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Infrared Structure of 2 + 1 Dimensional Quantum Electrodynamics *

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Abstract

The Bloch-Nordsieck approximation is used to study the infrared structure of Quantum Electrodynamics in 2 + 1 dimensions.

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Motivated by the recent interest in $2+1$ dimensional gauge theories, we investigate in this paper the infrared structure of QED_3. Our basic tool for this purpose will be the Bloch-Nordsieck (BN) approximation [1, 2, 3]. The situation is more involved than in QED_4 because, on shell, ultraviolet regularized Feynman amplitudes posses infrared singularities whose degree of divergence grows with the order of perturbation.

In this work, QED_3 will be approached as the zero mass limit of massive QED_3,

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\theta^2 A^\mu A_\mu - \frac{1}{2\lambda}(\partial_\mu A^\mu)(\partial_\nu A^\nu) \\ &+ \frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{i}{2}(\partial_\mu\bar{\psi})\gamma^\mu\psi + e\bar{\psi}\gamma^\mu A_\mu\psi - m\bar{\psi}\psi, \qquad (1)\end{aligned}$$

namely, the theory describing the coupling of fermions of mass m and electric charge e, to the vector meson A^μ of mass θ. As usual, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. Throughout this paper we use natural units ($c = \hbar = 1$). Our metric is $g_{00} = -g_{11} = -g_{22} = 1$, while for the γ – matrices we adopt the representation $\gamma^0 = \sigma_3, \gamma^1 = i\sigma^1, \gamma^2 = i\sigma^2$; $\sigma^i, i = 1,2,3$ are the Pauli spin matrices. Neither parity nor time-reversal are, separately, symmetries of the model.

The BN approximation, consists in replacing the γ^μ-matrices in (1) by the vector u^μ, with $u^2 = 1$. As consequence, the usual free fermion propagator is replaced by the retarded function

$$\begin{aligned}G_F(x-y) &\equiv \frac{1}{(2\pi)^3}\int d^3p \frac{e^{-ip\cdot(x-y)}}{u\cdot p - m + i\epsilon} \\ &= -iu^0 H(x^0-y^0) e^{-i\frac{m}{u^0}(x^0-y^0)} \delta^{(2)}[\vec{u}(x^0-y^0) - u^0(\vec{x}-\vec{y})], \qquad (2)\end{aligned}$$

where $H(x^0)$ is the Heaviside step function. Then, the interaction does not correct the vector meson propagator. Presently, this does not represent a serious drawback since vacuum polarization insertions do not alter the leading infrared behavior of a graph.

The Green functions of the theory will be computed by functionally differentiating the generating functional $U_0[J_\mu, \eta, \bar{\eta}]$ with respect to the external sources. In this paper, J_μ and η, $\bar{\eta}$ denote the vector meson and fermion external sources, respectively. After integration on the fermionic degrees of freedom one finds

$$U_0[J_\mu,\eta,\bar\eta] = \mathcal{N}\int[\mathcal{D}A_\mu]e^D\exp\{iS[A]-\frac{i}{2\lambda}\int d^3x(\partial_\mu A^\mu)^2 + i\int d^3xJ_\mu(x)A^\mu(x) - i\int d^3x\int d^3y\bar\eta(x)G[A|x,y]\eta(y)\}, \quad (3)$$

where e^D is the fermionic determinant, $S[A]=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ and $G[A|x,y]$ is the fermion two point Green function in an external A_μ field.

The solubility of the model in the BN approximation is partly due to the fact that the A_μ propagator is not corrected by the interaction and partly due to the factorization of $G[A|x,y]$. In fact, one can convince oneself that the differential equation

$$\left[iu^\mu\left(\partial_{x_\mu}-ieA_\mu(x)\right)-m\right]G[A|x,y]=\delta^{(3)}(x-y), \quad (4)$$

is solved by

$$G[A|x,y]=h[A|x]G_F(x-y)h^{-1}[A|y], \quad (5)$$

where

$$h[A|x]=\exp\{ie\int d^3z\xi^\mu(x-z)A_\mu(z)\} \quad (6)$$

and

$$\xi^\mu(x)=\frac{i}{(2\pi)^3}\int d^3k\frac{u^\mu}{(u\cdot k)}e^{-ik\cdot x}. \quad (7)$$

One can check that expression (5) can be casted in a form analogous to that quoted in ref. [2] for the case of QED_4.

Formally, the computation of D yields $D=\int_0^e de'\int d^3xG[A|x,x]u^\mu A_\mu(x)$, which in view of (5) reduces to

$$D=e\int d^3xG_F(x,x)u^\mu A_\mu(x), \quad (8)$$

showing that D can, at most, depend linearly on A_μ. However, as seen from (2), $G_F(x,x)$ is ambiguous. In the real model (γ^μ instead of u^μ), Lorentz invariance demands the vanishing of the tadpole contribution to the fermionic determinant and we shall therefore take, from now on, $D=0$.

The complete two point fermion Green function can now be readily found

$$G(x,y;m,\theta)\equiv\frac{1}{U_0[0]}\frac{1}{i^2}\frac{\delta^2U_0[J_\mu=0,\eta,\bar\eta]}{\delta\bar\eta(x)\,\delta\eta(y)}\Big|_{\eta=\bar\eta=0}=iG_F(x-y) \times\exp\{-\frac{i}{2}e^2\int d^3z\int d^3z'S_\mu(x,y;z)\Delta^{\mu\nu}(z,z')S_\nu(x,y;z')\}, \quad (9)$$

where

$$S^\mu(x,y;z)\equiv\xi^\mu(x-z)-\xi^\mu(y-z) \quad (10)$$

and $i\Delta^{\mu\nu}$ is the vector meson propagator. From (3) follows that, in momentum space,

$$i\tilde\Delta^{\mu\nu}(k,\theta) = -\frac{i}{k^2-\theta^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right) - i\lambda\frac{k^\mu k^\nu}{(k^2)(k^2-\lambda\theta^2)}. \quad (11)$$

From now on we shall always be working in Landau gauge $\lambda=0$. For the Fourier transform $G(p;m,\theta)\equiv\int d^3x\exp(ipx)G(x;m,\theta)$ one gets, after some algebra,

$$G(p;m,\theta)=\int_0^\infty d\nu\exp[i\nu(u\cdot p-m+i\epsilon)+f(\nu,\theta)], \quad (12)$$

where

$$f(\nu,\theta)=-\frac{ie^2}{(2\pi)^3}\int d^3k[1-cos\nu(u\cdot k)]U(k,\theta) \quad (13)$$

and

$$U(k,\theta)\equiv\frac{u^\mu\tilde\Delta_{\mu\nu}(k,\theta)u^\nu}{(u\cdot k)^2}. \quad (14)$$

By power counting one concludes that $f(\nu,\theta)$ is a regular function of θ at $\theta=0$. This, of course, does not imply that $G(p,m,\theta)$ is finite at $\theta=0$ for all values of p. On the other hand, the behavior of the integrand in equation (13) at the limit $k\to\infty$, but such that $u\cdot k$ remains constant, tell us that $f(\nu)$ develops a logarithmic ultraviolet divergence. This divergence can be absorbed into a mass renormalization. The two point Green function G in terms of the renormalized mass m_R turns out, then, to be given by

$$G(p;m_R,\theta)=\frac{1}{\theta}\int_0^\infty dv\exp\Big\{iv\Big[i\epsilon+\frac{Q}{\theta}-\frac{e^2}{4\pi\theta}\Big(\ln\frac{M}{\theta} + \frac{e^{-iv}-1}{iv}+2\frac{e^{-iv}-1+iv}{(iv)^2}+Ei(-iv)\Big)\Big]\Big\}, \quad (15)$$

where $Q\equiv u\cdot p-m_R$, $Ei(z)$ is the exponential integral function,

$$m_R=m+\frac{e^2}{4\pi a^2}\ln\left(\frac{\Lambda}{M}\right), \quad (16)$$

Λ is an ultraviolet cutoff and M plays the role of a subtraction point. In particular, if only terms up to the order e^2 are retained in (16) one finds that

$$\begin{aligned} G^{(2)}(p;m_R,\theta) &= \frac{i}{Q}+i\frac{e^2}{4\pi}\frac{1}{Q^2}\Big[-2\frac{Q}{\theta} \\ &- 2\left(\frac{Q}{\theta}\right)^2 \ln\left(\frac{Q-\theta}{Q}\right) - \ln\left(\frac{\theta-Q}{M}\right)\Big]. \end{aligned} \tag{17}$$

Therefore, $G^{(2)}$ develops branch cuts starting at $Q=0$ and at $Q=\theta$ and only becomes unbounded at the branch point $Q=\theta$.

We now turn into studying the limit $\theta\to 0$ of (15) and (17). As $\theta\to 0$ (17) reduces to

$$G^{(2)}(p;m_R)=\frac{i}{Q}-i\frac{e^2}{4\pi}\frac{1}{Q^2}\ln\left(-\frac{Q}{M}\right). \tag{18}$$

Hence, an on mass shell logarithmic divergence has emerged as a result of performing the limit $\theta\to 0$ in (17). Power counting alone tell us that the infrared divergences become of higher degree as the order of perturbation increases.

It remains to be investigated whether these divergences sum up to a finite limit when one considers the complete two point fermion Green function. By letting $\theta\to 0$ in (15) and after performing a Wick like rotation one arrives at

$$G(p;m_R)=\frac{i}{Q}\int_0^\infty dv\,\exp\Big\{-v+\frac{e^2}{4\pi Q}v\Big[C+\ln\left(-\frac{M}{Q}v\right)\Big]\Big\}, \tag{19}$$

for negative Q. On the other hand, if Q is positive one similarly finds that

$$G(p;m_R)=-\frac{i}{Q}\int_0^\infty dv\,\exp\Big\{v-\frac{e^2}{4\pi Q}v\Big[C+\ln\left(\frac{M}{Q}v\right)\Big]\Big\}, \tag{20}$$

although in this last case the Wick like rotation is only allowed if $e^2\neq 0$. We remark that the expressions (19) and (20) are well defined for generic values of Q but contain an essential singularity at $Q=0$. This result should be compared with the corresponding one in QED_4, where the two point fermion Green function exhibits a power law behavior in the variable Q [2].

The results presented in this note may be relevant in computing the radiative corrections for the recently derived [4] effective fermion–fermion potential in the Maxwell–Chern–Simons theory.

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