UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-976

THEORY OF FREE WAVE ACCELERATION

M.S. Hussein and **M.P. Pato**
Instituto de Física, Universidade de São Paulo

A.K. Kerman
Center for Theoretical Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139, U.S.A.

Março/1992

THEORY OF FREE WAVE ACCELERATION

M.S. Hussein* and M.P. Pato*

Nuclear Theory and Elementary Particle Phenomenology Group
Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, S.P., Brazil

and

A.K. Kerman**
Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, U.S.A

Abstract

Laser acceleration of electrons is considered. A general formulation of the problem is developed which would allow the recently proposed NAIBEA accelerator to function with any state of polarization and shape of the laser pulse, and any kind of applied field. Analytical estimates as well as numerical analysis for the case of an applied magnetic field are made. It is found that electrons, injected with few tens of MeV's, can be accelerated to GeV energies using a NAIBEA accelerator of a length of few meters.

* Supported in part by the CNPq.
** Supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under contract # DE–ACO2–76ERO3069.

March/1992

Laser acceleration of particles, has been the focus of significant attention in the last few years. Recently, two of us[1)] have proposed a method of linearly accelerating electrons with the aid of a powerful laser coupled to an optimally determined static electric field which is made to change sign at appropriate places along the accelerator. With initial electron energy of 35 MeV, a laser power of 10^{15} W/cm^2 for a wavelength of 10 μm and an applied field intensity of $\sim 10^{-5}$ that of the laser intensity, we predicted a final electron energy of, say, $\sim$ 400 MeV if three inversions of the static field are made along an accelerator tube of a length of about 3 meters! The advantages of such a machine obviously calls for a more general discussion concerning how optimum the set up is and the dependence on the laser parameters as well as the nature of the applied field.

In the present paper we develop a very general formulation of the problem that allows both semi–analytical and numerical answers to the above questions. Before entering into details, we first give some general remarks concerning the new acceleration principle which we called non–linear amplification of inverse bremsstrahlung electron acceleration (NAIBEA) in ref. 1). First, the new machine is meant to be a very high energy booster; the initial particle energy should already be highly relativistic. The reason is very simple. What sets the macroscopic scale of the machine is basically the Doppler shift of the laser wavelength as seen in the particle rest frame. Calling the laser wavelength λ_0 and, the velocity of the particle v_0 then the shifted wavelength obtained with the appropriate Lorentz transformation is

$$\lambda = \lambda_0 \left[\frac{1 + \beta_0}{1 - \beta_0}\right]^{1/2} , \qquad \beta_0 = v_0/c \quad . \tag{1a}$$

Whereas λ_0 is microscopic (μm's), λ becomes macroscopic (cm's) if β_0 is close to unity. The distance Δz travelled by the electron in the laboratory during a one cycle encounter with the laser is then

$$\Delta z = v_0(\lambda/c)\gamma = \lambda_0 \frac{\beta_0}{1-\beta_0} \quad . \tag{1b}$$

Clearly Δz is one of the quantities that would decide the size of the accelerator.

Secondly, the rate of change of the particle energy is determined by $\vec{v}\cdot\vec{E}$ and thus to keep the particle gaining energy from the laser, $\vec{v}\cdot\vec{E}$ should be made always positive. This calls for another degree of freedom to couple to the system. A possible simple form for this extra degree of freedom, was found in Ref. 1) to be an array of applied static electric field with interchanging signs placed at appropriate positions along the accelerator. Clearly a more general form for this degree of freedom can be found that would make the machine more efficient. A combination of electric and magnetic fields, not necessarily static could be one of the choices. To better determine the nature of the applied field, one needs an equation, which we now turn our attention to its derivation.

To simplify the presentation, we consider the units of mass in mc^2, the vector potential A in mc/e, the distance x in 1/k, where k is the wave number and time, t in $1/\omega$; ω being the frequency. We take the direction of wave propagation and particle, acceleration to be along z. The Hamiltonian of the system, particle + laser + applied field is (note that we are working in the temporal gauge)

$$H = \sqrt{1 + P_z^2 + (P_x - A_x)^2 + (P_y - A_y)^2} \equiv \gamma \tag{2}$$

in our units.

In Eq.(2) $\vec{A} = \vec{A}^{(0)}(t-z) + \vec{A}_{app}(t,z)$. Then $\vec{E}_{app} = -\dot{\vec{A}}_{app}$ and $\vec{B}_{app} = \vec{\nabla}\times\vec{A}_{app}$ are the external applied fields which add to those of the traveling laser pulse $\vec{A}^{(0)}(t-z)$.

Hamiltons equations follow from Eq.(1), i.e.

$$\dot{P}_z = -\frac{\partial\gamma}{\partial z} \; ; \quad \dot{P}_x = 0 \quad , \quad \dot{P}_y = 0$$

$$\dot{\gamma} = \frac{\partial\gamma}{\partial t} = \frac{\vec{P}-\vec{A}}{\gamma}\cdot\left[\frac{\partial\vec{A}^{(0)}}{\partial t} + \frac{\partial\vec{A}_{APP}}{\partial t}\right] \quad . \tag{3}$$

Futher reduction of $\dot{P}_z$ gives

$$\dot{P}_z = \frac{\vec{P}-\vec{A}}{\gamma}\cdot\left[\frac{\partial\vec{A}^{(0)}}{\partial t} + \frac{\partial\vec{A}_{APP}}{\partial z}\right] \quad . \tag{4}$$

Combining $\dot{\gamma}$ and $\dot{P}_z$ we obtain

$$\dot{\gamma} - \dot{P}_z = \frac{1}{\gamma}\left(\left(\vec{A}\times\vec{B}_{app}\right)_z + \vec{A}\cdot\vec{E}_{app}\right) \tag{5}$$

which, when integrated yields

$$\gamma = P_z + u \; ; \quad u = u_0 + \int_{-\infty}^{t} \frac{\left(\left(\vec{A}\times\vec{B}_{app}\right)_z + \vec{A}\cdot\vec{E}_{app}\right)}{\gamma}\,dt' \quad , \tag{6}$$

$$u_0 = \frac{1}{\gamma_0}(1+\beta_0)^{-1} = \left[\frac{1-\beta_0}{1+\beta_0}\right]^{1/2}$$

At this point, we remark, as was done in Ref. 1), that since $\vec{A}^{(0)}$ will be the dominant field, it is more convenient to use the phase $\varphi = t-z$ as an integration variable. Then, since $\dot{\varphi} = 1-\dot{z}$ and $\dot{z} = \partial\gamma/\partial P_z = P_z/\gamma$ and from (6), $1-\dot{z} = u/\gamma$, we have $d\varphi/u = dt/\gamma$. Further, since $\dot{u} = (1-\dot{z})\,du/d\varphi = (u/\gamma)\,du/d\varphi$, we obtain from Eq.(6)

the following

$$u^2(\varphi) = u_0^2 + 2\int_{-\infty}^{\varphi} \left((\vec{A} \times \vec{B}_{app})_z + \vec{A} \cdot \vec{E}_{app} \right) d\varphi' \quad , \tag{7}$$

and from $\gamma^2 = (P_z + u)^2 = 1 + P_z^2 + \vec{A}^2$ (Eq. 2) ,

$$\gamma(\varphi) = \frac{1 + \vec{A}^2 + u^2}{2u} \quad ; \quad P_z = u\frac{dz}{d\varphi} = \frac{1 + \vec{A}^2 - u^2}{2u} \tag{8}$$

$$z(\varphi) = \int_{-\infty}^{\varphi} \left[\frac{1 + \vec{A}^2 - u^2}{2u^2} \right] d\varphi' = \int_{-\infty}^{\varphi} \left[\frac{\gamma(\varphi')}{u(\varphi')} - 1 \right] d\varphi' \ . \tag{9}$$

The x and y coordinates of the particle are determined from the equations $\dot{P}_x = 0$ and $\dot{P}_y = 0$, which yield for the canonical momenta, $P_x = 0$ and $P_y = 0$, and thus the physical momenta are given by

$$p_x = \gamma\dot{x} = -A_x \quad \text{and} \quad p_y = \gamma\dot{y} = -A_y \tag{10}$$

or

$$x(\varphi) = -\int_{-\infty}^{\varphi} \frac{A_x(\varphi')d\varphi'}{u(\varphi')} \tag{11}$$

$$y(\varphi) = -\int_{-\infty}^{\varphi} \frac{A_y(\varphi')d\varphi'}{u(\varphi')} \ . \tag{12}$$

The set of equations (7–11) constitutes the Free Wave Accelerator (FWA) equations. These equations are important generalizations of the NAIBEA equations of Ref. 1 in that: 1) the laser could be in any state of polarization, and have any pulse shape, and 2) the applied EM field E_{app} and B_{app} is quite general. Note that since $\vec{A}$ is defined to within an arbitrary constant, we have here full freedom in giving the electron non zero initial value of p_x and/or p_y (Eq. 10). The trajectory parameter, Q , which was introduced in Ref. 1 is here generalized to be a vector in the x–y plane and is defined by the equation

$$\frac{d\vec{Q}}{d\varphi} = -\vec{A} \ . \tag{13}$$

It is a simple matter to show that the second derivative of $\vec{Q}$ can be written as

$$\frac{d^2\vec{Q}}{d\varphi^2} = \frac{-d\vec{A}^{(0)}(\varphi)}{d\varphi} + \frac{\gamma}{u}\vec{E}_{app} + \left[\frac{\gamma}{u} - 1\right]\vec{B}_{app} \tag{14}$$

and the rate change of γ with respect to φ

$$\frac{d\gamma}{d\varphi} = (p\cdot\vec{E})/u = -\frac{1}{u}\vec{A}\cdot\vec{E} = \frac{1}{u}\frac{d\vec{Q}}{d\varphi}\cdot\vec{E}(\varphi) \ . \tag{15}$$

For γ to increase with φ, $\frac{d\vec{Q}}{d\varphi}\cdot\vec{E}(\varphi)$ must always be positive (notice that $u(\varphi) > 0$)

$$\frac{d\vec{Q}}{d\varphi}\cdot\vec{E} > 0 \ . \tag{16}$$

The above is the condition that guarantees that when $E_i(\varphi_j) = 0$, $p_i(\varphi_j) = \frac{dQ_i}{d\varphi_j}$ is also zero, then $\frac{d\gamma(\varphi_j)}{d\varphi} = 0$ and $\frac{d^2\gamma(\varphi_j)}{d\varphi^2} = 0$. Having such an inflection point in γ at φ_j

(instead of a maximum) guarantees that γ keeps increasing for $\varphi > \varphi_j$. If B_{app} is taken to be zero as Ref. 1, then $\vec{p}\cdot\vec{E} = \vec{p}\cdot\vec{E}^{(0)} + \vec{p}\cdot\vec{E}_{app}$. If $\vec{E}$ is taken in the y–direction, then we have $p_y E_y^{(0)} + p_y E_{app} > 0$. Since $E_y^{(0)}$ is the dominant field, except when passing through zero, the above condition says that we use E_{app} to "fine tune" the sign of p_y so that it is always the same as that of $E_y^{(0)}$. The fundamental role of the applied field is to guarantee the validity of Eq.(16). This can happen even if $E_{app}/E^{(0)}$ or $B_{app}/B^{(0)}$ or both are much smaller than unity. The injection of electrons with a non–zero $p_x(0)$ or $p_y(0)$, albeit very small, is very important to set the machine to work. This is so since in the x–y plane the motion of the electron must be oscillatory in accordance with Eq.(16).

We turn now to specific choices of the accelerator. We consider a linearly polarized pulse with $\vec{A}$ taken to be along the y direction. We first consider a constant applied electric field, E_{app}, then

$$\vec{A} = \left[A_y^{(0)}(\varphi) - E_{app}\, t - p_y(0)\right] \hat{j} \quad . \tag{17}$$

With the $\vec{A}$ above used in the FWA equations we recover the NAIBEA equation of Ref. 1. Inversions of E_{app} are made at appropriate values of z to assure the validity of Eq. (16), namely $p_y(n\pi) = 0$. This means that the applied field is inverted at φ_j's such that $\left.\frac{dp_y(\varphi)}{d\varphi}\right|_{\varphi_j = \frac{n\pi}{2}} = 0$.

We now replace E_{app} by a constant magnetic field along x. Then

$$\vec{A} = \left[A_y^{(0)}(\varphi) - B_{app}\, z - p_y(0)\right] \hat{j} \quad . \tag{18}$$

The resulting NAIBEA equations are almost identical to those of Ref. 1 except for a

change in sign of the second term in Eq.(8) of that reference (with E_{app} replaced by B_{app}). Further, Eq.(7) reduces to

$$u^2 = u_0^2 - 2\, B_{app}\, Q \quad . \tag{19}$$

The y–component of the momentum is given by (Eq. 10)

$$p_y(\varphi) = p_y(0) - A_y^{(0)}(\varphi) + B_{app}\, z \quad . \tag{20}$$

The continuous acceleration of the particle results if B_{app} is chosen so that $p_y(n\pi) = 0$. This requires changing the sign of B_{app} at appropriate places ($\varphi \simeq \frac{n}{2}\pi$).

We consider the following numerical example. The initial value of γ, $\gamma_0 = 70$, $P = \frac{3}{2}\nu_0^2\, 10^{+15}$ W/cm^2 for $\lambda_0 = 10^{-3}$ cm. The parameter $\nu_0 = 1$ refers to the maximum value of electric field of the pulse in our units. We also take nine cycles within the pulse. The shape of the pulse is taken to be a Gaussian, $\bar{A}_0^2 = \exp(-\varphi^2/\Delta^2)$, Fig. 1, with $\Delta = 3\pi$. The applied field intensity is taken to be 2.34 teslas which corresponds to $\sim 5 \times 10^{-5}$ that of the laser. The electrons are injected at an angle of 0.6° with respect to the z–axis $\left(p_y(0) \approx \frac{0.6}{180}\pi\,\gamma_0\right)$. We consider, as an example nine changes in the sign of the modulated applied magnetic field. Since we have not self consistently chosen the position of the field reversal we have not actually optimized the decrease of u in Eq.(6). In figure 2 we show the change of γ vs. z. The gain in energy is a factor of 35 over a distance (accelerator length) of seven meters! The accelerator length could be made smaller if full optimization is accomplished. The corresponding trajectory of the particle, confined in the (z,y)–plane is shown in figure 3. The undulation in y(z) is a consequence of the changes in the sign of B_{app} and it guarantees that the particle remains in the path of the laser if one insists that

$$y = -\int_{-\infty}^{\varphi} \frac{A_y^{(0)}(\varphi')\,d\varphi'}{u(\varphi')} \quad , \tag{21}$$

is contained within the laser transversal extension. As we see from figure 2, at the end of the acceleration the dispersion in y is ~ 2 mm which could be within available laser transversal dimensions[2].

Before ending we comment briefly on the case of circularly polarized laser pulse,

$$\vec{A}^{(0)} = \bar{A}(\varphi)\,[\hat{i}\cos\varphi + \hat{j}\sin\varphi] \quad , \tag{22}$$

$$\bar{A}(\varphi) = \nu_0 \exp(-\varphi^2/2\Delta^2) \quad .$$

Thus, clearly $\vec{A}^2$ is just $\bar{A}^2(\varphi)$, which does not oscillate. Then we get approximately for the accelerator length

$$\Delta z = \frac{1-u_0^2}{2u_0^2}\,T + \frac{\nu_0^2}{2u_0^2}\,T \approx (2\gamma_0^2)(1+\nu_0^2)\,T \tag{23}$$

where T is $\Delta\varphi$ of the pulse defined as $\int_{-\infty}^{\infty} A^2\,d\varphi' = \nu_0^2\,T \equiv 2\pi\,\nu_0^2\,n$, where n is the number of oscillations in the pulse. Thus Δz is determined by three large factors (γ_0^2, ν_0^2, n) which can easily make it macroscopic even though it is in units of $\frac{\lambda}{2\pi}$. This generalizes the simple Doppler shift argument concerning the size of the accelerator.

In conclusion, we have developed here a general formalism for Free Wave Acceleration. A combined powerful laser pulse plus an optimally determined EM field, which could be time–dependent, was found to be able to accelerate electrons to very high

energies with relatively small accelerator dimensions. Radiation damping has not been included in our theory and it may modify some of our conclusions, even in setting a limit to the final energy of the particle[3,4]. After the completion of this paper we learned of related work done in Refs. 5) and 6).

REFERENCES

1) M.S. Hussein and M.P. Pato, *Phys. Rev. Lett.* 68, 1136 (1992) and references there in; M.S. Hussein and M.P. Pato, to be published.

2) P.W. Milonni and J.H. Eberly, "Lasers", (John Wiley & Sons, New York, 1988) pg. 484.

3) L.D. Landau and E.M. Lifshitz, "The Classical Theory of Fields", (Addison–Wesley, Reading, Massachusetts, 1962).

4) M.S. Hussein, M.P. Pato and A.K. Kerman, to be published.

5) P.K. Kaw and R.M. Kulsrud, *Physics of Fluids* 16, 321 (1973).

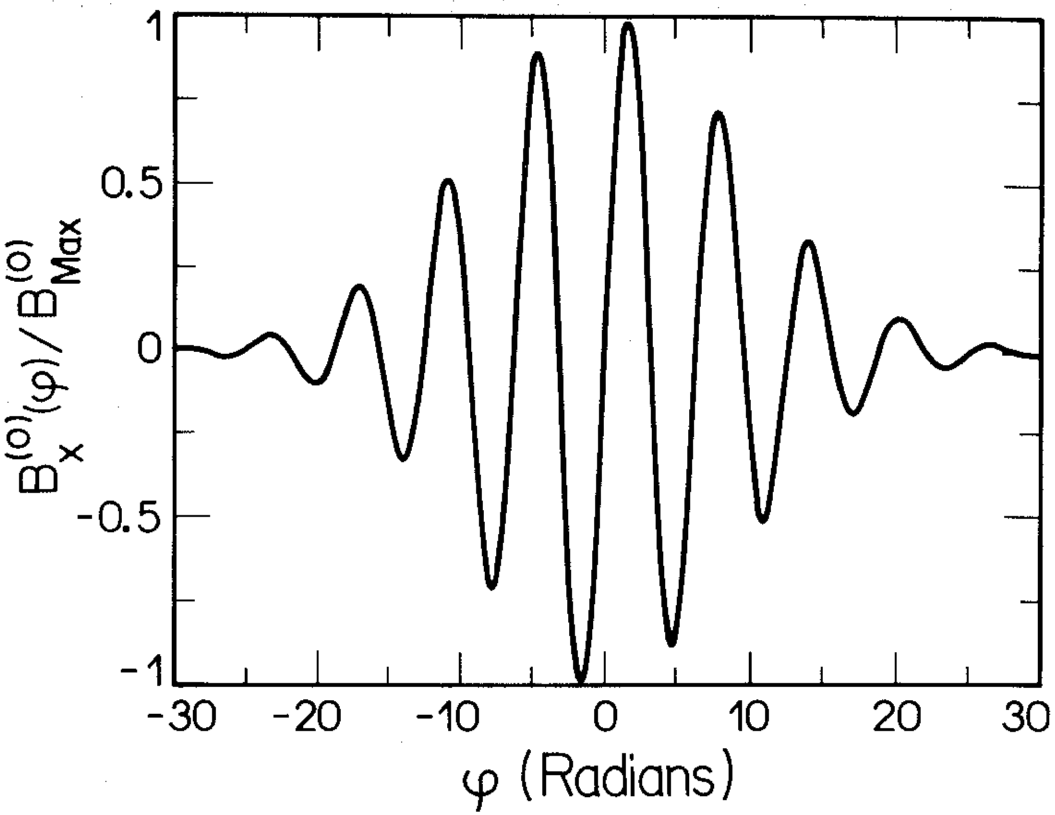
6) A.A. Chernikov, G. Schimidt and A.I. Neishtadt, *Phys. Rev. Lett.* 68, 1507 (1992).

FIGURE CAPTIONS

Figure 1. The laser pulse vs. φ used in our calculation the width $\Delta = 3\pi$.

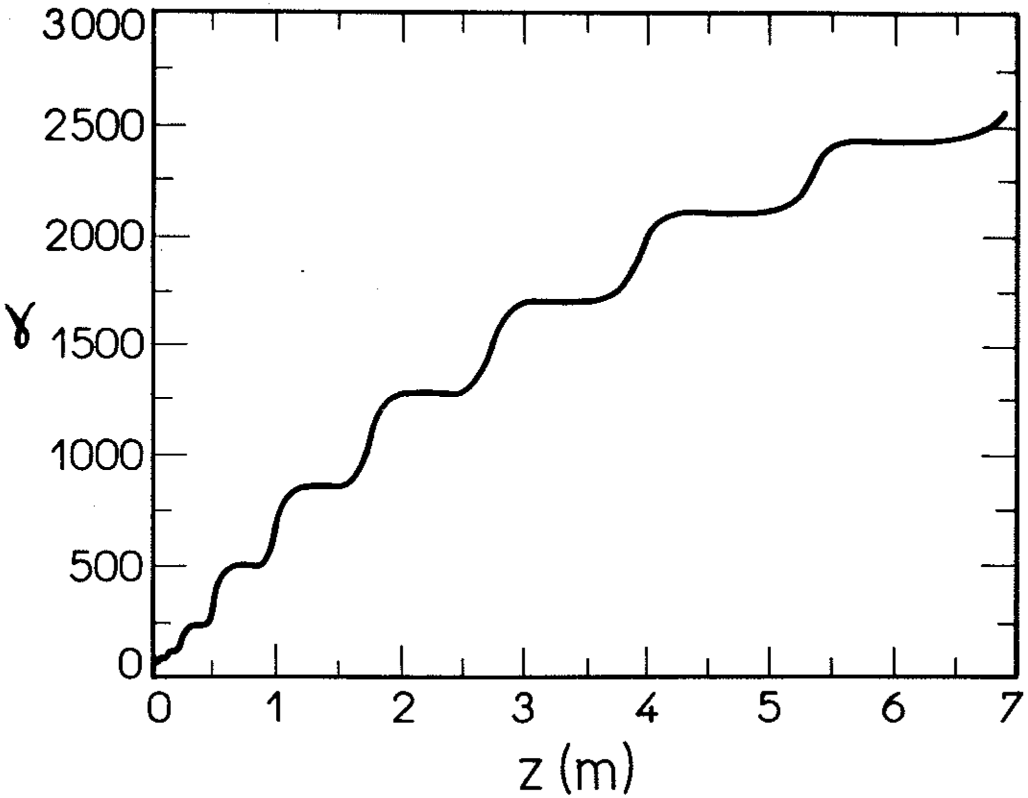
Figure 2. The energy of the particle in units of mc^2 vs. the traveled distance z .

Figure 3. The trajectory of the particle in the y–z plane. Inset: scale of y in mm .



1
0.5
0
-0.5
-1
$B_x^{(0)}(\varphi)/B_{Max}^{(0)}$
-30
-20
-10
0
10
20
30
φ (Radians)

Fig. 1



3000
2500
2000
1500
1000
500
0
γ
0
1
2
3
4
5
6
7
z (m)

Fig. 2

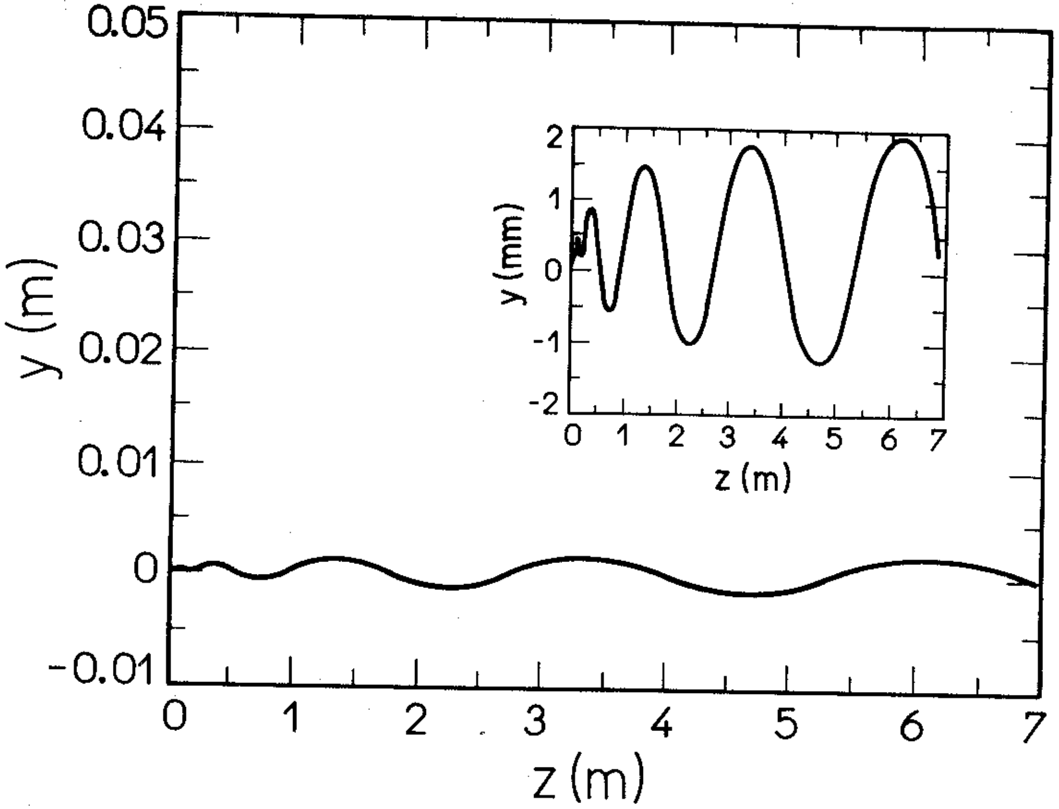


Fig. 3