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THE BACKGROUND FIELD METHOD AND THE NON-RENORMALIZABILITY OF THE NON-LINEAR SIGMA MODEL IN THREE DIMENSIONS

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The Background Field Method and the non-renormalizability of the non-linear Sigma Model in Three dimensions.

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We show that 2+1 dimensional bosonic and supersymmetric non-linear sigma models with an arbitrary Riemannian manifold as target space are non-renormalizable. The perturbative calculations of the counterterms through two loop order are worked out using the background field method and the normal coordinate expansion.

I. INTRODUCTION

Non linear sigma models have originally been studied in the context of current algebra[1], but later, they have been proved an excelent laboratory, since in two dimensional space time they are similar to four dimensional Yang-Mills theory[2]. Afterwards, sigma models have been extensively used to obtain important informations from string theory[3]. In this last case, with the background field method[4] one is able to obtain the counterterms as functions of the gravitational fields. Conformal invariance restraints these counterterms to zero, definning quantum corrections to the Einstein equations[5]. It is natural to extend these methods to three dimensional space time for several reasons. First, they are models for membranes[6], in the same sense as the two dimensional cases are models for strings. They have been used to study fermi boson transmutation and superconductivity[7]. It is weel known that renormalizability may be achieved in the large N expansion for $O(N)$ and $SU(N)$ invariant models[8]. Our work deals with the question of whether sigma models in three dimensional space time may be perturbatively renormalizable or not. We shall consider one and two loop counterterms for the sigma model and supersymmetric extension, using normal coordinates[9, 4].

II. THE BOSONIC NON LINEAR SIGMA MODEL.

The purely bosonic non linear sigma model is defined by the Lagrangian

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j \tag{1}$$

The background field method using normal coordinates applied to this model is well known[4]; using the background quantum splitting $\phi = \varphi + \pi$, where φ is the classical background and π the quantum field, expanded in terms of normal coordinates ξ[4] the following expansion is already standard

$$
\begin{aligned}
S &= S^{(0)}[\varphi] + S^{(2)}[\varphi] + S^{(3)}[\varphi] + S^{(4)}[\varphi] \cdots \\
S^{(2)} &= \frac{1}{2}\int d^3x \Big\{ \Big[R_{iabj}(\varphi)\partial_\mu\varphi^i\partial^\mu\varphi^j + \frac{\partial S}{\partial\varphi^i}\Gamma^i_{ab} \Big]\xi^a\xi^b - \\
&\quad - A^{ab}_\mu \xi^a \partial_\mu \xi^b + A^{ab}_\mu A^{\mu ac}\xi^b\xi^c \Big\} \\
S^{(3)} &= \frac{1}{2}\int d^3x \Big\{ \Big[\frac{1}{3} D_a R_{ibcj}\partial_\mu\varphi^i\partial^\mu\varphi^j + \frac{4}{3} R_{iabd} A^{dc}_\mu \partial^\mu\partial^i \Big]\xi^a\xi^b\xi^c + \\
&\quad + \frac{4}{3} R_{iabc}\partial_\mu\varphi^i\xi^a\xi^b\partial^\mu\xi^c \Big\} \\
S^{(4)} &= \frac{1}{2}\int d^3x \Big\{ \Big[\frac{1}{3} D_a R_{ibce} A^{ed}_\mu \partial^\mu\varphi^i + \frac{1}{3} R_{eabf} A^{ec}_\mu A^{\mu fd} + \\
&\quad + \frac{1}{12} D_a D_b R_{icdj}\partial_\mu\varphi^i\partial^\mu\varphi^j + \frac{1}{3} R^m_{abi} R_{mcdj}\partial_\mu\varphi^i\partial^\mu\varphi^j \Big]\xi^a\xi^b\xi^c\xi^d + \\
&\quad + \frac{1}{2} D_a R_{ibcd}\partial_\mu\varphi^i\xi^a\xi^b\xi^c\partial^\mu\xi^d + \frac{1}{3} R_{cabe} A^{ed}_\mu \xi^a\xi^b\partial^\mu\xi^c\xi^d + \\
&\quad \frac{1}{3} R_{cabd} A^{cc}_\mu \xi^a\xi^b\xi^c\partial^\mu\xi^d + \frac{1}{3} R_{cabd}\xi^a\xi^b\partial_\mu\xi^c\partial^\mu\xi^d \Big\}
\end{aligned} \tag{2}
$$

We should note that naively, the theory defined in equation is perturbatively non renormalizable; therefore we must allow terms of higher dimensions, which are discarded in the two dimensional case[4]. In the above equation, we define

$$
\begin{aligned}
A^{ab}_\mu &= \omega^{ab}_i \partial_\mu \varphi^i \\
(D_\mu \xi)^a &= \partial_\mu \xi^a + \omega^{ab}_i \partial_\mu \varphi^i \xi^b
\end{aligned} \tag{3}
$$

and ω^{ab}_i is the spin connection, defined by

$$
\begin{aligned}
e^a_{j;i} &= \partial_i e^a{}_j + \omega^{ab}_i(e) e_{bj} - \Gamma^k_{ji} e^a{}_k = 0 \\
\omega^{ab}_i &= -e^{bj}\nabla_i e^a{}_j = -e^{bj}\partial_i e^a{}_j + e^{bj}\Gamma^k_{ij} e^a{}_k
\end{aligned} \tag{4}
$$

At the one loop level, we have contributions from $S^{(2)}[\varphi]$. The relevant diagrams are displayed in figure (1).

We do not use dimensional regularization, since in odd dimensional space time it is in fact a renormalization prescription, which deletes all divergent contributions

automatically, rendering the theory finite. Thus, if we wish to study the regularization effects in detail, we must make the counterterm structure explicit. We therefore choose a Pauli-Villars regularization (subtracting the infinities with the use of a regulator mass). The one loop result sumarized in figure (1) gives the result

$$
\delta S^{(1)} = \frac{\Lambda}{4\pi^2}\int d^3x C_{aa} \tag{5}
$$

This counterterm may be absorbed in a redefinition of the metric, as

$$
g^R_{ij} = g_{ij} + \frac{\Lambda}{4\pi^2} R_{ij} \tag{6}
$$

At the two loop order we have several contributions. The diagrams are displayed in figure (2). There we see the following results. For the first diagram, we have the square of a one loop diagram, which is easily computable. The result is

$$
\begin{aligned}
\delta\mathcal{L}^{(2a)} &= \frac{\Lambda}{4\pi^4}\Big[\frac{1}{4}[D_e R_{ic} - 3D_c R_{ie}]\omega^{ec}_j - \frac{1}{6} R_{ef}\omega^{ec}_i\omega^{fc}_j + \frac{1}{3} R_{e(ab)f}\omega^{eb}_i\omega^{fa}_j + \\
&\quad + \frac{1}{12} D^a D_i R_{aj} - \frac{1}{8} D_a D^a R_{ij} + \frac{1}{4} R_{iabc} R_j{}^{abc} + \frac{1}{6} R_{ib} R_j{}^b \Big]\partial_\mu\varphi^i\partial^\mu\varphi^j - \frac{\Lambda}{72\pi^2} R
\end{aligned} \tag{7}
$$

This implies still a redefinition of the vacuum. It corresponds to a "cosmological term". It is in essence a one loop counterterm. The same is valid for $(2b)$ and $(2c)$, which result in

$$
\begin{aligned}
\delta\mathcal{L}^{(2b+2c)} &= \frac{1}{192\pi^4}\left[\frac{\pi\Lambda^3}{3\mu} + 4\Lambda^2 \right]\left[R_{ab}\omega^{ma}_i\omega^{mb}_j + R^{ab}R_{iabj} \right]\partial_\mu\varphi^i\partial^\mu\varphi^j + \\
&\quad + \frac{1}{32\pi^4}\frac{\Lambda}{\mu} X_{ijpq}\partial_\mu\varphi^i\partial^\mu\varphi^i\partial_\nu\varphi^p\partial^\nu\varphi^q
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
X_{ijpq} &\equiv \frac{1}{4}\Big[[D_e R_{ic} + D_i R_{ec} - 2D_c R_{ie}] R_{p(cd)q} + [D_d R_{ibce} + \\
&\quad + D_b R_{idce} + D_c R_{ibde}] R_{p(bc)q} \Big]\omega^{ed}_j + \frac{1}{3}\Big[R_{e(bc)f} R_{p(bd)q} \times \\
&\quad \times [\omega^{ec}_i \omega^{fd}_j [\omega^{ed}_i \omega^{fc}_j] - \frac{1}{2} R_{ef} R_{pcdq} [\omega^{ec}_i\omega^{fd}_j + \frac{1}{2} R_{ebdf} R_{pbdq} [\omega^{ec}_i\omega^{fc}_j \Big] + \\
&\quad + \frac{1}{48}\Big[2d^2 R_{iodj} R_{podq} + 4[D_a D_i R_{jd} - 5 D_a D_d R_{ij}] R_{padq} +
\end{aligned}
$$

$$+\frac{1}{3}\Big[R_{cadi}[R_{c(ac)j}R_{p(cd)q}+R_{e(bd)j}R_{p(ab)q}]-R_{ei}R_{ecdj}R_{p(cd)q}\Big]=$$
$$=\Big(\Sigma(DR)R+\Sigma(DR)R\omega+\Sigma(DR)R\omega\omega+\Sigma(DD)RR+\Sigma RRR\Big)_{ijpq} \quad (9)$$

Contribution $(2d)$ is new. It contains the first really "non renormalizable" counterterm. We divide it into two pieces. The first analogous to the previous contributions is given by

$$\delta\tilde{\mathcal{L}}(2d-1)=\frac{1}{2(2\pi)^6}Y_{ijpq}\partial_\mu\varphi^i\partial^\mu\varphi^j\partial_\nu\varphi^p\partial^\nu\varphi^q\int\frac{d^3kd^3l}{(k^2+\mu^2)(l^2+\mu^2)[(k+l+p)^2+\mu^2]} \quad (10)$$

$$Y_{ijpq}=(\Sigma DRDR+\Sigma RDR\omega+\Sigma RR\omega\omega)_{ijpq} \quad (11)$$

and the second one, also non renormalizable, given by the following expression in momentum space:

$$\delta\tilde{\mathcal{L}}(2d-2)=\frac{2}{9(2\pi)^6}\left[R_{iabc}R_j^{\ abc}+R_{iabc}R_j^{\ bac}\right]\partial^\mu\varphi^i\partial^\nu\varphi^j\times$$
$$\times\int\frac{d^3kd^3l\,[2l_\mu l_\nu+l_\mu(k+p)_\nu]}{[k^2+\mu^2][l^2+\mu^2][(k+l+p)^2+\mu^2]} \quad (12)$$

leading to higher derivative counterterms, that is

$$\delta\mathcal{L}^{(2d)}=-\frac{1}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}Y_{ijpq}\partial_\mu\varphi^i\partial^\mu\varphi^j\partial_\nu\varphi^p\partial^\nu\varphi^q-$$
$$-\frac{1}{192\pi^4}\Big\{\Big[8\Lambda^2-\frac{1}{5}p^2(\ln\frac{\Lambda^2}{\mu^2}-\frac{8}{3}\frac{\Lambda}{\pi\mu}-\frac{4}{3\pi^2}\frac{\Lambda^2}{\mu^2})\Big]\eta^{\mu\nu}+$$
$$+\frac{1}{5}\pi^2p^\mu p^\nu\Big[3\ln\frac{\Lambda^2}{\mu^2}-\frac{4}{3\pi}\frac{\Lambda}{\mu}-\frac{4}{3\pi^2}\frac{\Lambda^2}{\mu^2}\Big]\Big\}R_{iabc}R_j^{\ abc}\partial_\mu\varphi^i\partial_\nu\varphi^j \quad (13)$$

We have computed explicitely the counterterms in the following cases:

1.Ricci-flat spaces ($R_{ij}=0$);

2.Locally symmetric spaces, where $D_mR_{jkln}=0$; this includes the $O(n)$ and CP^{n-1} models.

In the first case we have finiteness at one loop level, but non renormalizability at two loops; the counterterm is given by

$$\delta\mathcal{L}^{(2)}=-\frac{\Lambda^2}{8\pi^4}\Big[R\omega\omega-\frac{1}{2}RR\Big]_{ij}\partial_\mu\varphi^i\partial^\mu\varphi^j$$
$$-\frac{1}{96\pi^4}\Big\{\Big[4\Lambda^2-\frac{\pi^2}{10}p^2(\ln\frac{\Lambda^2}{\mu^2}-\frac{8}{3\pi}\frac{\Lambda}{\mu}-\frac{4}{3}\frac{\Lambda^2}{\mu^2})\Big]\eta^{\mu\nu}+$$
$$+\frac{\pi^2}{10}p^\mu p^\nu\Big[3\ln\frac{\Lambda^2}{\mu^2}-\frac{4}{3\pi}\frac{\Lambda}{\mu}-\frac{4\Lambda}{3\pi\mu}\Big]\Big\}\times R_{iabc}R_j^{\ abc}\partial_\mu\varphi^i\partial_\nu\varphi^j$$
$$-\frac{1}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\Big[\Sigma(DR)(DR)+\Sigma RDR\omega+\Sigma RR\omega\omega\Big]_{ijpq}\times\partial_\mu\varphi^i\partial^\mu\varphi^j\partial_\nu\varphi^p\partial_\nu\varphi^q$$
$$-\frac{1}{32\pi^2}\frac{\Lambda}{\mu}\Big[\Sigma(R)(DR)\omega+\Sigma RR\omega\omega+\Sigma(D^2R)R+\Sigma RRR\Big]_{ijpq}\times$$
$$\times\partial_\mu\varphi^i\partial^\mu\varphi^j\partial_\nu\varphi^p\partial_\nu\varphi^q \quad (14)$$

is not of the form of the original Lagrangian. In the second case we have a renormalizability at one loop order, and again, new infinities at two loop order. Specifically, in the $O(n)$ model, where the metric and the curvature are given by

$$g_{ij}(\varphi)=\delta_{ij}+\frac{\varphi_i\varphi_j}{1-|\varphi|^2}$$
$$R_{ijkl}=g_{ik}(\varphi)g_{jl}(\varphi)-g_{il}(\varphi)g_{jk}(\varphi) \quad (15)$$

we have

$$\delta\mathcal{L}_{O(n)}=-\frac{1}{48\pi^4}\Big\{\Big[(3n-n^2+4)\Lambda^2+\frac{\pi}{12}(n^2-5n+6)\frac{\Lambda^3}{\mu}\Big]\eta^{\mu\nu}+$$
$$+\frac{\pi^2}{10}\Big[(3\ln\frac{\Lambda^2}{\mu^2}-\frac{4\Lambda}{3\pi\mu})p^\mu p^\nu-$$
$$-(\ln\frac{\Lambda^2}{\mu^2}-\frac{8\Lambda}{3\pi\mu}-\frac{4\Lambda^2}{3\pi^2\mu^2})p^2\eta^{\mu\nu}\Big](n-2)\Big\}\times\int d^3x g_{ij}\partial_\mu\varphi^i\partial_\nu\partial^j$$
$$+\frac{1}{96\pi^3}\frac{\Lambda}{\mu}\int d^3x\Big[(n^2-3n-3)g_{ij}g_{pq}+(n+3)g_{ip}g_{jq}\Big]\partial_\mu\varphi^i\partial^\mu\varphi^j\partial_\nu\varphi^p\partial_\nu\varphi^q$$
$$-\frac{n+1}{72\pi^2}\ln\frac{\Lambda^2}{\mu^2}\int d^3x g_{ij}g_{pq}\partial_\mu\varphi^i\partial^\mu\varphi^j\partial_\nu\varphi^p\partial_\nu\varphi^q \quad (16)$$

3. The supersymmetric non linear sigma model

We shall consider now the supersymmetric extension of Lagrangian , which reads, in terms of superfields

$$\mathcal{L}\left[\phi\right] = \frac{1}{4i} g_{ij}\left(\Phi^k\right)\overline{D}\Phi^i D\Phi^j \tag{17}$$

or, in terms of components

$$\mathcal{L} = \frac{1}{2}\left[g_{ij}\left(\phi\right)\partial_\mu\phi^i\partial^\mu\phi^j + ig_{ij}\left(\phi\right)\overline{\psi}^i\gamma^\mu D_\mu\psi^j + \frac{1}{6}R_{ijkl}\left(\overline{\psi}^i\psi^k\right)\left(\overline{\psi}^j\psi^l\right)\right] \tag{18}$$

where

$$\left(D_\mu\psi\right)^j = \partial_\mu\psi^j + \Gamma^j_{kl}\partial_\mu\phi^k\psi^l \tag{19}$$

The background field method works well, as in the previous case[10]. We consider the fermi fields ψ^i to be quantum fields, avoiding background quantum splitting for anticommuting variables. We obtain

$$g_{ij}\left(\phi\right)\overline{\psi}^i \not{D}\psi^j = \left(g_{ij}(\varphi) + \frac{1}{3}R_{iklj}\xi^k\xi^l\right)\overline{\psi}^i \not{D}\psi^j + \frac{1}{2}R_{ijkl}\partial_\mu\varphi^l\xi^k\left(\overline{\psi}^i\gamma^\mu\psi^j\right). \tag{20}$$

We can now write all relevant objects in terms of tangent space variables, using

$$\xi^a = e^a_i\xi^i\,,\quad \psi^a = e^a_i\psi^i\,,\quad \left(D_\mu\psi\right)^a = \partial_\mu\psi^a + \omega_i^{ab}\partial_\mu\varphi^i\psi^b. \tag{21}$$

Gathering together all relevant informations, we obtain

$$S\left[\psi\right] = S^{(0)}\left[\psi\right] + S^{(1)}\left[\psi\right] + S^{(2)}\left[\psi\right] + S^{(3)}\left[\psi\right] + S^{(4)}\left[\psi\right] + \cdots \tag{22}$$

$$S^{(0)}\left[\psi\right] = \frac{1}{2}\int d^3x i\overline{\psi}^a\left(\gamma^\mu\partial_\mu + m\right)\psi^a \tag{23}$$

$$S^{(1)}\left[\psi\right] = \frac{1}{2}\int d^3x i\overline{\psi}^a\gamma^\mu A_\mu^{ab}\psi^b \tag{24}$$

$$S^{(2)}\left[\psi\right] = \frac{1}{6}\int d^3x R_{acdb}\xi^c\xi^d i\overline{\psi}^a\gamma^\mu D_\mu\psi^b \tag{25}$$

$$S^{(3)}\left[\psi\right] = \frac{1}{4}\int d^3x R_{abci}\partial_\mu\varphi^i\xi^c i\overline{\psi}^a\gamma^\mu\psi^b \tag{26}$$

$$S^{(4)}\left[\psi\right] = \frac{1}{12}\int d^3x R_{abcd}\overline{\psi}^a\psi^c\overline{\psi}^b\psi^d \tag{27}$$

where a cutoff mass has been introduced again in order that we obtain infrared finite results. Using the Pauli Villars regulator, we obtain a vanishing result at one loop (see figure 3). At two loop order, we have the contributions shown in figure 4. In the first diagram we have a contribution arising from $S^{(2)}\left[\psi\right]$, given by

$$\delta S^{(a)}\left[\psi\right] = -\frac{1}{6}\int d^3x R_{acdb}\left\langle T\left(\xi^c\xi^d\overline{\psi}^a\left(\gamma^\mu\partial_\mu\psi^b + i\gamma^\mu A_\mu^{bc}\psi^c\right)\right)\right\rangle \tag{28}$$

Upon contracting the ξ's and the ψ's, we obtain

$$\delta S^{(a)}\left[\psi\right] = \frac{\Lambda^2}{36\pi^2}\int d^3x R(x) \tag{29}$$

which is analogous to previous computations (see eq.*****); due to a factor of 2, there is no cancellation between these terms. Note that diagrams (b) and (c) do not contribute. Finally, for the last contribution we have

$$\begin{aligned}\delta S^{(a)}\left[\psi\right] &= -\frac{1}{32}\int d^3x d^3y R_{abcd}(x)\partial_\mu\varphi^d(x)R_{efgh}(y)\partial_\nu\varphi^h(y)\times \\ &\quad\times\left\langle T\left[\left(\xi^c\overline{\psi}^a\gamma^\mu\psi^b\right)(x)\left(\xi^g\overline{\psi}^e\gamma^\nu\psi^f\right)(y)\right]\right\rangle \\ \delta S^{(a)}\left[\psi\right] &= -\frac{1}{192\pi^4}\left\{\left[\Lambda^2 - \frac{\pi^2}{5}p^2(3\ln\frac{\Lambda^2}{\mu^2} - \frac{4\Lambda}{\pi\mu} + \frac{8\Lambda^2}{3\pi^2\mu^2})\right]\eta^{m\mu\nu} + \right. \\ &\quad\left.\frac{2}{15}\pi^2 p^\mu p^\nu\left[\ln\frac{\Lambda^2}{\mu^2} + \frac{\Lambda}{4\pi^2\mu}\right]\right\}\int d^3x R_{iabc}R_i{}^{abc}\partial_\mu\varphi^i\partial^\nu\varphi^j.\end{aligned} \tag{30}$$

III. CONCLUSIONS

The conclusions we draw from this computation, though negative in the sense that we do not find any sensible renormalizable theory in any simple case, is important in view of the many applications of sigma models. Moreover, it is important to point out the different result one obtains from the perturbation theory used here, and other perturbative results based on the large n behavious[12][11], which defines a renormalizable theory. Thus, we conclude that several of the infinites we found are fake infinities produced by perturbation theory, or else, the theory has different phases.

From the $1/n$ perturbation of the CP^{n-1} model one learns that the model has two phases (not those under speculation above), one having a massive n-plet and a massless abelian gauge field, and another with a massless $(n-1)$-plet and a gauge field displaying no pole in the propagator. In these sigma models, cancellation of divergencies are a consequence of the definition of the auxiliary field propagator[13], and the identity shown in figure 5.

In the supersymmetric case, cancellation between bosons and fermions is not enough to render the model renormalizable. We think that the same continues to be true for higher supersymmetry (we worked out explicitely the case $N=2$) Restrictions of the manifold may result in the fact that some counterterms might be zero, but not all of them.

We should also make some remarks concerning general 4 dimensional non linear sigma models. Although already studied many years ago[14], it is not difficult to obtain the first few counterterms using the background field method. Indeed, the Lagrangian

$$\mathcal{L} = g_{ij}\overline{\psi}^i \not{D}\psi^j + g_{ij}D_\mu\varphi^i D^\mu\varphi^j \tag{31}$$

has a background-quantum expansion given by

$$\mathcal{L} = \mathcal{L}_{cl}(\varphi^a,\psi^a) + R_{iabj}\left(\partial_\mu\varphi^i\partial^\mu\varphi^j + \frac{1}{3}\overline{\psi}^i \not{D}\psi^j\right)\xi^a\xi^b \tag{32}$$

with a gauge field $A_\mu^{ab} = \omega_i^{ab}\partial_\mu\varphi^i$.

The diagram with two, three and four external $A_m u$ legs cannot be made to vanish, and we need a counterterm $F^2_{\mu\nu}$, which is non renormalizable already at one loop level.

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FIGURES

FIG. 1. One loop order contributions

FIG. 2. Two loop order contributions

FIG. 3. Vanishing contribution upon use of gauge invariant regularization

FIG. 4. Two loop contribution for the supersymmetric case

FIG. 5. Cancellation mechanism in the $1/n$ expansion

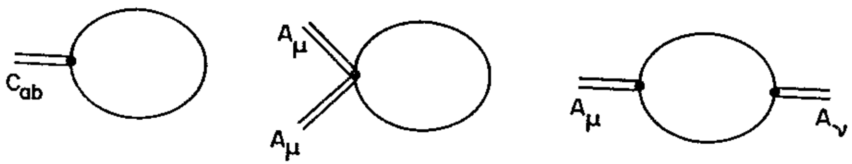

Figure 1

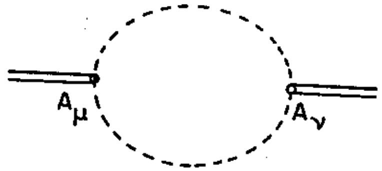

Figure 3

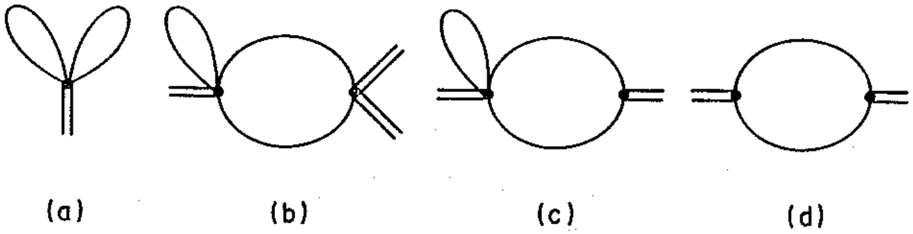

Figure 2

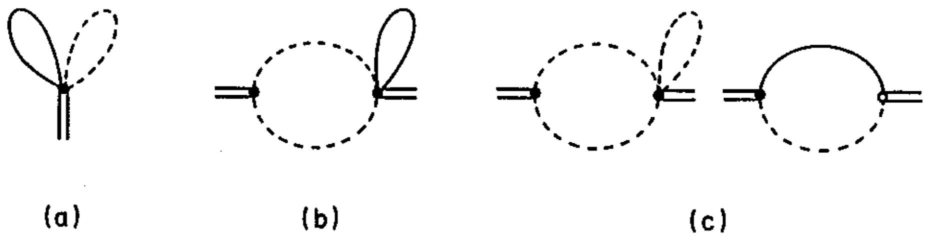

Figure 4

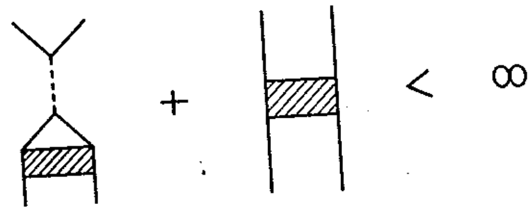

Figure 5