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QUANTUM ALGEBRAIC DESCRIPTION OF THE MOSZKOWSKI MODEL

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Abstract

In this letter we investigate the behaviour of the Moszkowski model within the context of quantum algebras. The Moszkowski hamiltonian is diagonalized for different numbers of particles and for various values of the deformation parameter of the quantum algebras. We also include cranking in our system and observe its modification in function of deformation parameters.

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The Moszkowski model [1] is a two level model, each of them being N-fold degenerate with two different kinds of particles, i.e., N_a particles of type a and N_b particles of type b. The model is the two-dimensional analog of the Elliott model [2], which includes also a one-body spin-orbit term. The $su(2) \times su(2)$ hamiltonian which describes the model reads

$$H = \epsilon(J_z(a) - J_z(b)) + V(J_x^2 + J_y^2) \tag{1}$$

where ϵ is the energy difference between both levels, V is the interaction,

$$J_i = J_i(a) + J_i(b), \qquad J^2 = \sum_i J_i^2, \qquad i = x, y, z \tag{2}$$

and

$$J_x(\alpha) = \frac{1}{2}(J_+(\alpha) + J_-(\alpha)), \qquad J_y(\alpha) = \frac{-i}{2}(J_+(\alpha) - J_-(\alpha)), \qquad \alpha = a, b. \tag{3}$$

The hamiltonian can be rewritten in terms of J_+, J_- and J_z operators yielding

$$H = \epsilon(J_z(a) - J_z(b)) + \frac{V}{2}(J_+(a)J_-(a) + J_-(a)J_+(a) + J_+(b)J_-(b) + J_-(b)J_+(b)$$

$$+2J_+(a)J_-(b) + 2J_+(b)J_-(a)). \tag{4}$$

Notice that the above hamiltonian commutes with J_z, but does not commute with J^2. The basis of states on which the Moszkowski hamiltonian can be diagonalized are given by

$$|\psi_{ab}> = |\frac{N_a}{2} m_a > |\frac{N_b}{2} m_b > \tag{5}$$

where

$$m_a = -\frac{N_a}{2}, -\frac{N_a}{2} + 1, ..., \frac{N_a}{2}, \qquad m_b = -\frac{N_b}{2}, -\frac{N_b}{2} + 1, ..., \frac{N_b}{2}. \tag{6}$$

Some features of the Moszkowski model have already been extensively discussed [3], but our aim in this work is somewhat different. Here we discuss the behaviour of the model within the context of quantum algebras [4]. Quantum algebras, also known as QUE (quantum universal enveloping) algebras are generalizations of the usual Lie algebras, differing from them in the associativity condition. Instead of the usual Jacobi identity necessary to identify a Lie algebra, the quantum algebras are required to satisfy a Yang-Baxter equation, also known as braid-Jacobi equation. QUE algebras are also called Hopf algebras. From the physical point of view, they can describe deformation of systems previously studied within the context of Lie algebras, i.e., they can describe perturbations from some underlying symmetry structure. Stretching effects are taken into account when one allows the algebra to deviate from the usual Lie algebra limit by means of a deformation parameter. It is worth pointing out that the connection of QUE algebras to q-groups is quite different from the usual link established between Lie algebras and Lie groups.

Applications of quantum algebras to systems obeying a Lie algebra structure may help us to understand some features as symmetry breaking or phase transitions. In this paper we study the behaviour of a system described by the Moszkowski hamiltonian when deviations from the $su(2) \times su(2)$ algebra are introduced.

The generators J_+, J_- and J_z of the quantum algebra $su_q(2)$ satisfy the following ***quommutation*** relations

$$[J_z, J_+] = J_+, \qquad [J_z, J_-] = -J_-, \qquad [J_+, J_-] = [2J_z], \tag{7}$$

where

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}} \tag{8}$$

and q is the ***deformation*** parameter of the algebra. When $q \to 1$, $[x] = x$. Within this formalism, the application of the raising, lowering and J_Z operators to a generic ket of the basis $|JM>$ gives

$$J_Z|JM>= M|JM>$$

$$J_+|JM>= \sqrt{[J-M][J+M+1]}|JM+1>$$

$$J_-|JM>= \sqrt{[J+M][J-M+1]}|JM-1>. \tag{9}$$

We then rewrite eq. (4) in terms of the deformation parameter q, and obtain the hamiltonian we name $(su(2) \times su(2))_q$ Moszkowski hamiltonian, which can be diagonalized with the help of eq. (9).

In figure 1 we show the difference between the first excited state and the ground state in function of NV/ϵ, where $N = N_a + N_b$ for $N_a = N_b = 4$ for q =1.0, 1.2 and 2.0 and in figure 2 for $N_a = N_b = 8$. In figure 3 we have $N_a = N_b = 30$ and show curves for q = 1.0, 1.1, 1.5 and 2.0 and in figure 4 $N_a = N_b = 50$ and the curves are shown for q =1.0, 1.1 and 1.5. In all figures one may observe that, independently of the number of particles considered, when the interaction V is turned off, $\frac{E_1 - E_0}{\epsilon}$ is always equal to 1.0. This fact can be easily understood since, in this case, the hamiltonian just depends on $J_z(a)$ and $J_z(b)$ and the energy difference between the first excited state and the ground state is always 1.0.

It also appears to us that the inclusion of deformation anticipates a ***kind of*** phase transition that would happen in the $N_a + N_b$ system for larger interactions. One may also notice that for larger systems, i.e., with more particles, this phase transition is more pronounced for the same value of the deformation parameter. It is observable that there is a critical q_c from which, as soon as the interaction is turned on, the gap $E_1 - E_0$ increases steadily, as can be seen in figures 3 and 4. This critical deformation parameter is reached faster (smaller q_c) in systems with more particles. We believe this is due to the fact that when q_c is reached, the interaction makes all particles very strongly correlated.

From this point on, we try to break some implicit symmetries of the systems described by the Moszkowski hamiltonian. One way of performing a symmetry breaking is by means of the inclusion of different number of particles of type a and b. In figure 5 we draw curves for $N_a = 2$ and $N_b = 4$ for $q = 1.0$ and 2.0, from where we notice discontinuities. For the same values of q, discontinuities are also observed in figure 6 for $N_a = 6$ and $N_b = 4$.

Another way of introducing a symmetry breaking follows the idea developed in [2], where cranking is considered. For this purpose we add the term $-\omega J_x$ to the hamiltonian shown in eq. (4). This choice was made in contrast to the one in ref.[2] $(-wJ_z)$ because in our model, J_z is already the symmetry axis (i.e., $[H, J_z] = 0$). The *cranked* hamiltonian is given by

$$H_{crank} = H - \frac{\omega}{2}(J_+(a) + J_-(a) + J_+(b) + J_-(b)). \tag{10}$$

The cranked hamiltonian, including $N_a = N_b = 8$ particles, is diagonalized for different values of ω and the difference between the first excited state and the ground state in function of NV/ϵ is shown in figure 7 for $q = 1.0$ and in figure 8 for $q = 2.0$. For $\omega = 0$, the solid curves in figures 7 and 8 are the same as the ones in solid and dotted lines drawn in figure 2. It is worth pointing out that for the deformed case ($q = 2.0$, fig.8) we can see well pronounced minima when $\omega = 0.5$ and $\omega = 1.0$ are introduced. This feature is not seem in the *non-deformed* case (vide fig.7).

Another investigated point was the behaviour of the ground states in function of the J_z projection for different values of q. The results we obtained are shown in figures 9 and 10 for $N_a = N_b = 4$ and $N_a = N_b = 30$ particles respectively. The general trend of the curves seems to remain the same but, for larger $q's$ we obtain higher minima. The already mentioned critical deformation parameter q_c also plays its role here. When it is reached, both sides of the curve close together, as in fig 10, when $q = 2.0$. Unfortunately, plotting the effects of the deformation on the yrast line does not make sense in our problem since the Moszkowski hamiltonian does not commute with J^2.

The aim of this work was to show the effects of deformation upon the Moszkowski model and, whenever possible, explain them.

Some work in the very same line as this one has already been done, namely the application of quantum algebras to the $su(2)$ Lipkin model [5]. Generalizations from the classical to the quantum $su(N)$ algebra have also been studied [6] and they can be useful in helping to extend the above application to the $su(3)$, or even $su(N)$ Lipkin model. This work is already under investigation.

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Figure captions

Figure 1. - Difference between the first excited state and the ground state in function of NV/ϵ, where $N = N_a + N_b$ and $N_a = N_b = 4$ particles. The solid line shows the function for $q = 1.0$, the dashed line for $q = 1.2$ and the dotted line for $q = 2.0$.

Figure 2. - The same as in fig. 1 but for $N_a = N_b = 8$.

Figure 3. - Difference between the first excited state and the ground state in function of NV/ϵ, where $N = N_a + N_b$ and $N_a = N_b = 30$ particles. The solid line shows the function for $q = 1.0$, the dashed line for $q = 1.1$, the dot-dashed line for $q = 1.5$ and the dotted line for $q = 2.0$.

Figure 4. - Difference between the first excited state and the ground state in function of NV/ϵ, where $N = N_a + N_b$ and $N_a = N_b = 50$ particles. The solid line shows the function for $q = 1.0$, the dashed line for $q = 1.1$ and the dotted line for $q = 1.5$.

Figure 5. - Difference between the first excited state and the ground state in function of NV/ϵ, where $N = N_a + N_b$ and $N_a = 2$ and $N_b = 4$ particles. The solid line shows the function for $q = 1.0$ and the dashed line for $q = 1.2$.

Figure 6. - Difference between the first excited state and the ground state in function of NV/ϵ, where $N = N_a + N_b$ and $N_a = 6$ and $N_b = 4$ particles. The solid line shows the function for $q = 1.0$ and the dashed line for $q = 1.2$.

Figure 7. - Difference between the first excited state and the ground state in function of NV/ϵ, where $N = N_a + N_b$ and $N_a = N_b = 8$ particles for $q = 1.0$. The solid line shows the function for $\omega = 0.0$, the dashed line for $\omega = 0.5$ and the dotted line for $\omega = 1.0$.

Figure 8. - The same as in fig. 7 but for $q = 2.0$.

Figure 9. - Ground state versus M, which is the projection of the J_z operator, for $q = 1.0$ (solid line), $q = 1.5$ (dashed line) and $q = 2.0$ (dotted line) for $N_a = N_b = 4$ particles and $V = +2$.

Figure 10. - Ground state versus M, which is the projection of the J_z operator, for $q = 1.0$ (solid line), $q = 1.1$ (dashed line) and $q = 2.0$ (dotted line) for $N_a = N_b = 30$ particles and $V = +2$.

Figure 1

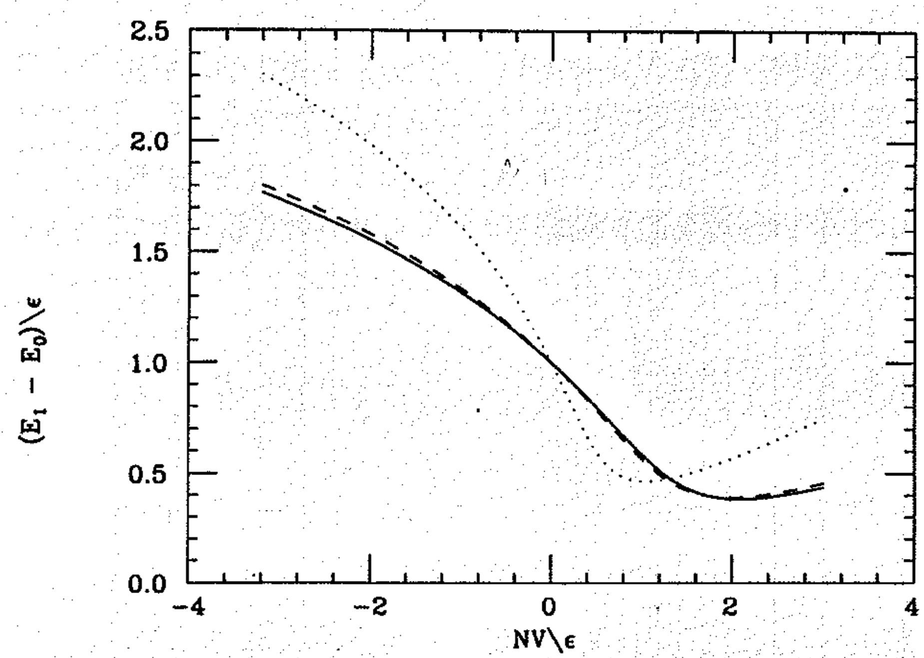
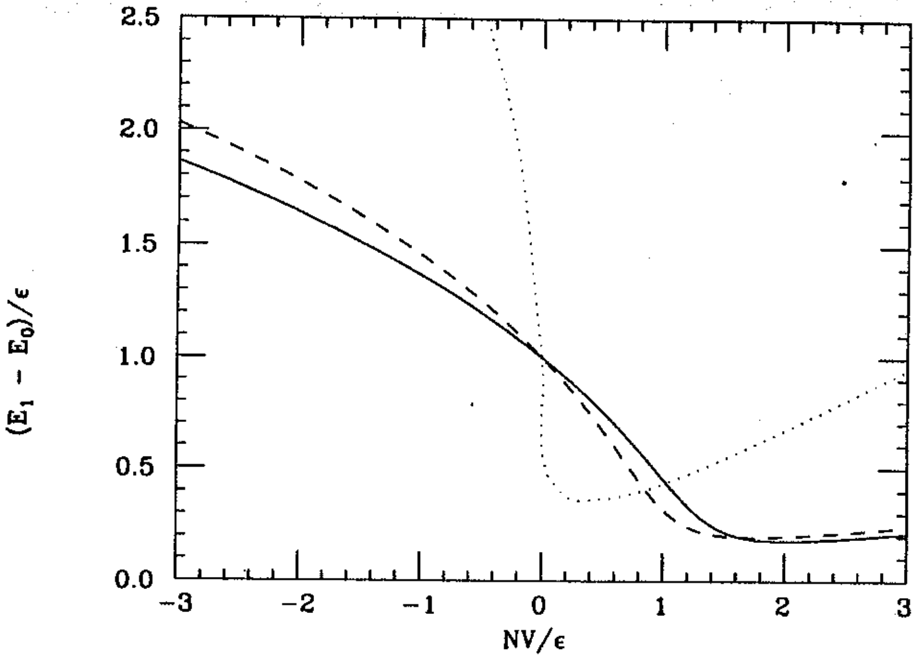


Figure 2



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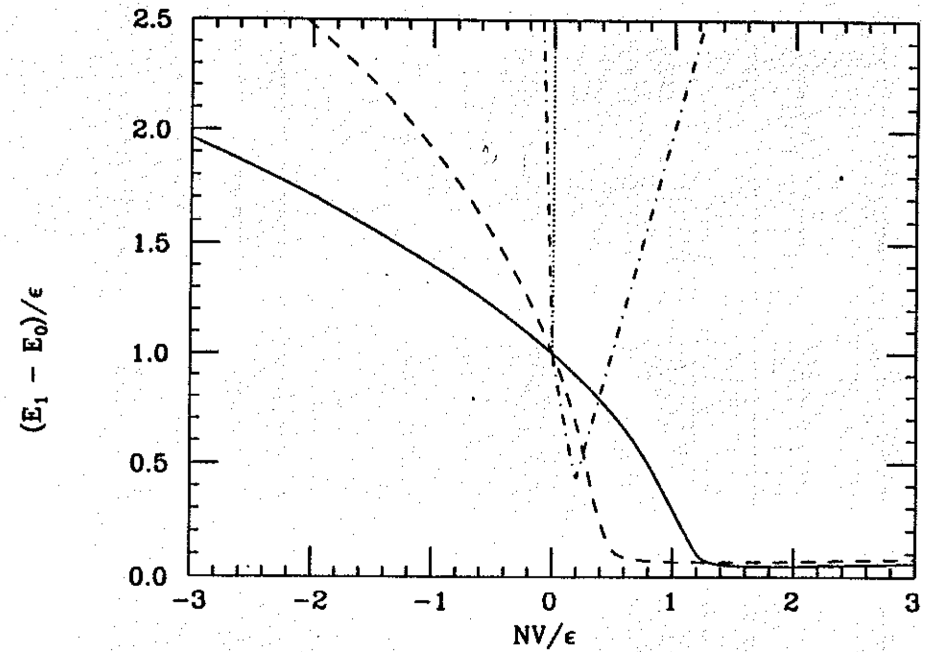

Figure 4

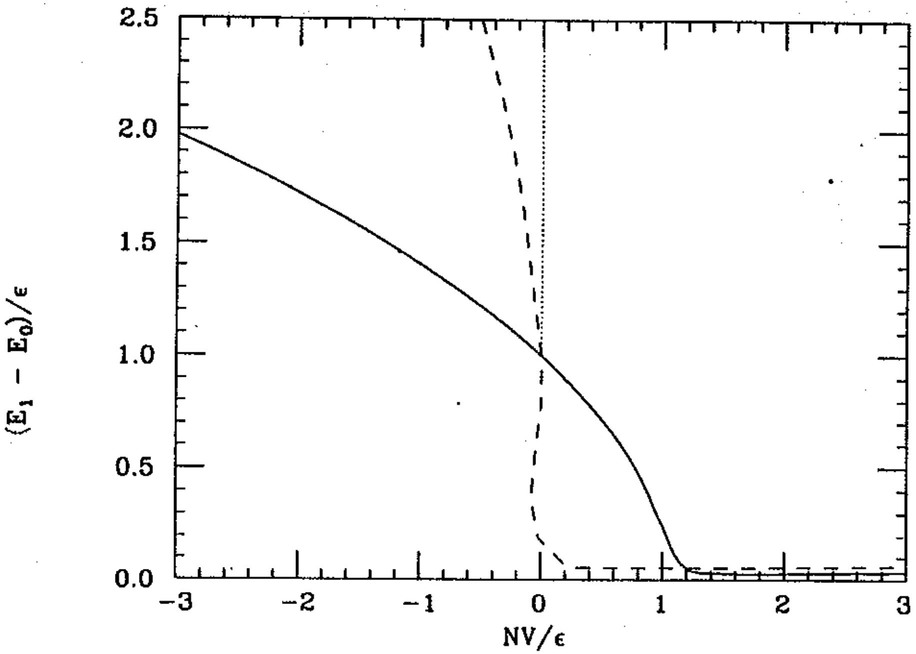

Figure 5

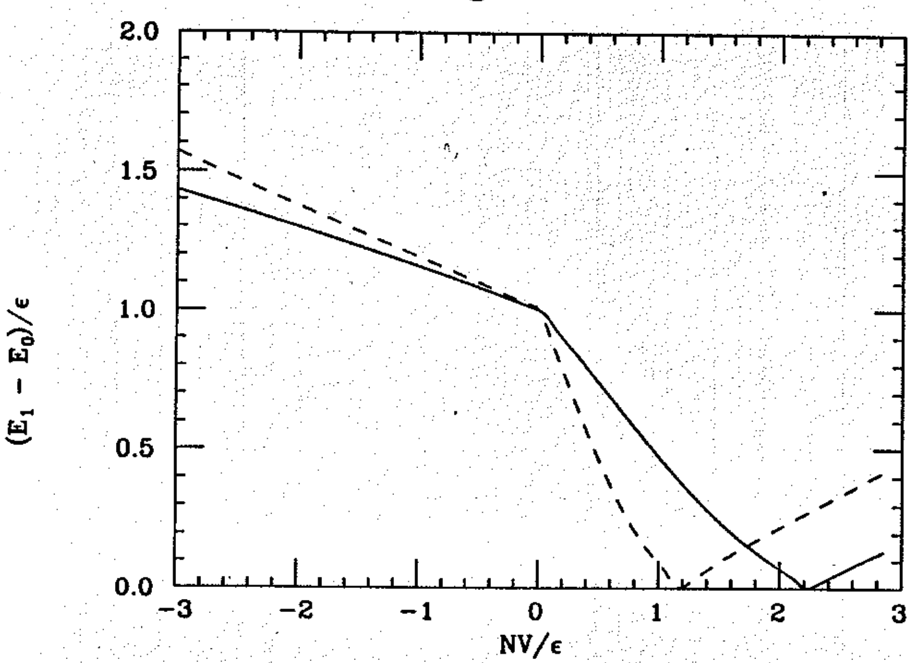

Figure 6

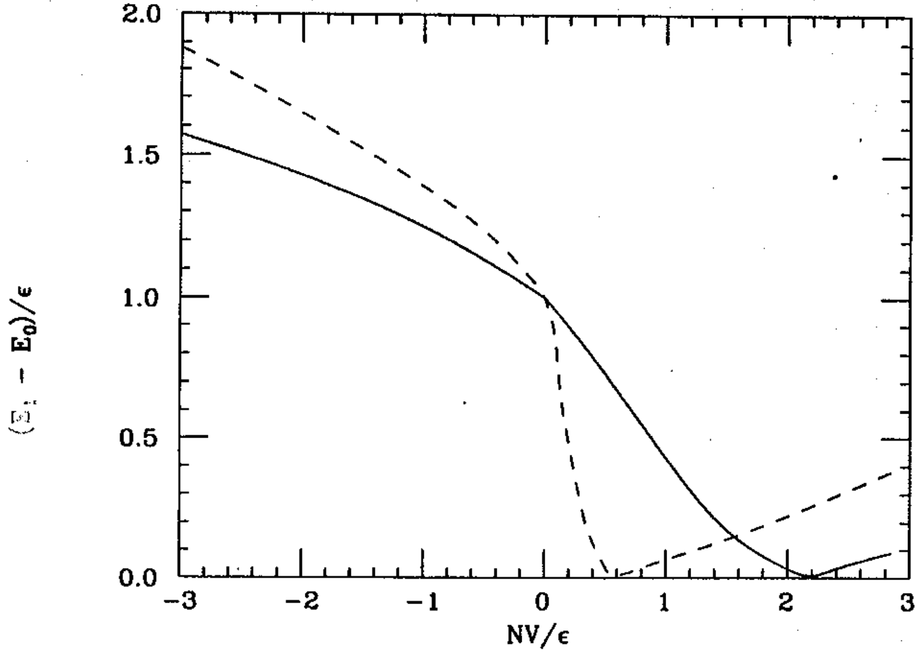

Figure 7

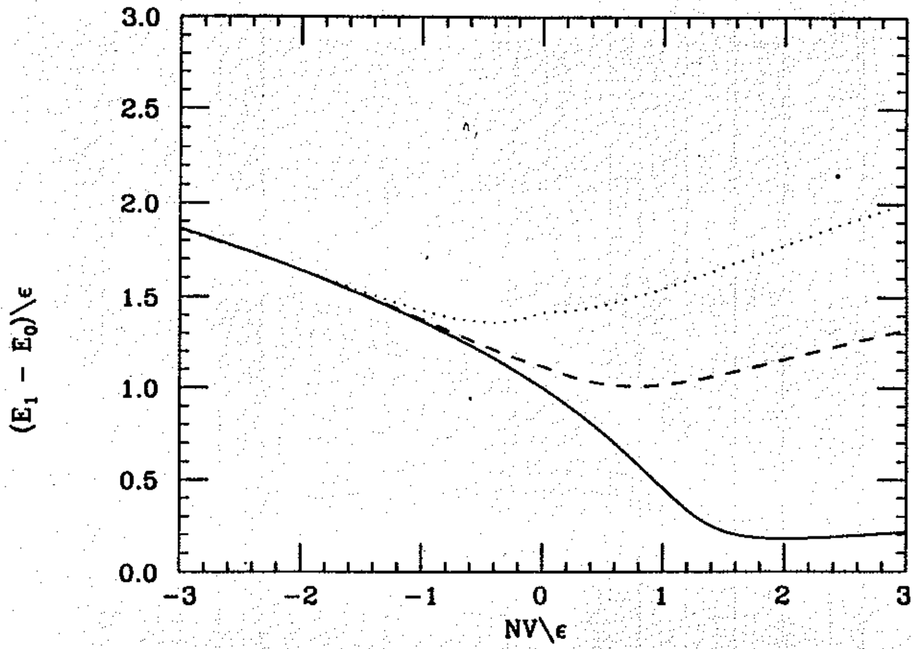

Figure 8

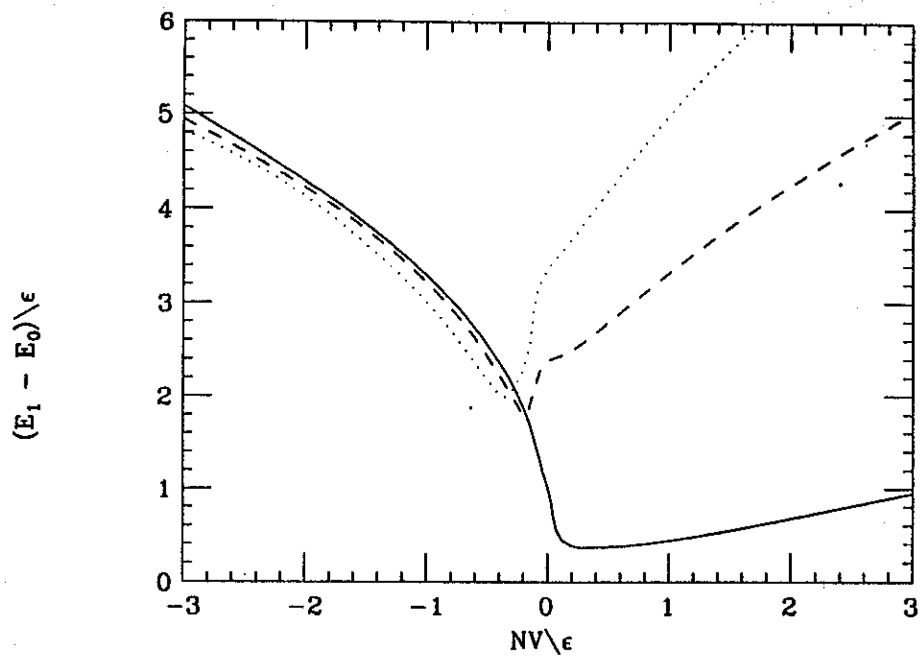

Figure 9

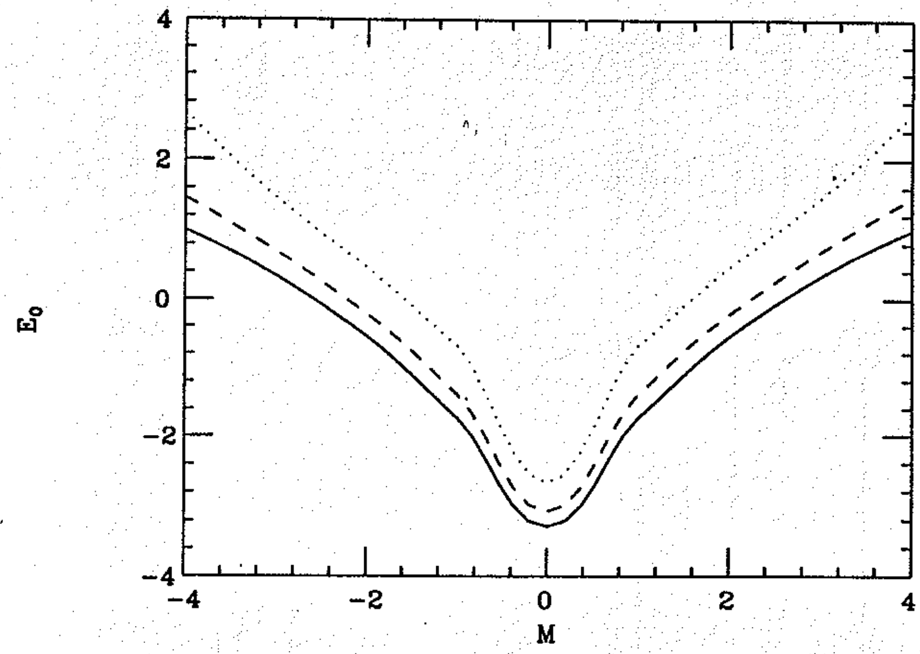

Figure 10