}} +$$

$$+ \frac{2.1 \times 10^{4}}{\alpha^{7/9}} \frac{(\dot{M}_{17}\, S)^{7/18}}{(M_{34}\, r^{3/2})^{25/18}}\, T_{ic}^{1/9} . \tag{48}$$

Differently from the $\xi \ll 1$ case, there is solution to equation (48) for any accretion rate. It should be remarked, however, that these solutions will be physically reasonable only for $r \gg 1$.

For the pure bremsstrahlung cooling we shall consider only the $\xi \ll 1$ limit. In that case we obtain

$$\xi = \frac{1.14 \times 10^4}{\alpha^4} \left[\frac{\dot{M}_{17}\ S}{M_{34}\ r^{3/2}} \right]^2 T_e^{-5/2} \quad , \tag{49}$$

$$A = \frac{1.5 \times 10^{28}}{\alpha^4} \frac{(\dot{M}_{17}\ S)^2}{(M_{34}\ r^{3/2})^3} T_e^{-2} \quad , \tag{50}$$

$$B = \frac{10^{24}\ T_e^3}{M_{34}\ r^{3/2}} \tag{51}$$

and

$$T_e^5 - 4.8 \frac{\dot{M}_{17}\ S}{M_{34}\ r^{3/2}} T_e^2 + 1.3 \times 10^4 \left[\frac{\dot{M}_{17}\ S}{M_{34}\ r^{3/2}} \right]^2 = 0 \quad . \tag{52}$$

Again, we find a critical accretion rate given by

$$\dot{M}_{17}^c = 2 \times 10^{-12} \frac{\alpha^2\ M_{34}\ r^{3/2}}{S} \quad . \tag{53}$$

For accretion rates greater than that given by (53) there is no solution to equation (52) and the system acquires temporal behavior.

The existence of critical accretion rates for both the inverrse unsaturated comptonization and pure bremsstrahlung models is quite surprising, because using the

α–standard accretion disk theory, we obtain stationary solutions for both models, for any accretion rate. We must, then, conclude that thermal conduction, even not dominating the cooling process, will be responsible for the temporal behavior of the disk, for negligeable radiation presssure.

It is interesting to remark that if we use the usual criterion for the existence of a two–temperature regime, i.e., t_c (cooling time)/t_f (transit time) greater than 1, we would obtain, for $\xi \ll 1$,

$$2.6 \times 10^{-3} \frac{\dot{M}_{17}\ S}{M_{34}^2\ r^{1/2}} T_{ic}^{1/2} < \xi < 1 \quad , \tag{54}$$

which, using eqs. (38) and (39), changes to

$$2.26 \times 10^3 \frac{\dot{M}_{17}\ S}{\alpha^2\ M_{34}^2\ r^{3/2}} < T_{ic} < \frac{3.8 \times 10^3 (\dot{M}_{17}\ S)^{5/8}}{\alpha^{7/4}\ M_{34}^{3/8}\ r^{19/16}} \quad . \tag{55}$$

Clearly, the condition for the existence of a solution to inequality (55) is

$$\dot{M}_{17} < 4\ \alpha^{2/3} \frac{M_{34}^{5/3}\ r^{5/6}}{S} \quad . \tag{56}$$

Comparing this expression with the critical accretion rate, eq.(43), we see that account of thermal conduction greatly reduces the region in $\dot{M}$ space for which there exists stationary solution to the two–temperature soft photon comptonized accretion disk.

IV. The temporal behavior of the two–temperature soft photon comptonized accretion disk: the $\xi \gg 1$ limit

In this section we shall obtain the behavior of the two–temperature soft photon comptonized accretion disk against perturbations in the radial direction. We shall not go into details and, for a better treatment, one is referred to the works of Shakura & Sunyaev 1976, Piran 1978 and Meirelles 1991 (a,d).

If we define the matter surface density U as

$$U = 2\ell \int_0^1 \rho \, dx \quad , \tag{57}$$

the continuity equation will be

$$\frac{\partial U}{\partial t} = \frac{1}{2\pi r} \frac{\partial}{\partial r} \dot{M} \quad , \tag{58}$$

which with the help of equations (3) and (4) changes to

$$\frac{\partial}{\partial t} U = \frac{2}{9r} \frac{\partial}{\partial r} \frac{1}{\Omega r} \frac{\partial}{\partial r} \frac{\alpha^2}{1+\alpha^2} \Omega^2 r^2 U \ell^2 \quad . \tag{59}$$

Integration of the thermal equation (equation (28) in Meirelles 1991(c)) yields the energy equation

$$\frac{1}{4} \frac{\partial}{\partial t} \left[\frac{1 + \beta + \beta \alpha^2}{1 + \alpha^2} \right] U \Omega^2 \ell^2 + \frac{1}{6} U \Omega^2 \ell^2 \frac{\partial}{\partial t} \ell =$$

$$= \frac{2}{9} \frac{\alpha^2}{(1+\alpha^2)^2} \frac{\partial}{\partial r} \left\{ \frac{5 + 3\beta + \alpha^2(3\beta + 2)}{12} \right\} \Omega^2 \ell^2 \frac{\partial}{\partial t} \Omega^2 \ell^2 + \tag{60}$$

$$+ \frac{V_r}{6} \frac{\partial}{\partial r} U \Omega^2 \ell^2 + \frac{\alpha^2}{4(1+\alpha^2)} U \Omega^3 \ell^2 - \frac{c m_H}{\sigma_T} \beta \ell \Omega^2 (1+q) \quad .$$

To obtain eq. (60), we have used for the internal energy density

$$\varepsilon = \frac{3}{2} \left[\frac{1 + \beta + \beta \alpha^2}{1 + \alpha^2} \right] P \quad , \tag{61}$$

and for the ratio of pressures

$$\xi = \frac{\beta \, (1+\alpha^2)}{1-\beta \, (1+\alpha^2)} \quad . \tag{62}$$

To study the temporal behavior of U and ℓ, we write

$$U = U_0(1 + u)$$
$$\ell = \ell_0(1 + h) \tag{63}$$

where the subscript 0 stands for unperturbed values.

Compared to the other terms, the one proportional to V_r is of order superior on ℓ/r and will be neglected.

Linearization of equations (59) and (60), with the help of (63), results respectively in

$$\frac{\partial}{\partial t} u = \frac{2}{9} \alpha^2 \Omega \ell_0^2 \frac{\partial}{\partial r^2} \{u + 2h\} \tag{64}$$

and

$$\frac{1}{4}(1+\beta_0)\, U_0 \Omega^2 \ell_0^2 \frac{\partial}{\partial t}(u+2h) + \frac{1}{4} U_0 \Omega^2 \ell_0^2 \frac{\partial}{\partial t} \beta_1 + \frac{1}{6} U_0 \ell_0^2 \Omega^2 \frac{\partial}{\partial t} h =$$

$$= \frac{2}{9}\alpha^2 \left[\frac{5 + 3\beta_0}{12}\right] \Omega^3 \ell_0^4 U_0 \frac{\partial^2}{\partial r^2}(u+2h) + \frac{\alpha^2}{4} U_0 \ell_0^2 \Omega^3 (u+2h) +$$

$$- \frac{c m_H}{\sigma_T} \beta_0 \ell_0 \Omega^2 - F_c^0 \left[\frac{\delta F_c}{F_c^0} - h\right] , \tag{65}$$

where we have split the variables into unperturbed and perturbed parts, i.e.,

$$\begin{aligned} \beta &= \beta_0 + \beta_1 \\ F_c &= F_c^0 + \delta F_c \\ T_{ic} &= T_{ic}^0 + \delta T_i \\ T_{ec} &= T_{ec}^0 + \delta T_e . \end{aligned} \tag{66}$$

Using equations (2), (10), (11) and (14) we obtain

$$\frac{\beta_1}{\beta_0} = \frac{7}{2} u (1-\beta_0) , \tag{67}$$

$$\frac{\delta F_c}{F_c^0} = \frac{7}{2} \frac{\delta T_i}{T_0} - h . \tag{68}$$

$$\frac{\delta T_i}{T_{ic}^0} = 2h - \frac{7}{2} \beta_0 u . \tag{69}$$

$$\frac{\delta T_e}{T_{ec}^0} = -u . \tag{70}$$

Assuming a temporal behavior of the kind $e^{\omega t}$. we substitute equations (67), (68), (69) and (70) into equation (65) to obtain

$$4(1+q_0)(4+3\beta_0)\,\omega h + (1+q_0)(-4 + 21\beta_0 - 21\beta_0^2)\,\omega u =$$

$$= 3\alpha^2 \Omega \left\{u\left(-5 + 2q_0 + 7\beta_0 + 49 q_0 \beta_0\right) + 2h(1-4q_0)\right\} . \tag{71}$$

We now look for a solution, for both u and h, of the type $\sin r/\lambda$, where λ is the perturbation wave length. Making the substitution $\psi = u + 2h$, we obtain from (64)

$$u = -\frac{2}{9} \frac{\alpha^2 \Omega}{\omega} \left[\frac{\ell_0}{\lambda}\right]^2 \psi , \tag{72}$$

$$h = \frac{\psi}{2}\left[1 + \frac{2}{9} \frac{\alpha^2 \Omega}{\omega} \left[\frac{\ell_0}{\lambda}\right]^2\right] . \tag{73}$$

Substituting back into equation (71) results in the following dispersion relation

$$6(1+q_0)\,(4+3\beta_0)\,\omega^2 + \alpha^2\,\Omega\,\omega\left\{9(4q_0-1) + \left[\frac{\ell_0}{\lambda}\right]^2 (8 - 10\beta_0 + 14\beta_0^2)\right\} +$$

$$+\left[\frac{\alpha^2\,\Omega\,\ell_0}{\lambda}\right]^2 \left\{-6 + 6q_0 + 7\beta_0 + 49q_0\,\beta_0\right\} = 0 \quad . \tag{74}$$

We see from (74) that stability of the disk depends heavily on the value of q_0. Using equations (33) and (34) we obtain for the upper branch

$$q_0 = \frac{8}{7}\,\frac{B}{A} \tag{75}$$

and the stability conditions will be

$$32B \geq 7A \quad , \tag{76}$$

$$(-42 + 49\beta_0)A + 8(6 + 49\beta_0)B \geq 0 \quad . \tag{77}$$

As we are considering the case $\xi \gg 1$, we must have $\beta_0 \approx 1$. Therefore, in that limit the disk will be stable as long as inequality (76) is satisfied. Using eqs. (46) and (47), we see that will occur whenever

$$T_{ic} \geq 2M_{17}^{18/61} \tag{78}$$

or

$$r \geq \frac{28.8}{\alpha^{4/3}}\,\dot{M}_{17}^{-1}\,M_{34}^{-2/3} \quad . \tag{79}$$

The stability conditions we just met are valid for the upper branch. For the lower branch we should remind that equations (36) and (75) change respectively to

$$D = 121B + 5.25A \tag{80}$$

and

$$q_0 = 23\,\frac{B}{A} \quad . \tag{81}$$

Therefore, the stability conditions will be

$$92B \geq A \tag{82}$$

and inequality (77).

Proceeding in the some way as we done before, we obtain

$$T_{ic} \geq 1.3\,\dot{M}_{17}^{18/61} \tag{83}$$

or

$$r \geq \frac{12.8}{\alpha^{4/3}}\,\dot{M}_{17}^{-1}\,M_{34}^{-2/3} \quad . \tag{84}$$

We now apply these results to Cygnus X–1. From Liang and Nolan 1984, $M_{39} \approx 3$, $\dot{M}_{17} \approx 0.6$, $T_e \approx 1\text{–}2$. As we are assuming magnetic fields as the source of viscosity, for consistency reasons (Meirelles 1991(a)) we must have $\alpha^2 \ll 1$, say $\alpha^2 \approx 0.1$. We then conclude that the disk will stable for $r \geq 6.4$ (2.9 for the lower branch) and thermally unstable inside this region. It should be noticed that as the temperature grows with r, in that model the X–ray are produced far from the hole. One interesting consequence of that is possible shorter variability for photons in less energetic parts of the spectrum. Besides this, as the upper and lower branches do not differ much in energy, the system may not tune correctly and spend some time in one or another. This possibility would explain the states of high and low luminosity of Cygnus X–1.

V. Conclusions

The results we have obtained here do improve ealier results obtained for the conductive energy transport in accretion disks with isothermal atmosphere (in z–direction) (Meirelles 1991(b)).

One of the main differences is related to the amount of energy transported by conduction. In the isothermal (z) atmosphere conductions is by far ($q \sim 10^4$) the dominant transport process. Here, it may dominate only in the outer parts of the inner region. Another point that deserves some comments is the absence of solution for the two–temperature soft photon comptonized accretion disk for canonical values of the accretion rates in the case of X–rays binaries, for negligeable radiation pressures.

It is worth to remark that conduction, even to being the dominant energy transport mechanism, makes narrower the region in $\dot{M}$–space for which there exists a stationary solution for both the two–temperature and pure bremsstrahlung accretion disks.

For systems with dominant radiation pressure we have found that the solution is double–valued, slightly different temperatures and strongly different ratio conductive–radiative transport.

Applying these results to Cygnus X–1, we see that the model decouples in a very sensible way the region of maximum energy output (the innermost one) from the region of X–ray emission (outermost). This may have observational consequences in terms of variability. This model may explain the states of high and low luminosity of Cygnus X–1.

According to this model Cygnus X–1 would be stable fo $r \geq 6.4$ (2.9 for the lower branch) and thermally unstable inside this radius.

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