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CHAOS AND ORDER IN THE ATOMIC NUCLEUS: THE CASE OF A DEFORMED GAUSSIAN ORTHOGONAL ENSEMBLE

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CHAOS AND ORDER IN THE ATOMIC NUCLEUS: THE CASE OF A DEFORMED GAUSSIAN ORTHOGONAL ENSEMBLE*

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I. Introduction

In recent years, the debate conserving the transition between chaos and order que quantum level has been intensive. The Wigner level repulsion, a major ingredient for chaotically behaved quantum systems has attracted a great amount of discussion as to the way it behaves as the system becomes gradually more ordered. The Wigner Surmise (WS) asserts that the slope of the varition of the probability distribution of the level spacing is positive linear in the level spacings, s , at small s . If the system is made less chaotic (mixing levels with different quantum numbers), how would the above general criterion of the Wigner chaos change? Berry and Robnik[1], have adressed this question, within a semiclassically motivated modification of the WS. Their results, however, were shown to be erred, in the sense that they predicted that the WS–based distributions loose level repulsion very gradually. The true distribution, briefly discussed by Robnik[2], exhibits level repulsion all the way until a sharp Wigner–Poisson transition occurs.

Recently, we[3] have derived a new distribution law for the level and amplitude statistics. The result of our work, based on a conveniently deformed GOE corroborates the conclusion of Robnik. Numerically, our results are close to the empirical Brody distribution[4]. We have applied our theory to 2×2 and 3×3 matrices. Further, a new fluctuation cross section for nuclear reactions intermediate between compound (GOE) and direct is also derived.

In the following, the transparencies of my talk at the Workshop on Nonlinear Dynamics in Nuclear and Accelerator Physics, held at MSU, October 27, 1991 are reproduced. Further details can be found in the references.

References

1) M.V. Berry and M. Robnik, *J. Phys.* A17 (1984) 2413.

2) M. Robnik, *J. Phys.* A20 (1987) 1495.

3) C.E. Carneiro, M.S. Hussein and M.P. Pato, Deformed GOE for the Description of Systems Intermediate Between Chaos and Order, to appear in the proceedings of the Miniworkshop on Quantum Chaos, Trieste, June 1990; M.S. Hussein and M.P. Pato, IFUSP/p–831.

4) T. Brody et al., *Rev. Mod. Phys.* 53 (1981) 385.

Deformed GOE For The Description of Systems Intermediate Between Chaos and Order

Or:

How to describe the statistical Behaviour of a Complex Quantum System

M. P. Pato and M. S. H.

M. S. Hussein
Univ. São Paulo

Applications

Nuclear Physics

Structure : H (Hamiltonian)

Reactions : $S(H)$ (S-matrix)

Solid State Physics

Disordered Metals : Transfer Matrix $R(H)$

Use The GOE for H

A convenient and elegant way to derive the GOE distributions is with the use of the max. entropy principle.

Maximize

$$S = -\int dH \; P(H) \ln P(H)$$

Subject to the constraints

1) $\langle \mathrm{tr}\, H^2 \rangle = \int dH \; P(H) \, \mathrm{Tr}(H^2) \equiv \mu$

2) $\langle 1 \rangle = \int dH \; P(H) = 1$

$$\delta \left[S - \alpha \langle \mathrm{tr}\, H^2 \rangle - \lambda \langle 1 \rangle \right] = 0$$

Get

$$P(H) = \frac{1}{2^{N/2}} \left(\frac{\pi}{2\alpha} \right)^{-N(N+1)/4} \exp\left[-\alpha \, \mathrm{Tr}\, H^2 \right]$$

The GOE !

If $\quad a_k = \langle i | E_k \rangle \quad ; \; k = 1, 2, \cdots, N$

$$\mathrm{Tr}\, H^2 = \sum_{i=1}^{N} E_i^2$$

The GOE distribution becomes

$$P_{GOE}(E's\, ; a's) = \underbrace{2\,\frac{\Gamma\left(\frac{N}{2}+1\right)}{\pi^{N/2}}\,\delta\left(1-\sum_i a_i^2\right)}_{P(a's)} \;\underbrace{\mathcal{N}_N \prod_{j>i}^{N} |E_j - E_i|\, e^{-\alpha\left(E_1^2+E_2^2+\cdots E_N^2\right)}}_{P(E,s)}$$

normalization

Integrating over all the a's and all the spacings <u>except one</u>, one gets

$$P_{GOE}(s) \simeq \frac{\pi}{2D^2}\cdot s \cdot e^{-\frac{\pi^2}{4}\frac{s^2}{D}}$$

$$D = \langle s \rangle = \sqrt{\frac{8\pi}{\alpha}}$$

Wigner's

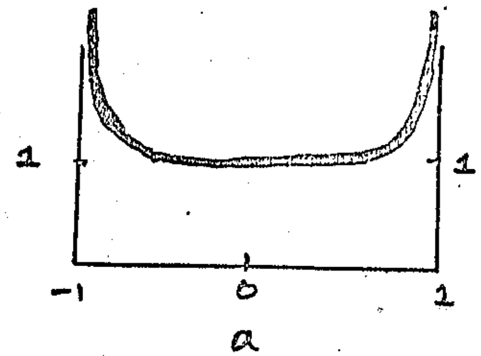
2) Amplitude distribution

$$|E_k\rangle = \sum_i a_i^k\, |i\rangle$$

basis

For 2×2 matrices

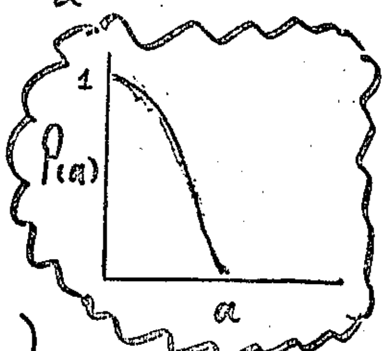
$$P(a) = \frac{1}{\sqrt{1-a^2}}$$



For $N\times N$, $N \to \infty$

$$P(a) = \left(\frac{N}{2\pi}\right)^{1/2} \exp\left[-\frac{Na^2}{2}\right]$$

Porter-Thomas



For $a^2 \quad \left(|\langle a | V | c\rangle|^2\right)$

$$P(a^2) = \left(2\pi\, a^2 \langle a^2\rangle\right)^{-1/2} e^{-\frac{a^2}{2\langle a^2\rangle}}$$

Generalization To Intermediate Cases (Chaos – Order)

– Study of symmetry breaking, deviations from HF etc.

Use another (or other) constraints

$$\langle \mathrm{tr}\, PH\underbrace{(1-P)}_{Q}HP \rangle = \nu$$

$$p = |i\rangle\langle i|$$

$$\Rightarrow \quad \langle \sum_{j\neq i} H_{ij}^2 \rangle = \nu$$

The NEW distribution becomes

$$P(H) = P_{GOE}(H)\, e^{-\beta\, \mathrm{tr}(PHQHP)} \left[1+\frac{\beta}{2\alpha}\right]^{(N-1)/2}$$

Mixes the a and E distributions

This new constraint can be understood by looking at the second moment

In GOE

$$\langle H_{ij} H_{i'j'} \rangle = \left(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{ji'}\right)\frac{1}{\alpha}$$

no dependence on label

In DGOE, the variance depends on the label (state)

Eg 2×2

$$\langle H_{11}^2 \rangle = \frac{2}{\alpha}, \quad \langle H_{22}^2 \rangle = \frac{2}{\alpha}$$

$$\langle H_{12}^2 \rangle = \frac{1}{\alpha+\beta}$$

increasing β makes $\langle H_{21}^2 \rangle$ smaller.

Secondly, the joint distribution $P(a, s)$ is factorized in GOE and it is NOT in DGOE

The spacings and amplitude distribution

1) for 2x2 matrices

$$P(s) = \alpha \left(1+\frac{\beta}{2\alpha}\right)^{1/2} s\, e^{-\left(\frac{\alpha}{2}+\frac{\beta}{8}\right)s^2} \cdot I_0\left(\frac{\beta}{8} s^2\right)$$

Bessel function

For $s \to 0$

$P(s) \to \left(1+\frac{\beta}{2\alpha}\right)^{1/2} \alpha s$

Repulsion

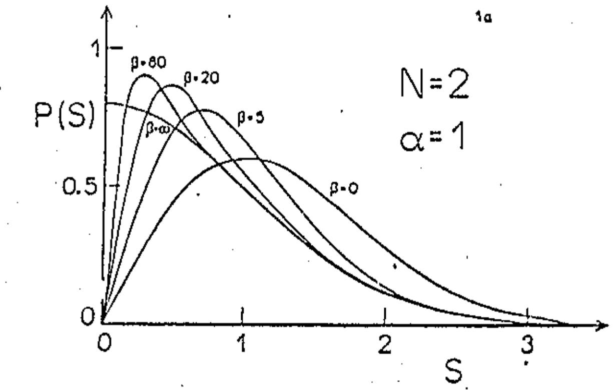
Slope changes

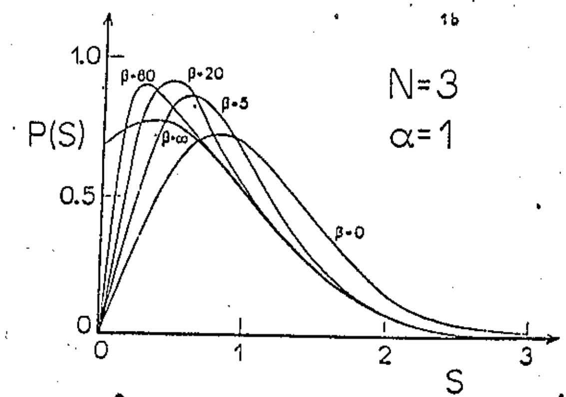
$$P(a) = \frac{\alpha}{2\pi}\left[1+\frac{\beta}{2\alpha}\right]^{1/2} \frac{1}{\sqrt{1-a^2}} \frac{1}{\left[\frac{\alpha}{2}+\beta a^2(1-a^2)\right]}$$

If $\beta \longrightarrow \infty$

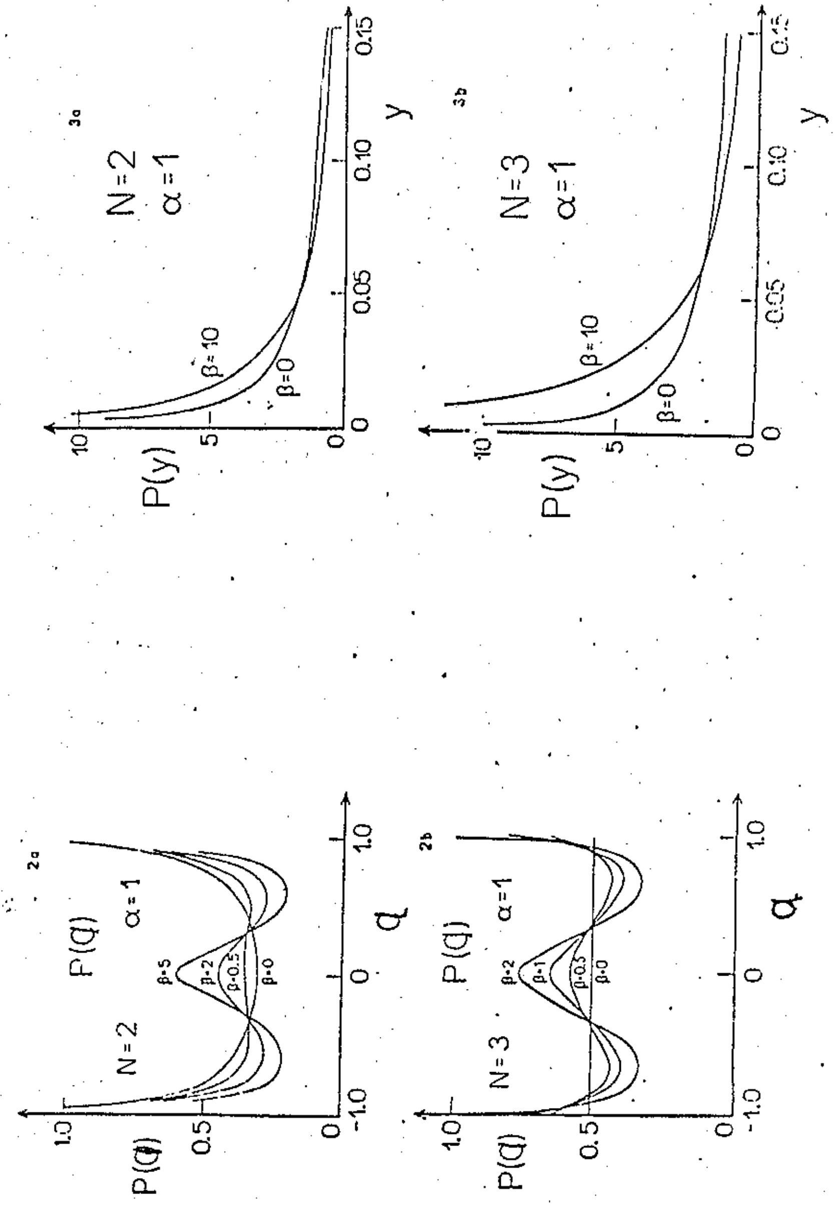
$$P(s) = \frac{2\alpha}{\pi} e^{-\frac{\alpha s^2}{2}} \quad \text{—— Poisson}$$

$$P(a) = \frac{1}{2}\left[\delta(a) + \delta(a-1) + \delta(a+1)\right]$$





[In going from Wigner's to normal (by increasing β) the transition is gradual as is always accompanied by level repulsion]



$Y = \alpha^2$

Statistical Nuclear Reactions

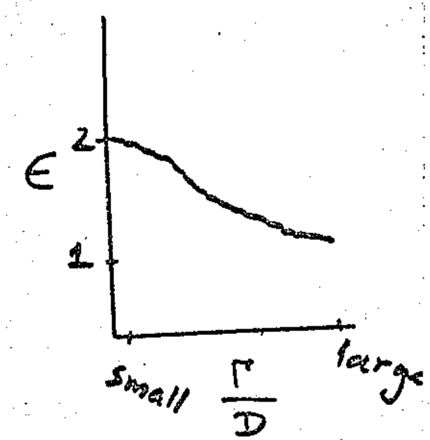
Hauser-Feshbach

$$\overline{\sigma}_{ab} = \xi_a \, \xi_b \left(\epsilon \, \delta_{ab} + 1\right)$$

ϵ : elastic enhancement factor

$$\xi_a = \left(\frac{T_a}{\sum_c T_c} \right)^{1/2}$$

Bohr's hypothesis
(Cons. of GOE ?)



Deviations:

$$S_{ab} = \delta_{ab} - 2\pi i \, W_{a\mu} G_{\mu\nu} W_{b\nu}$$

$$G_{\mu\nu} = \left(E - \underset{\text{GOE}}{H} - i\pi W^{+} W \right)^{-1}_{\mu\nu}$$

Calculate

$|S_{ab}|^2$

For GOE Get ~HF

For deformed GOE

should be testable

$$|S_{ab}|^2 \sim \frac{T_a \, T_b}{\sum_c T_c} f(\beta, T_a, T_b)$$