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SPHERICAL AND DEFORMED TARGETS

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NEAR-BARRIER FUSION OF  $^{11}\text{Li}$  WITH  
HEAVY SPHERICAL AND DEFORMED TARGETS\*

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Abstract

The cross-sections for the fusion of  $^{11}\text{Li}$  with  $^{208}\text{Pb}$  and  $^{238}\text{U}$  are calculated at near-barrier energies. The coupling of the entrance channel to the soft giant dipole resonance in  $^{11}\text{Li}$  is taken into account together with the coupling to the break-up channel  $^9\text{Li} + 2n$ . The deformation of  $^{238}\text{U}$  is also considered. The cross-section is found to exhibit important structure around the barrier.

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Recently, the low energy fusion of radioactive beams, such as  $^{11}\text{Li}$ , with heavy target nuclei has been discussed<sup>1-4</sup>). The principal motivation is twofold: i) the enhancement of the fusion cross-section  $\sigma_f$  that arises from the existence of the halo neutrons can be used to further understand these exotic nuclei, ii) the potential production of superheavy cold compound nuclei, with a reasonably measurable cross-sections.

In the calculations made so far, two features of the halo are taken into account: the lowering of the static Coulomb barrier and the coupling of the entrance channel to the low lying soft giant dipole resonance (the pygmy resonance). Both of these effects lead to an enhanced fusion cross-section.

The existence of the pygmy resonance at about 1.2 MeV has recently been firmly established through the study of the double charge exchange reaction  $^{11}\text{B}(\pi^-, \pi^+)^{11}\text{Li}$ <sup>5</sup>). One anticipates, on general ground, that this state has a large width due to the very low binding energy of the dineutron ( $\sim 0.2$  MeV). Thus it is of great importance in any fusion calculation to consider the finite lifetime of the pygmy resonance. This leads to an enhancement of  $\sigma_f$  reduced with respect to that already reported. The purpose of this letter is to consider the above effect by coupling the pygmy resonance to the break-up channel in the calculation of the fusion cross-section.

In a coupled channel description of a heavy-ion reaction, the fusion cross-section can be calculated from the total reaction cross-section as<sup>6</sup>)

$$\sigma_f = \sigma_R - \sigma_D \quad (1)$$

where  $\sigma_D$  is the direct reaction cross-section and  $\sigma_R$  is given by

$$\sigma_R = \frac{k}{E} \langle \Psi_{\mathbf{k}}^{(+)} | -\text{Im} V | \Psi_{\mathbf{k}}^{(+)} \rangle \quad (2)$$

where  $\langle \mathbf{r} | \psi_{\mathbf{k}}^{(+)} \rangle$  is the wave function that describes the elastic scattering and  $V$  is the optical potential that generate  $\langle \mathbf{r} | \Psi_{\mathbf{k}}^{(+)} \rangle$ . It can be shown<sup>6</sup>) that the cross-section  $\sigma_f$  (Eq.(1)) can be written in the form

$$\sigma_f = \frac{k}{E} \sum_i \langle \Psi_{\mathbf{k}_i}^{(+)} | -\text{Im} \hat{V}_i | \Psi_{\mathbf{k}_i}^{(+)} \rangle \quad (3)$$

where  $\hat{V}_i$  is the bare optical potential in channel  $i$  (no channel coupling) and  $|\Psi_{\mathbf{k}_i}^{(+)}\rangle$  is the exact scattering wave function in that channel. Eq. (3) has been used by several authors to calculate  $\sigma_f$  using coupled channels codes<sup>7</sup>). Other models based on this equation but with the further assumption of infinite absorption once the barrier is penetrated have also been developed<sup>8</sup>). Here we generalize the second class of models by incorporating the effect of the break-up channel (included in  $|\Psi_{\mathbf{k}_i}^{(+)}\rangle$ ).

To be more specific, we deal here with a case involving the coupling of the elastic channel to a resonant state in the projectile. If this resonant state is approximated by an excited state whose width is very small, then  $\sigma_f$  can be written as (ignoring the excitation energy of the state)<sup>8</sup>)

$$\sigma_f = \frac{1}{2} \left[ \hat{\sigma}_f(+F) + \hat{\sigma}_f(-F) \right] \quad (4)$$

where  $\hat{\sigma}_f$  is the one-channel fusion cross-sections and  $F$  is the channel coupling potential evaluated at the barrier radius. The  $+/-$  sign indicates addition/subtraction to/from the barrier height.

To include the break-up channel coupling effect, in Eq. (4), namely the non-zero width of the excited state, it is convenient first to express the cross-section as a sum of partial wave contributions

$$\sigma_f = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) T_{\ell}^f \quad (5)$$

$$T_{\ell}^f = \left[ 1 + \exp \left[ \frac{2\pi}{\hbar\omega} \left[ V_B + \frac{\hbar^2 \ell(\ell+1)}{2\mu R_B^2} - E_{c.m.} \right] \right] \right]^{-1}$$

In the above  $R_B$  and  $V_B$  are the Coulomb barrier radius and height, respectively. When incorporating the break-up channel coupling effect, the partial fusion probability,  $T_{\ell}^f$ , has to be multiplied by the break-up survival probability,  $(1 - T_{\ell}^{bu})$ . Thus

$$\sigma_f^{ob} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) (1 - T_{\ell}^{bu}) T_{\ell}^f \quad (6)$$

And therefore we have finally

$$\begin{aligned} \sigma_f &= \frac{1}{2} \frac{\pi}{k^2} \left[ \sum_{\ell=0}^{\infty} (2\ell+1) (1 - T_{\ell}^{bu}) T_{\ell}^f(+F) + \sum_{\ell=0}^{\infty} (2\ell+1) (1 - T_{\ell}^{bu}) T_{\ell}^f(-F) \right] = \\ &= \frac{1}{2} \left[ \sigma_f^{ob}(+F) + \sigma_f^{ob}(-F) \right] \quad (7) \end{aligned}$$

Here the Coulomb break-up does not contribute since it is significant only at  $\ell$  larger than those for which fusion is relevant. The nuclear break-up transmission factor,  $T_{\ell}^{bu}$ ,

has been recently calculated for several radioactive systems<sup>9,10</sup>. The major conclusion of these studies is that the dynamic polarization potential which enters in the evaluation of  $T_{\ell}^{bu}$  via

$$T_{\ell}^{bu} = 1 - \exp \left[ -2 \int_{\rho_0}^{\infty} \frac{\text{Im } V_{pol}/E_{c.m.}}{\sqrt{1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2}}} d\rho \right] \quad (8)$$

is very sensitive to the binding energy of the break-up cluster. In Eq. (8)  $\eta$  is the Sommerfeld parameter and  $\rho_0$  is the distance of closest approach, multiplied by the wave number  $k$ , obtained from  $1 - 2\eta/\rho_0 - \ell(\ell+1)/\rho_0^2 = 0$ . A closed form expression for  $T_{\ell}^{bu}$  was derived in Ref. 10 and it reads

$$T_{\ell}^{bu} = 1 - \exp \left[ -\frac{4\mathcal{F}_0^2}{E^2} \left| S_{\ell}^{(1)} \right| I_{\ell}^2(\eta, s) \right] \quad (9)$$

where  $\mathcal{F}_0$  is a coupling strength factor, which was found to be 4.859 MeV for  $^{11}\text{Li} + ^{208}\text{Pb}$ .  $|S_{\ell}^{(1)}|$  is the modulus of the elastic S-matrix in the break-up channel and  $I_{\ell}(\eta, s)$  is a Coulomb radial integral evaluated and discussed in Ref. 10. The sensitivity of  $V_{pol}$  and  $T_{\ell}^{bu}$  to the binding energy of the dineutron in  $^{11}\text{Li}$  resides in the  $\ell$ -dependence of  $I_{\ell}(\eta, s)$ .

In the following, we use the fusion calculation of Takigawa and Sagawa<sup>4</sup> as a background for the study of the effect of the coupling to the break-up channel. We took the height of the Coulomb barrier ( $V_B = 26$  MeV) its radius ( $R_B = 11.1$  fm) and curvature ( $\hbar\omega = 3$  MeV) from Figure 1 of Ref. 4 and used these in our Hill-Wheeler transmission coefficients of Eq. (5). The strength  $F$  was adjusted to reproduce the values

of  $\sigma_f$  of Ref. 4. We found  $F \sim +3.0$  MeV. The break-up effect was then investigated, through the modified fusion cross-section, Eq. (7), with the help of Eq. (9) and taking  $|S_\ell^{(1)}| = [1 - T_\ell^f(E_{c.m.} - 0.2)]^{1/2}$ . The result of our calculation is shown in Figure 1. It is clear that the inclusion of the break-up coupling and thus the life-time of the pygmy resonance, reduces the enhancement of  $\sigma_f$  by as much as a factor of 100 at energies slightly below the barrier. More important is the fact that the break-up of the projectile renders the fusion cross-section *lower* than the one-dimensional calculation at energies extending from above the barrier ( $26 \text{ MeV} < E_{c.m.} < 45 \text{ MeV}$ ) to slightly below the barrier ( $24 \text{ MeV} < E_{c.m.} < 26 \text{ MeV}$ ). At energies less than 24 MeV the enhancement sets in. The increase of the enhancement with increasing  $E_{c.m.}^{-1}$  is, however, much slower than the case without break-up, Eq. (4). Only at energies  $E_{c.m.} \leq 10 \text{ MeV}$ , does the break-up effect subside completely, letting the pygmy resonance acts as a complete vibrational enhancer.

Figure 2 exhibits more clearly the above features through the behaviour of the enhancement factor,  $\epsilon$ , defined as the ratio of the fusion cross-section to the one-dimensional barrier penetration cross-section. The break-up effect is contained in the interval  $10 \text{ MeV} < E_{c.m.} < 45 \text{ MeV}$ . Further there is a sharp dip at the barrier. This dip is easily understood. At energies above the barrier, the nuclear break-up process inhibits fusion. This inhibition becomes less effective as the energy approaches the barrier, which acts as a natural threshold. At sub-barriers energies, the increase in the distance of closest approach leads to a further reduction in the break-up effects. These striking features arising from the halo should be easy to verify experimentally. The saturation value of  $\epsilon$  is 250 and it represents simply the value of  $1/2 \exp(2\pi/\hbar\omega F)$  (see Eq. (4)), which is attained at much lower energies ( $\sim 10 \text{ MeV}$ ) than predicted by Takigawa and Sagawa<sup>4</sup>.

We have repeated the above calculation for the deformed nucleus  $^{238}\text{U}$ . Here the enhancement of  $\sigma_f$  arises both from the coupling to the pygmy resonance of the projectile

and the coupling to the states of the target rotor. Taking only the  $0^+$  and  $2^+$  states of  $^{238}\text{U}$  into consideration, the fusion formula reads<sup>3</sup>) (the sudden limit is assumed)

$$\begin{aligned} \sigma_f &= \frac{1}{2} \left\{ 0.562 \left[ \overset{\circ}{\sigma}_f (F + 0.73 \beta_2 f(R_B)) + \overset{\circ}{\sigma}_f (-F + 0.73 \beta_2 f(R_B)) \right] + \right. \\ &= \left. + 0.348 \left[ \overset{\circ}{\sigma}_f (F - 1.37 \beta_2 f(R_B)) + \overset{\circ}{\sigma}_f (-F - 1.37 \beta_2 f(R_B)) \right] \right\} \end{aligned} \quad (10)$$

where  $f(R_B)$  is the rotational coupling form factor given approximately by

$$f(R_B) = \frac{1}{\sqrt{4\pi}} V_B \frac{R_2}{R_B} \left[ 1 - \frac{3}{5} \frac{R_2}{R_B} \right] \quad (11)$$

In Eqs. (10) and (11)  $R_2$  is the radius of  $^{238}\text{U}$  (7.4 fm) and  $\beta_2$  is the deformation parameter ( $\beta_2 \approx 0.27$ ), and we estimate  $V_B$  to be about 29 MeV, so that  $f(R_B) = 3.3 \text{ MeV}$ .

In Figure 3 we present the result of our calculation of  $\sigma_f$ , according to Eq. (5) (one-dimensional barrier penetration model), Eq. (10) (pygmy resonance vibration and target rotation coupling model) and with the inclusion of the break-up survival probability in Eq. (10), obtained by replacing  $\overset{\circ}{\sigma}_f$  by  $\overset{\circ b}{\sigma}_f$  (full curve). We find here a fusion behaviour similar to that of the  $^{11}\text{Li} + ^{208}\text{Pb}$  system except that the enhancement is a factor of 11 larger. The corresponding enhancement factors are shown in Figure 4, which shows very similar behaviour as Figure 2. The saturation value of  $\epsilon$  is 1000, which is attained at about  $E_{c.m.} = 13 \text{ MeV}$ . Again one sees the sharp dip of  $\epsilon$  at the barrier (2.9 MeV).

In conclusion, we have calculated the influence of the non-zero width of the pygmy resonance on the fusion of  $^{11}\text{Li}$  with heavy spherical and deformed nuclei at close-to-barrier energies. This is accomplished by taking into account in the multi-dimensional fusion calculation, the effect of the break-up channel  $^{11}\text{Li} \rightarrow ^9\text{Li} + 2n$ .

The usual vibrational and vibrational + target rotational enhancement of the sub-barrier fusion cross-section is appreciably reduced. Further, the enhancement factor  $\epsilon$  is found to exhibit non-trivial structure around the barrier. This is clearly related to the halo neutrons in  $^{11}\text{Li}$  and should be easily verified experimentally.

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## Figure Captions

Figure 1. Excitation function for the fusion cross-section of  $^{11}\text{Li} + ^{208}\text{Pb}$ . The dotted curve is the one-dimensional Hill-Wheeler cross-section (Eq. (5)), the dashed curve is the pygmy resonance enhanced cross-section (Eq. (4)) and the full curve represents the result with inclusion of the break-up coupling (Eq. (7)) (see text for details).

Figure 2. The enhancement factor  $\epsilon$  vs.  $E_{\text{c.m.}}$  for  $^{11}\text{Li} + ^{208}\text{Pb}$ . The dashed curve is Eq. (6)/Eq. (5) while the full curve represents Eq. (7)/Eq. (4).

Figure 3. Same as Fig. 1 for  $^{11}\text{Li} + ^{238}\text{U}$  (dotted curve). The dashed curve is the pygmy resonance + target rotation enhanced cross-section (Eq. (10)). The full curve includes the effect of the  $^{11}\text{Li}$  break-up on Eq. (10).

Figure 4. The enhancement factor  $\epsilon$  vs.  $E_{\text{c.m.}}$  for  $^{11}\text{Li} + ^{238}\text{U}$ . The dashed curve is Eq. (10)/Eq. (4) while the full curve includes the break-up effect. See text for details.



