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NON–LINEAR AMPLIFICATION OF INVERSE–BREMSSTRAHLUNG ELECTRON ACCELERATION*

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Abstract

The acceleration of electrons using a laser and a static electric field perpendicular to the former is considered. The coupled particle–field equations are reduced to a second–order non–linear inhomogeneous equation which determines the trajectory of the particle. The particle energy equation is considered and found to exhibit fold catastrophes. At these catastrophes, which may occur whenever the wave phase is $n\pi$ if the direction of the applied weak field is reversed at $(2n-1)\pi/2$, further net acceleration occurs. The trajectory equation is found amenable to analytical treatment.

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December/1991

Laser acceleration of particles without plasma has been proposed as a means of gaining, energy without the difficulty of plasma control[1–3]. In this method one uses a strong laser field (typically the power is $\sim 10^{15}$ W/cm^2 for a wavelength of 10 μm) to set the particles in motion, and a weak perpendicular static electric or magnetic field (with a intensity of 10^{-5} that of the laser field). The resulting slightly deformed EM field allows for the acceleration of the particles. Clearly the laser field alone only sets the kinetic energy of the particle to oscillate without any net gain[4].

In a recent paper, Kawata et al.[5] discussed in details the optimal conditions for the electron acceleration with a laser + small perpendicular static electric field, E_{app}. Through single–particle computation and a particle simulation, they found that the electron is accelerated in both half wavelengths of the wave, and its relativistic factor γ increases by as much as a factor of 3.

In this letter we analytically analyse the work of ref.5 and show that the coupled electron + EM field system is governed by a set of equations containing one nonlinear and several linear ones. The non–linear equation can be solved exactly and it governs the bulk of the physics, the linear equations are then solved from the knowledge of the solution to the non–linear one. We predict, among other things, that the increasing in γ can be made much larger than given in ref.5 by applying, at optimally determined positions, an array of E_{app}'s with interchanging signs.

The starting point of our analysis is the coupled particle–field equations (see Fig. 1)

$$\frac{dP_x}{dt} = -e\,\beta_y\,E_y \tag{1}$$

$$\frac{dP_y}{dt} = -e(1-\beta_x)\,E_y - e\,E_{app} \tag{2}$$

where E_{app} is the applied field intensity, $\beta_x = v_x/c$, $\beta_y = v_y/c$, $B_z = E_y = -E_0 \sin\varphi$

with $\varphi = k(ct-x)$. P_x and P_y are the x and y components of the linear momentum of the particle. The energy equation can be easily derived from Eqs. 1 and 2 by multiplying Eq. 1 by v_x and (2) by v_y and add to obtain

$$\frac{d\varepsilon}{dt} = -e E_y v_y \quad , \quad \varepsilon(t) = m c^2 \gamma(t) + e E_{app} y(t) \tag{3}$$

where γ is the time varying relativistic factor which relates P_x and P_y to v_x and v_y through $P_x = m \gamma v_x$ and $P_y = m \gamma v_y$.

From (1), (2) and (3) one can obtain the solutions

$$P_x c = m c^2 \gamma + e E_{app} y + K_1 \tag{4}$$

$$P_y c = -\frac{e}{kc} E_{y_0} \cos\varphi - e E_{app} t + K_2 \tag{5}$$

where K_1 and K_2 are constants determined from the initial conditions. We now show that the dynamical equations (1), (2) and (3) can be reduced to one–dimensional problem through changing the independent variable from the time to the phase φ. This is accomplished by introducing the new coordinate Q ,

$$Q \equiv -\frac{k}{2c}\left(e E_{app} y^2 + 2K_1 y\right) \tag{6}$$

such that $P_y = dQ/d\varphi$. Using the fact that $\dot{\varphi} = k(c-v_x) = -k/mc\gamma\left(e E_{app} y + K_1\right)$ and from (4),

$$m c^2 \gamma = \frac{1}{2}\,\frac{m^2 c^4 + P_y^2 c^2}{\left[K_1^2 - \frac{2c\, e\, E_{app} Q}{k}\right]^{1/2}} + \frac{1}{2}\left[K_1^2 - \frac{2c\, e\, E_{app} Q}{k}\right]^{1/2} \quad . \tag{7}$$

Taking the derivative with respect to the phase φ of Eqs. (4) and using (5) and (7) we finally obtain

$$Q'' = -\frac{c}{2k}\, e E_{app}\, \frac{m^2 c^2 + Q'^2}{K_1^2 - \frac{2c}{k} e\, E_{app} Q} - \frac{e E_{app}}{2k\, c} + \frac{e}{kc} E_{y_0} \sin\varphi \tag{8}$$

and

$$\varepsilon' = \frac{ce\, E_{y_0}}{k}\, \frac{1}{\left[K_1^2 - \frac{2c\, e\, E_{app} Q}{k}\right]^{1/2}}\, Q' \sin\varphi \quad . \tag{9}$$

Equations (8) and (9) are the Inverse–Bremsstrahlung Electron Acceleration (IBEA) equations. Equation (8) is a second order non–linear inhomogeneous one that determines the trajectory of the particle since x and y are explicitly given in term of Q as

$$x = \frac{1}{k}\left[kc\,\frac{1}{e\, E_{app}}\left(-Q' - \frac{e}{kc} E_{y_0} \cos\varphi + K_2\right) - \varphi\right] \tag{10}$$

$$y = -\frac{1}{e\, E_{app}}\left[\left[K_1^2 - \frac{2ce\, E_{app}\, Q}{k}\right]^{1/2} + K_1\right] \quad . \tag{11}$$

The acceleration equation, Eq.(9), is easily solved once a first integration of (8) is done. The important feature to be emphasized here is that ε' is proportional to $Q'\sin\varphi$.

Before discussing the solution of Eq.(8), we analyse the expected behaviour of the energy as a function of φ (or t). Clearly whenever $\varphi = n\pi$, ε' is zero (ε is maximum or minimum). If, say, at $\varphi = \pi$, $Q' = P_y$ is also zero, then ε is at an inflection point, $\varepsilon'' = 0$. This behaviour is commonly referred to as fold catastrophe, according to the classification of Thom[6]. This catastrophe also characterizes the phenomenon of rainbow. This behaviour is shown in figure 2, where a case similar to that of Ref.5 is considered, namely the amplitude of the EM wave is $E_{y_0} = 0.1\ E_0$ where $E_0 = \frac{mc^2}{(e\lambda/32)} = \frac{1.636\times10^7}{\lambda}\left[\frac{V}{cm}\right]$, λ is the wavelength in cm and $E_{app}/E_{y_0} = 4.28 \times 10^{-5}$. The laser power is $3.5 \times 10^{15}\ \frac{W}{cm^2}$ for $\lambda = 10\ \mu m$. The initial electron velocity is $v_0 = 0.999c$. The electrons are injected at an angle of 0.608° with respect to the laser direction (along x). The figure exhibits $\varepsilon(\varphi)$ and $Q(\varphi)$. The inflection point alluded to above is clearly shown (indicated by the arrow). At later times ε reaches a maximum at $\varphi = 2\pi$ and then just oscillates along with the wave. If the applied field is reversed at $3\pi/2$, the original maximum in ε at $\varphi = 2\pi$ becomes an inflection point and the particle energy is then pushed up to another maximum at 3π after which the oscillation set in again. The net gain in energy after the first kick is about 300% whereas after the second kick is 600%. Thus one can double the gain by reversing the direction of the applied electric field at the appropriate time (or x) In figure 3 we show $\varepsilon(t)$ vs. t which exhibits the stair structure of the acceleration quite clearly.

The mechanism responsible for this doubling of the gain in the particle energy is governed by non–linear equations. We therefore coin it the Non–linear Amplification of Inverse–Bremsstrahlung Electron Acceleration (NAIBEA). Clearly the NAIBEA can be repeated several times by merely alternating the sign of the applied field at the appropriate phases of the wave. A simple estimate of the net gain in energy after the elapse of $n\pi$ in φ, with accompanying changes in the sign of E_{app} is $\sim n(\Delta\varepsilon)$ where $\Delta\varepsilon$ is the gain after the first kick (in our case $\Delta\varepsilon$ is about 150 MeV). The way to accomplish this is by arranging an array of E_{app} with interchanging signs at appropriate positions along the x–direction. These locations are obtained from Eq.(10). The position x_1 at which the first inflection point in ε occurs is

$$x_1 = \frac{E_{y_0}}{\pi\ E_{app}}\lambda + \frac{c\ P_y(0)}{e\ E_{app}} - \frac{\lambda}{2} \qquad (12)$$

$$\simeq 0.11\ m\ .$$

If the applied field is reversed at $\varphi = 3\pi/2$, which corresponds to the position x_1^r,

$$x_1^r = \frac{1}{k}\left[\frac{kc}{e\ E_{app}}\left(-Q'_{3\pi/2} + P_y(0) + \frac{e}{kc}E_{y_0}\right) - \frac{3\pi}{2}\right] \qquad (13)$$

$$\simeq 0.243\ m\ ,$$

then the position of the second kick or inflection point is

$$x_2 = x_1 + \frac{2|Q'_{3\pi/2}|c}{e\ E_{app}} - \frac{\lambda}{2} \qquad (14)$$

$$\simeq x_1 + 0.365\ m = 0.474\ m\ .$$

It is easy to show that the position of the n^{th} reversing of the static field is given by

$$x_n^r = \frac{K_2^{(n-1)} + \left|Q'_{\frac{2n+1}{2}\pi}\right|}{e\ E_{app}} - \frac{2n+1}{2k}\pi \qquad (15)$$

where $K_2^{(n-1)}$ is given by

$$K_2^{(n-1)} = K_2^{(n-2)} + 2\ Q'_{\frac{2n+1}{2}\pi} \qquad (16)$$

and corresponding following kick

$$x_{n+1} = x_n + \frac{-2\, Q'_{\frac{2n+1}{2}\pi}}{e\, E_{app}} - \frac{\lambda}{2} \; . \tag{17}$$

The values of Q' can be calculated from the trajectory equation, Eq.(8), to which we turn our attention below. Before doing this we mention that the trajectory of the electrons accelerated with the NAIBEA mechanism is well behaved. Figure 4 shows a typical case. The dispersion (oscillation) along the y direction is quite small.

Numerical integration of Eq.(8) is quite simple. However, to gain more insight into what to expect we develop in what follows a procedure through which analytical solution can be obtained albeit in an approximate way. To proceed we write the solution of $q \equiv Q/mc$, as

$$q = q_0 + B\, q_1 + B^2\, q_2 + \ldots \tag{18}$$

where B is the laser parameter

$$B = \frac{e\, E_{y_0}}{k\, mc^2} = 0.5093 \; . \tag{19}$$

In Eq.(18) q_0 satisfies the non–linear equation

$$q_0'' = \frac{1}{4}\, \frac{1 + q_0'^2}{q_0 - k_0} \tag{20}$$

whereas q_1 and q_2 are both given by linear equations

$$q_1'' + \frac{q_0'}{2(k_0 - q_0)}\, q_1' + \frac{1 + q_0'^2}{4(k_0 - q_0)^2}\, q_1 = -\frac{1}{2} BG + B \sin\varphi \tag{21}$$

$$q_2'' + \frac{q_0'}{2(k_0 - q_0)}\, q_2' + \frac{1 + q_0'^2}{4(k_0 - q_0)^2}\, q_1 = \frac{1}{4}\left[\frac{q_1'^2}{q_0 - k_0} - \frac{q_0'\, q_1\, q_1'}{2(q_0 - k_0)^2} + \frac{(1 + q_0'^2) q_1^2}{4(q_0 - k_0)^3} \right] \tag{22}$$

where

$$k_0 = \frac{k\, K_1^2}{2m\, c^2\, e\, E_{app}} \; , \qquad G = \frac{E_{app}}{E_{y_0}} \; . \tag{23}$$

Eq.(20) can be easily solved analytically, and from the solution, q_0, Eqs.(21) and (22) can be integrated. The details of the full solution will be presented elsewhere[7]. In figure 5 we present a comparison between the analytical method, based on Eq.(18) (dashed line) and the numerically generated one. We have here an agreement to a better than 7% at the maxima and minima and better than 2% on the average. Therefore, for all practical purposes, the set of equations (18), (20), (21) and (22) is an excellent substitute to the numerically generated solution.

In conclusion, we have further analysed the IBEA model of Kawata et al.[5], and found it to be amenable to analytical treatment. We also discovered a new way of increasing the gain in electron energy through the mechanism we coined NAIBEA. Our accelerator involves applying an array of E_{app} with interchanging signs at optimally determined positions. Further optimization can be obtained by adjusting (through magnets) the new "entrance" angle after every kick.

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7. M.S. Hussein and M.P. Pato, to be published.

Figure Captions

Figure 1. The coupled electron–laser–static field configuration.

Figure 2. a) The energy ε in units of the electron rest mass mc^2 vs. the laser phase (see text for details). b) The trajectory variable in units of mc vs. the laser phase.

Figure 3. The energy ε in units of mc^2 vs. the time in units of $1.01 \times 10^6 \frac{\lambda}{32c}$ (see text and Ref.5 for details).

Figure 4. The trajectory of the electron. y and x are given in units of $\frac{\lambda}{32}$ where λ is the wave length of the laser wave (see text for details).

Figure 5. Solid curve: numerically generated solution of Eqs.(8) and (9) in the one kick case of Ref. 5. Dashed curve: the result obtained from the analytical method, Eqs.(18), (20), (21) and (22) (see text for details).

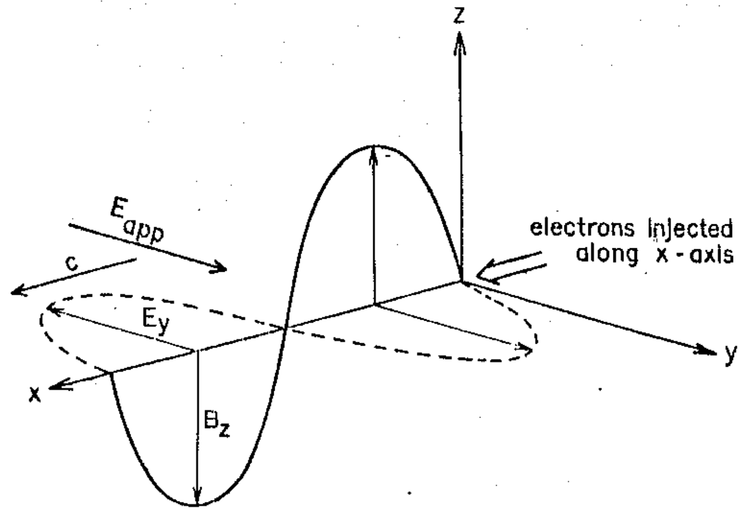

z
E_{app}
c
electrons injected
along x-axis
E_y
B_z
x
y

Fig. 1

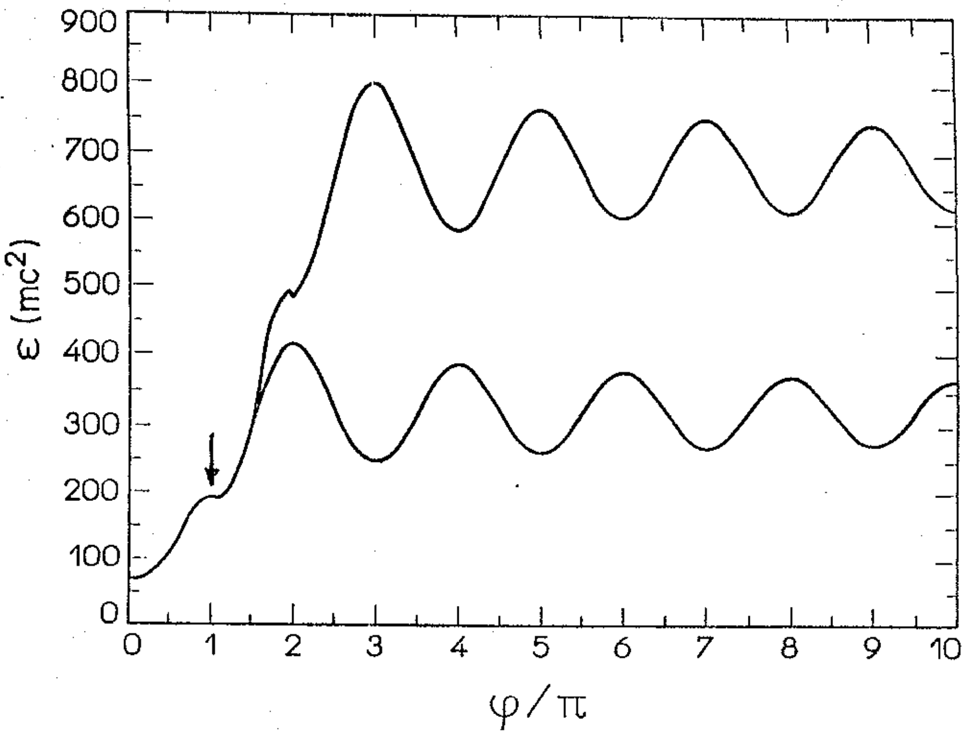
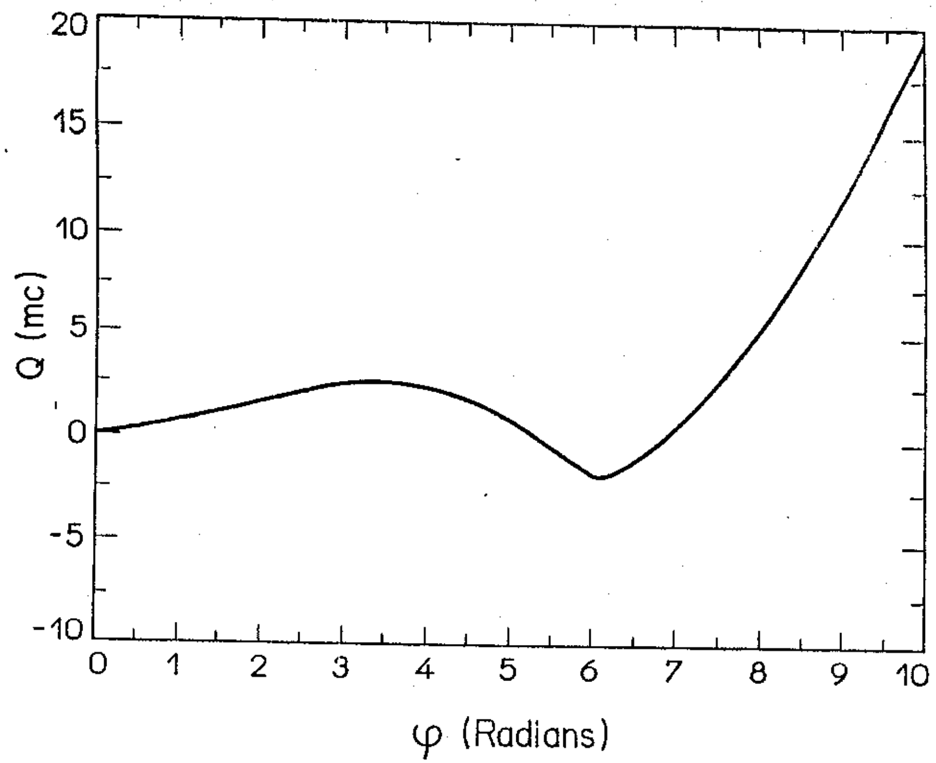
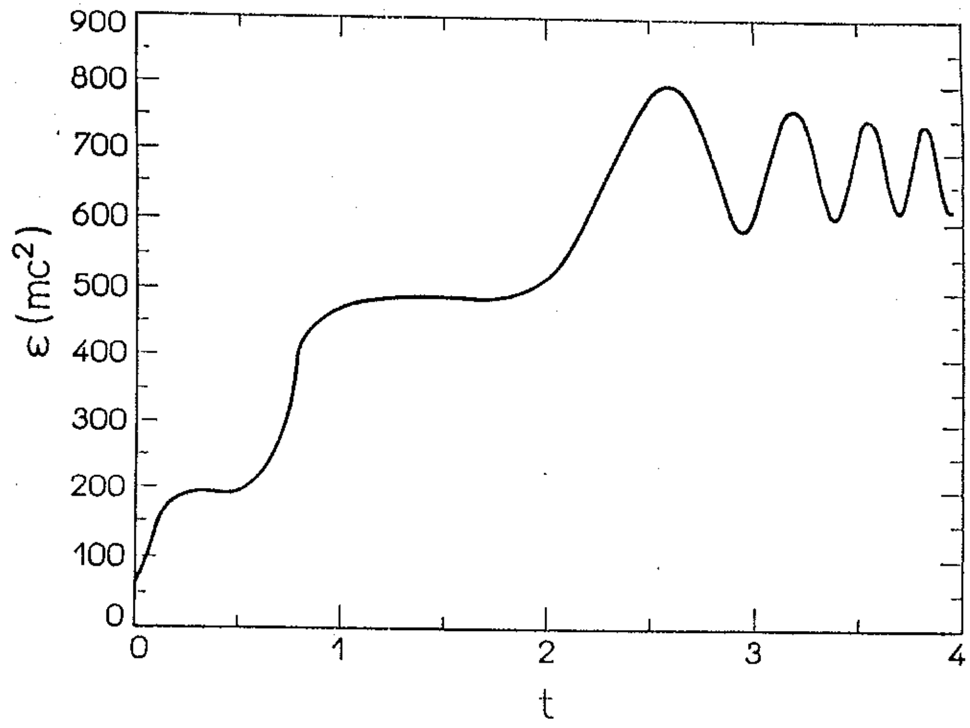

900
800
700
600
500
400
300
200
100
0
0
1
2
3
4
5
6
7
8
9
10
ε (mc²)
φ/π

Fig. 2a



20
15
10
5
0
-5
-10
Q (mc)
0
1
2
3
4
5
6
7
8
9
10
φ (Radians)

Fig. 2b



900
800
700
600
500
400
300
200
100
0
ε (mc²)
0
1
2
3
4
t

Fig. 3

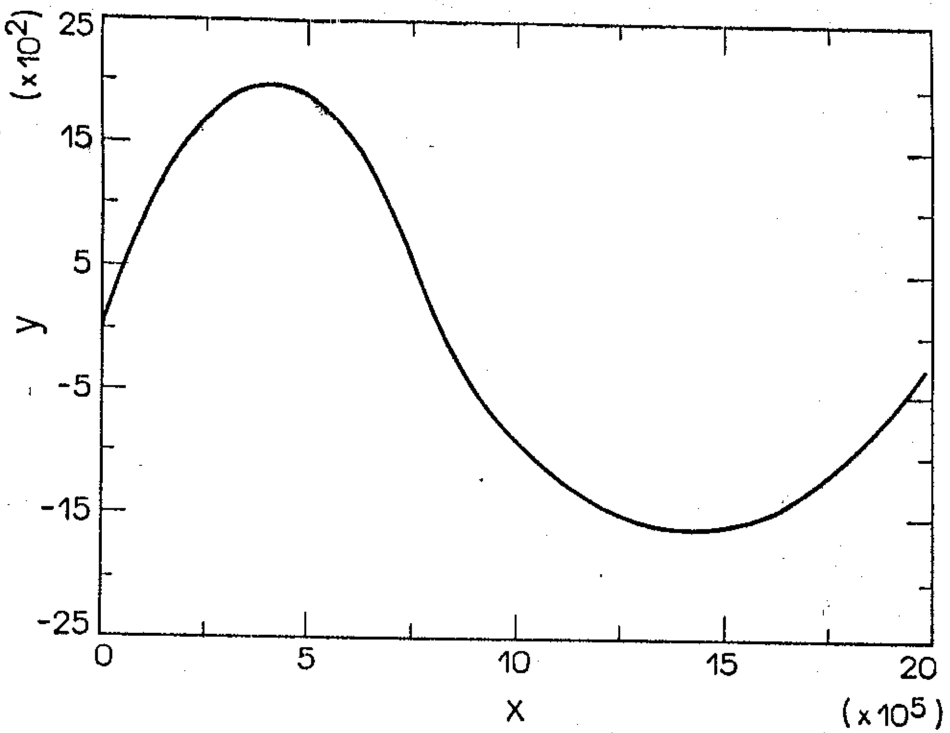
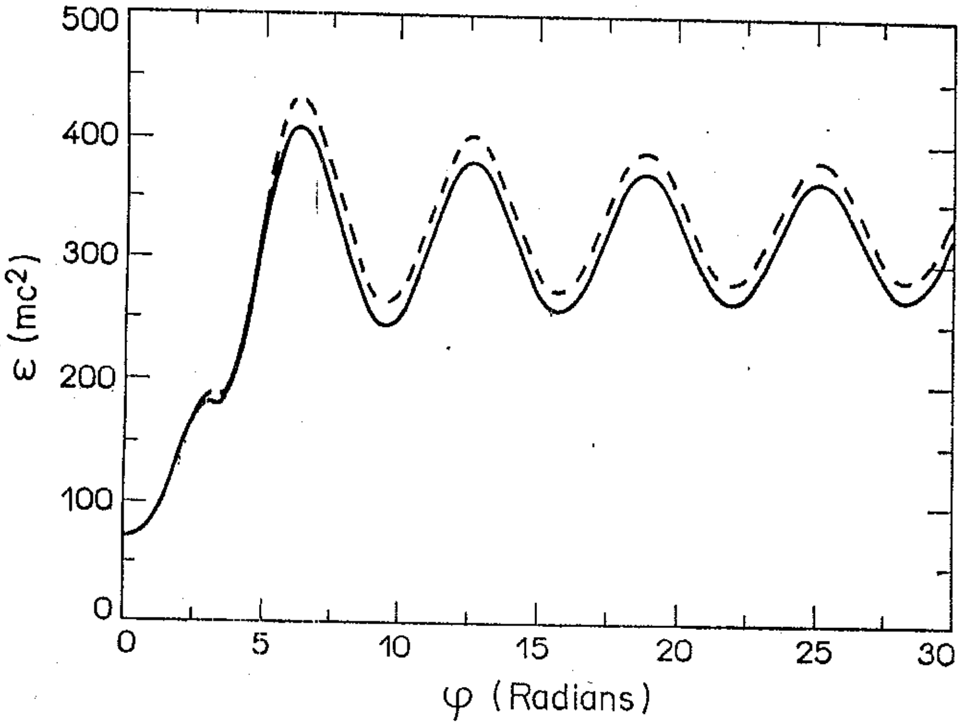

25
15
5
-5
-15
-25
$(\times 10^2)$
y
0
5
10
15
20
x
$(\times 10^5)$

Fig. 4


500
400
300
200
100
0
ε (mc²)
0
5
10
15
20
25
30
φ (Radians)