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PERTURBATIVE TREATMENT OF PARITY NON-CONSERVATION IN NEUTRON-NUCLEUS SCATTERING WITHIN THE OPTICAL MODEL

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PERTURBATIVE TREATMENT OF PARITY NON-CONSERVATION IN NEUTRON-NUCLEUS SCATTERING WITHIN THE OPTICAL MODEL*

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ABSTRACT

The questio of parity nonconservation in compou d nuclear reactions treated within the optic l model. This model supplies a descripti n of average cross-sec ns spin-polarization, spi -rotation, and the parity nonconservation direct testing quantit he longitudinal asymme ry. Taking for the OM potential a sum o a parity conserving ( ) strong complex potential and a parity nonconserving (PNC) w ak potential, the abo physical quantities are t en evaluated, with the PNC potential reated with perturbat theory. The compound lastic, absorption, and total components o all the measurable qu ities are discussed. Application is then made to the recent TRIPL data on the $n+^{232}$T systems at thermal neut ons energies.

I. INTRODUCTION

Recently there has been extensive work both theoretical[1] and experimental[2] on the question of parity non-conservation in low energy neutron-nucleus resonance reactions. It is by now widely accepted that the value of the matrix element of the parity non-conserving (PNC) interaction, extracted from first forbidden β-decay and from α-decay of unatural parity resonances is of the order of 1 eV [3]. Further, the longitudinal asymmetry measured in polarized proton scattering from very light targets (p, d and α) is of the order of 10^{-7}.

Further stringent test of PNC is supplied by the scattering of very low energy neutrons from heavy target nuclei. The level densities of the compound nuclei formed in these reactions are very high and consequently, it is suggested that the PNC effects are greatly enhanced by up to a factor of 10^4 for the longitudinal asymmetry. This has been confirmed experimentally by the TRIPLE collaboration[3]. The statistical theory which asserts that the δ-matrix elements are members of a Gaussian Orthogonal Ensemble (GOE). When considering PNC, two GOE's are considered coupled (+ and − parity)[4]. Several calculations have been performed and it is found that the above mentioned CN enhancement does indeed occur.

An important qualitative test of the statistical theory (ST) is supplied by looking at sign correlations among maxima of minima in the longitudinal asymmetry coefficient, ε. According to the ST, no sign correlations should be present. This has been recently tested by the TRIPLE collaboration[5] and the conclusion reached was that important sign correlations were observed in the fluctuations seen in ε of $n+^{232}$Th at thermal neutron energies.

Several models for a possible coherent mechanism were suggested to account for the observed sign correlations[6−8]. In particular, Aeurbach[8] has argued that in the entrance channel of $n+^{232}$Th a giant $J=0$ doorway is formed. Assuming that the PNC interaction can be approximated by a one-body operator of the type $\vec{\sigma}\cdot\vec{p}$, he accounts well for the sign correlation in ε. However some difficulties arise when he compares the doorw models with the single particle model.

Recently, Koonin, Johnson and Vogel[9] and Carlson and Hussein[10] have use he very simple optical model picture to describe the average observables. They used for the otential a sum of a strong parity-conserving (PC) complex potential and a weak PNC in raction, to account for the asymmetry within perturbation theory. The conclusions they ched is that the strength of effective PNC in $n+^{232}Th$ is several orders of magnitude la er than estimates based on standard meson-exchange models.

The purpose of this paper is to supply the details of our calculations summ ized in Ref. 10). In Section II we develop the theory of PC+PNC scattering using the opti l model Schrödinger equation plus the distorted wave Born approximation(DWBA). In S tion III we decompose the observable in our theory into compound elastic, absorption d total components. In Section IV we give a detailed analysis of the $n+^{232}$Th system. Finally, in Section V we present our concluding remarks.

II. SCATTERING THEORY OF SPIN-1/2 PARTICLE IN THE PRE ENCE OF PARITY CONSERVING AND PARITY NON-CONSERVING TER-ACTIONS

II.a. Introduction

We assume a Hamiltonian of the form

$$H = H_{\rm PC} + V_{\rm PNC} \tag{1}$$

where we take H_0 to be a typical spherically symmetric optical Hamiltonian

$$H_{\rm PC} = -\frac{\hbar^2}{\partial\mu}\vec{\nabla}^2 + U(r) + V_{\rm s.o.}(r)\,\vec{\ell}\cdot\vec{s} \tag{2}$$

in which $U(r)$ and $V_{s.o.)}(r)$ can be complex.

The parity violating term is V_π, which we take to be spherically symmetric and of the form

$$V_{PNC}(r) = \vec{\sigma}\cdot\vec{p}\,v(r) + v(r)\,\vec{\sigma}\cdot\vec{p} \quad . \tag{3}$$

The symmetrized form of $V_{PNC}(r)$ is Hermitian and turns time-reversal invariant, when $v(r)$ is real.

To estimate the effect of scattering of the term V_{PNC}, we will first analyze the problem of elastic scattering due to the Hamiltonian H_{PC}. We will then use the DWBA to estimate the effect of V_{PNC}.

II.b. Elastic Scattering of Neutron by an Optical Potential

As the Hamiltonian

$$H_{PC} = -\frac{\hbar^2}{2\mu}\vec{\nabla}^2 + U(r) + V_{s.o.}(r)\,\vec{\ell}\cdot\vec{s} \tag{4}$$

is spherically symmetric, we can expand the wavefunctions in eigenfunctions of the total spin

$$j = \vec{\ell} + \vec{s} \quad .$$

These are

$$\left\langle \hat{r}\middle|\ell\,\frac{1}{2}\;j\nu\right\rangle = \sum_{m\sigma}\left\langle \ell m\,\frac{1}{2}\,\sigma\middle|j\nu\right\rangle Y_{\ell m}(\hat{r})\,\chi_\sigma \tag{5}$$

where the χ_σ are the spin vectors

$$\chi_{1/2} = \begin{pmatrix}1\\0\end{pmatrix} \qquad \chi_{-1/2} = \begin{pmatrix}0\\1\end{pmatrix} \quad . \tag{6}$$

The $\left|\ell\,\frac{1}{2}\,j\nu\right\rangle$ are orthonormal

$$\left\langle \ell'\,\frac{1}{2}\,j'\nu'\middle|\ell\,\frac{1}{2}\;j\nu\right\rangle = \delta_{\ell\ell'}\,\delta_{jj'}\,\delta_{\nu\nu'} \quad . \tag{7}$$

As the spin-orbit coupling can be written as

$$\vec{\ell}\cdot\vec{s} = \frac{j(j+1)-\ell(\ell+1)-s(s+1)}{2} \tag{8}$$

they diagonalize the Hamiltonian H_0.

The scattering wave function can be written as

$$\psi^{(+)}_{\vec{k}}(\vec{r}) = 4\pi\sum_{j\ell\nu} i^\ell\,F^{(+)}_{\ell j}(r)\left\langle \hat{r}\middle|\ell\,\frac{1}{2}\;j\nu\right\rangle\left\langle \ell\,\frac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \tag{9}$$

where the radial functions $F^{(+)}_{\ell j}$ can be expressed in terms of the in- and out-going solutions as

$$F^{(+)}_{\ell j}(r) = \frac{1}{2}\left(H^{(-)}_{\ell j}(r) - H^{(+)}_{\ell j}\,S_{\ell j}\right) \quad . \tag{10}$$

As $r\to\infty$, the in and out solutions reduce to Hänkel functions, so that we have

$$\begin{aligned} F^{(+)}_{\ell j}(r) &\underset{r\to\infty}{\longrightarrow} \frac{i}{2}\left(h^{(-)}_\ell(r) - h^{(+)}_\ell(r)\,S_{\ell j}\right) \\ &= j_\ell(r) + h^{(+)}_\ell(r)\,\frac{S_{\ell j}-1}{2} \\ &= j_\ell(r) + h^{(+)}_\ell(r)\,t_{\ell j} \end{aligned} \tag{11}$$

$$t_{\ell j} = \frac{S_{\ell j}-1}{2} \quad .$$

Substituting the asymptotic form of the radial function into the expression for the wavefunction, and using the identities

$$4\pi\sum_{\ell m} i^\ell\,j_\ell(r)\,Y_{\ell m}(\hat{r})\,Y^*_{\ell m}(\hat{k}) = e^{i\vec{k}\cdot\vec{r}} \tag{12}$$

$$\sum_\sigma \chi_\sigma\,\chi^+_\sigma = \mathbf{1} \tag{13}$$

and

$$h_\ell^+(r) \underset{r\to\infty}{\longrightarrow} (-i)^\ell \, \frac{e^{ikr}}{kr} \quad , \tag{14}$$

we have

$$\psi_{\vec{k}}^{(+)}(\vec{r}) \underset{r\to\infty}{\longrightarrow} e^{i\vec{k}\cdot\vec{r}}\,\mathbf{1} + F_0(\theta)\,\frac{e^{ikr}}{r} \tag{15}$$

where

$$F_0(\theta) \;=\; \frac{4\pi}{k}\sum_{j\ell k} t_\ell^j \left\langle \hat{k}' \middle| \ell\,\frac{1}{2}\;j\nu \right\rangle \left\langle \ell\,\frac{1}{2}\,j\nu \middle| \hat{k} \right\rangle \quad . \tag{16}$$

we can rewrite this as

$$F_0(\theta) \;=\; f(\hat{k}\cdot\hat{k}') + i\,\vec{\sigma}\cdot\hat{k}\times\hat{k}'\; g(\hat{k}\cdot\hat{k}') \tag{17}$$

where

$$f(\hat{k}\cdot\hat{k}') \;=\; \frac{1}{k}\sum_\ell \left[(\ell+1)\,t_\ell^{j_\ell+\frac{1}{2}} + \ell\, t_\ell^{j_\ell-\frac{1}{2}}\right] P_\ell(\hat{k}\cdot\hat{k}') \tag{18}$$

and

$$g(\hat{k}\cdot\hat{k}') \;=\; \frac{1}{k}\sum_\ell \left[t_\ell^{j=\ell+\frac{1}{2}} - t_\ell^{j=\ell-\frac{1}{2}}\right] P_\ell'(\hat{k}\cdot\hat{k}') \quad . \tag{19}$$

Finally, to perform the DWBA calculation, we need the adjoint incoming wave solution

$$\tilde{\psi}_{\vec{k}}^{(-)}(\vec{r})^* \;=\; \left(\theta\psi_{\vec{k}}^{(+)}(\vec{r})\right)^+ \qquad , \qquad \left(\theta|\vec{k};I,m\rangle \;=\; (-)^{I-m}|-\vec{k},I,-m\rangle\right)$$

$$=\; 4\pi\sum_{j\ell\nu}(-i)^\ell F_{\ell j}^{(+)}(r)\left\langle \hat{k} \middle| \ell\,\frac{1}{2}\;j\nu \right\rangle \left\langle \ell\,\frac{1}{2}\,j\nu \middle| \hat{r} \right\rangle \quad . \tag{20}$$

Before turning directly to the DWBA calculation, let us calculate the matrix elements of $\vec{\sigma}\cdot\vec{\nabla}$.

We can decompose the reduced matrix elements as

$$\left(\ell'\,\frac{1}{2}\,j'\,\middle\|\vec{\sigma}\cdot\vec{\nabla}\middle\|\,\ell\,\frac{1}{2}\,j\right) \;=\; \sqrt{2(2\ell'+1)}\,(-)^{j-\ell'-\frac{1}{2}}\; W\left(\ell'\ell\,\frac{1}{2}\,\frac{1}{2};\,1j\right)\,\delta_{jj'}$$

$$\left(\frac{1}{2}\middle\|\,\sigma\,\middle\|\frac{1}{2}\right)(\ell'\|\nabla\|\ell) \tag{21}$$

where

$$\left(\frac{1}{2}\middle\|\,\sigma\,\middle\|\frac{1}{2}\right) \;=\; \sqrt{3}$$

and

$$(\ell'\|\nabla\|\ell) \;=\; (-)^{\ell'}\sqrt{2\ell+1}\begin{pmatrix}\ell' & 1 & \ell\\ 0 & 0 & 0\end{pmatrix}\left(\ell'\middle|\frac{d}{dr}+\frac{a}{r}\middle|\ell\right) \tag{22}$$

with

$$a \;=\; -\ell \quad , \quad \ell' \;=\; \ell+1$$
$$a \;=\; \ell+1 \quad , \quad \ell' \;=\; \ell-1 \;.$$

As

$$W\left(\ell'\ell\,\frac{1}{2}\,\frac{1}{2}\middle|1j\right)\begin{pmatrix}\ell' & 1 & \ell\\ 0 & 0 & 0\end{pmatrix} \;=\; (-)^{\ell-\frac{1}{2}-j}\,W\left(\ell'\middle|\,\frac{1}{2}\;;\,\ell\,\frac{1}{2}\right)\begin{pmatrix}\ell' & 1 & \ell\\ 0 & 0 & 0\end{pmatrix} \;=$$

$$=\; \frac{(-)^{\ell+\frac{1}{2}-j}}{\sqrt{3(2\ell'+1)}}\begin{pmatrix}1 & \frac{1}{2} & \ell\\ -\frac{1}{2} & \frac{1}{2} & 0\end{pmatrix} \;=\; \frac{(-)^{j+\frac{1}{2}}}{\sqrt{2\cdot 3\cdot(2\ell+1)(2\ell'+1)}} \tag{23}$$

we can put it all together to obtain

$$\left(\ell'\,\frac{1}{2}\,j\,\middle\|\vec{\sigma}\cdot\vec{\nabla}\middle\|\,\ell\,\frac{1}{2}\,j\right) \;=\; -\left(\ell'\middle|\frac{d}{dr}+\frac{a}{r}\middle|\ell\right) \tag{24}$$

where

$$a \;=\; -\ell \quad , \quad \ell' \;=\; \ell+1$$
$$a \;=\; \ell+1 \quad , \quad \ell' \;=\; \ell-1 \;.$$

Finally, we note that, since $\vec{\sigma}\cdot\vec{\nabla}$ is a rotational invariant, its matrix elements are identical to its reduced ones,

$$\left\langle \ell' \tfrac{1}{2} j \middle| \vec{\sigma}\cdot\vec{\nabla} \middle| \ell \tfrac{1}{2} j \right\rangle = \left(\ell' \tfrac{1}{2} j \middle\| \vec{\sigma}\cdot\vec{\nabla} \middle\| \ell \tfrac{1}{2} j \right) = \left(\ell' \middle| \frac{d}{dr} + \frac{a}{r} \middle| \ell \right) \quad . \tag{25}$$

II.c. The DWBA Matrix Element

To calculate the DWBA matrix element, we write the parity-violating term as

$$\frac{2\mu}{\hbar^2} V_{\rm PNC} = \vec{\sigma}\cdot\vec{p}\,V(r) + V(r)\,\vec{\sigma}\cdot\vec{p} \tag{26}$$

with

$$V(r) = \frac{2\mu}{\hbar^2} v(r) \quad .$$

The perturbation in the scattering amplitude is then

$$\begin{aligned} \delta F(\theta,\phi) &= -\frac{1}{4\pi} \left\langle \tilde{\psi}^{(-)}_{\vec{k}} \middle| \frac{2\mu}{\hbar^2} V_{\rm PNC} \middle| \psi^{(+)}_{\vec{k}} \right\rangle \\ &= -4\pi \sum_{\substack{j\ell k \\ j'\ell' k'}} (-i)^{\ell'} \left\langle \hat{k}' \middle| \ell' \tfrac{1}{2}\; j'\nu' \right\rangle \left\langle \ell \tfrac{1}{2}\, j\nu \middle| \hat{k} \right\rangle i^{\ell} \\ &= \int d^3r \left\langle \ell' \tfrac{1}{2}\; j'\nu' \middle| \hat{r} \right\rangle F^{(+)}_{\ell' j'}(r) \left(\vec{\sigma}\cdot\vec{p}\,v(r) + v(r)\,\vec{\sigma}\cdot\vec{p} \right) F^{(+)}_{\ell j}(r) \left\langle \hat{r} \middle| \ell \tfrac{1}{2}\; j\nu \right\rangle \end{aligned} \tag{27}$$

or

$$\delta F(\theta,\phi) = -\frac{4\pi}{k} \sum_{j\nu} i^{\ell-\ell'+1} \left\langle \hat{k}' \middle| \ell' \tfrac{1}{2}\; j\nu \right\rangle \left\langle \ell \tfrac{1}{2}\, j\nu \middle| \hat{k} \right\rangle t^{j}_{\ell'\ell} \tag{28}$$

with

$$t^{j}_{\ell+1\,\ell} = -t^{j}_{\ell\,\ell+1} = k \int r^2\,dr\,v(r) \left\{ \left(F^{(+)}_{\ell+1\,j} \frac{d}{dr} F^{(+)}_{\ell j} - F^{(+)}_{\ell j} \frac{d}{dr} F^{(+)}_{\ell+1\,j} \right) - \frac{2j+1}{r} F^{(+)}_{\ell+1\,j} F^{(+)}_{\ell j} \right\}$$

where in the last expression,

$$j = \ell + \frac{1}{2} \quad .$$

We can write the perturbation more explicitly as

$$\begin{aligned} \delta F(\theta,\phi) = \frac{4\pi}{k} \sum_{j\nu} t^{j}_{\ell,\ell+1} \Big(& \left\langle \hat{k}' \middle| \ell+1\, \tfrac{1}{2}\; j\nu \right\rangle \left\langle \ell \tfrac{1}{2}\, j\nu \middle| \hat{k} \right\rangle \\ & + \left\langle \hat{k}' \middle| \ell \tfrac{1}{2}\; j\nu \right\rangle \left\langle \ell+1\, \tfrac{1}{2}\, j\nu \middle| \hat{k} \right\rangle \Big) \end{aligned} \tag{29}$$

where we understand that the value of j corresponding to each ℓ is $j = \ell + \frac{1}{2}$.

II.d. The Scattering Amplitude

Combining the unperturbed scattering amplitude with the perturbation, we have

$$F(\theta,\phi) = F_0(\theta,\phi) + \delta F(\theta,\phi)$$

$$= \frac{4\pi}{k}\sum_{j\nu}\left(\left\langle \hat{k}'\middle|\ell\,\tfrac{1}{2}\,j\nu\right\rangle, \left\langle \hat{k}'\middle|\ell+1\,\tfrac{1}{2}\,j\nu\right\rangle\right)\begin{pmatrix} t^j_\ell & t^j_{\ell\ell+1} \\ t^j_{\ell\ell+1} & t^j_{\ell+1}\end{pmatrix}\begin{pmatrix}\left\langle \ell\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \\ \left\langle \ell+1\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle\end{pmatrix} . \tag{30}$$

We have written the amplitude in a matrix form to emphasize its matrix structure in angular momentum, we have

$$t^j = \begin{pmatrix} t^j_\ell & t^j_{\ell\ell+1} \\ t^j_{\ell\ell+1} & t^j_{\ell+1}\end{pmatrix} \tag{31}$$

with the off-diagonal elements due to the parity-violating term.

As mentioned before, the unperturbed amplitude takes the form

$$F_0(\theta,\phi) = \frac{4\pi}{k}\sum_{j\nu}\left(\left\langle \hat{k}'\middle|\ell\,\tfrac{1}{2}\,j\nu\right\rangle, \left\langle \hat{k}'\middle|\ell+1\,\tfrac{1}{2}\,j\nu\right\rangle\right)\begin{pmatrix} t^j_\ell & 0 \\ 0 & t^j_{\ell+1}\end{pmatrix}\begin{pmatrix}\left\langle \ell\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \\ \left\langle \ell+1\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle\end{pmatrix}$$

$$= f\left(\hat{k}'\cdot\hat{k}\right) + i\,\vec{\sigma}\cdot\hat{k}\times\hat{k}'\,g\left(\hat{k}'\cdot\hat{k}\right) . \tag{32}$$

Here we will complete the reduction by showing that the perturbation can be written as

$$\delta F(\theta,\phi) = \frac{4\pi}{k}\sum_{j\nu}\left(\left\langle \hat{k}'\middle|\ell\,\tfrac{1}{2}\,j\nu\right\rangle, \left\langle \hat{k}'\middle|\ell+1\,\tfrac{1}{2}\,j\nu\right\rangle\right)\begin{pmatrix} 0 & t^j_{\ell\ell+1} \\ t^j_{\ell+1\ell} & 0\end{pmatrix}\begin{pmatrix}\left\langle \ell\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \\ \left\langle \ell+1\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle\end{pmatrix}$$

$$= -\vec{\sigma}\cdot\left(\hat{k}+\hat{k}'\right)\,h\left(\hat{k}'\cdot\hat{k}\right) \tag{33}$$

with

$$h\left(\hat{k}'\cdot\hat{k}\right) = \frac{1}{k}\,\frac{1}{1+\hat{k}'\cdot\hat{k}}\sum_j \frac{(2j+1)}{2}\,t^j_{\ell+1\ell}\left(P_\ell\left(\hat{k}'\cdot\hat{k}\right) + P_{\ell+1}\left(\hat{k}'\cdot\hat{k}\right)\right)$$

To perform the reduction of δF, we take the trace of the product of F wi[illegible] $\vec{\sigma}\cdot\vec{k}$. We have

$$\mathrm{Tr}\left(\vec{\sigma}\cdot\hat{k}\,F\right) = -2\left(1+\hat{k}'\cdot\hat{k}\right)h\left(\hat{k}'\cdot\hat{k}\right) = \frac{4\pi}{k}\sum_{j\nu}$$

$$\begin{pmatrix} \left\langle \ell\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \vec{\sigma}\cdot\hat{k}\left\langle \hat{k}'\middle|\ell\,\tfrac{1}{2}\,j\nu\right\rangle t^j_\ell & \left\langle \ell+1\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \vec{\sigma}\cdot\hat{k}\left\langle \hat{k}'\middle|\ell\,\tfrac{1}{2}\,\nu\right\rangle t^j_{\ell+1\ell} \\ \left\langle \ell+1\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \vec{\sigma}\cdot\hat{k}\left\langle \hat{k}'\middle|\ell+1\,\tfrac{1}{2}\,j\nu\right\rangle t^j_{\ell+1\ell} & \left\langle \ell+1\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \vec{\sigma}\cdot\hat{k}\left\langle \hat{k}'\middle|\ell+\ \tfrac{1}{2}\,j\nu\right\rangle t^j_{\ell+1} \end{pmatrix}$$

Invariance of the trace under parity inversion permits us to discard the diagonal ter[illegible] (those involving t^j_ℓ and $t^j_{\ell+1}$). The off-diagonal terms (and the diagonal ones — whic[illegible] [illegible]ust be zero) involve the simplification of scalar products of the form

$$\sum_\nu \left\langle \ell\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \vec{\sigma}\cdot\hat{k}\left\langle \hat{k}'\middle|\ell'\,\tfrac{1}{2}\,j\nu\right\rangle .$$

To evaluate these, we first write the scalar product $\vec{\sigma}\cdot\hat{k}$ as

$$\vec{\sigma}\cdot\hat{k} = \sqrt{\frac{8\pi}{3}}\sum_{\sigma\sigma' n}\left\langle \tfrac{1}{2}\,\sigma\,\tfrac{1}{2}\,-\sigma'\middle|1n\right\rangle (-)^{\frac{1}{2}-\sigma'}\,Y^*_{1n}(\hat{k})\,\chi_\sigma\,\chi^+_{\sigma'} . \tag{35}$$

Substituting into the sum and using the definition of the $\left|\ell\,\tfrac{1}{2}\,j\nu\right\rangle$, we have

$$\sum_\nu \left\langle \ell\,\tfrac{1}{2}\,j\nu\middle|\hat{k}\right\rangle \vec{\sigma}\cdot\hat{k}\left\langle \hat{k}'\middle|\ell'\,\tfrac{1}{2}\,j\nu\right\rangle$$

$$= \sqrt{\frac{8\pi}{3}}\sum_{\substack{mm'\\ \sigma\sigma' \\ \nu n}} (-)^{\frac{1}{2}-\sigma'}\left\langle \ell m\,\tfrac{1}{2}\,\sigma\middle|j\nu\right\rangle\left\langle \ell' m'\,\tfrac{1}{2}\,\sigma'\middle|j\nu\right\rangle\left\langle \tfrac{1}{2}\,\sigma\,\tfrac{1}{2}\,-\sigma'\middle|1n\right\rangle$$

$$\cdot Y^*_{\ell m}(\hat{k})\,Y^*_{1n}(\hat{k})\,Y_{\ell' m'}(\hat{k}') . \tag{36}$$

We next simplify the sum over Clebsch-Gordon's:

$$\frac{1}{\sqrt{3}} \sum_{\sigma\sigma'\nu} (-)^{\frac{1}{2}-\sigma'} \left\langle \ell m \, \frac{1}{2} \, \sigma \middle| j\nu \right\rangle \left\langle \ell' m' \, \frac{1}{2} \, \sigma' \middle| j\nu \right\rangle \left\langle \frac{1}{2} \, \sigma \, \frac{1}{2} \, - \sigma' \middle| 1n \right\rangle$$

$$= (2j+1)(-)^n \sum_{\sigma\sigma'\nu} (-)^{\frac{1}{2}-\sigma'} \begin{pmatrix} \ell & j & \frac{1}{2} \\ m & \nu & \sigma \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ \sigma' & n & -\sigma \end{pmatrix} \begin{pmatrix} j & \frac{1}{2} & \ell' \\ \nu & \sigma' & m' \end{pmatrix}$$

$$= -\frac{(2j+1)}{\sqrt{2\ell'+1}} \langle \ell m |n| \ell' m \rangle \, W\left(\ell j 1 \frac{1}{2} ; \frac{1}{2} \, \ell'\right) \quad . \tag{37}$$

This leaves us with

$$\sum_{\nu} \left\langle \ell \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \vec{\sigma} \cdot \hat{k} \left\langle \hat{k}' \middle| \ell' \, \frac{1}{2} \, j\nu \right\rangle = -\sqrt{8\pi} \, \frac{(2j+1)}{\sqrt{2\ell'+1}} \, W\left(\ell j 1 \frac{1}{2} ; \frac{1}{2} \, \ell'\right)$$

$$+ \sum_{\substack{mm' \\ n}} \langle \ell m |n| \ell' m' \rangle \; Y^*_{\ell m}(\hat{k}) \, Y^*_{1n}(\hat{k}) \, Y_{\ell' m'}(\hat{k}') \quad . \tag{38}$$

Performing the remaining sums, we have

$$\sum_{\substack{mm' \\ n}} \langle \ell m |n| \ell' m' \rangle \; Y^*_{\ell m}(\hat{k}) \, Y^*_{1n}(\hat{k}) \, Y_{\ell' m'}(\hat{k}')$$

$$= \left(\frac{3 \cdot (2\ell+1)}{4\pi(2\ell'+1)} \right)^{1/2} \langle \ell 010 | \ell' 0 \rangle \sum_{m'} Y^*_{\ell' m'} \left\langle \hat{k} \left| Y_{\ell' m'} \right| \hat{k}' \right\rangle$$

$$= \left(\frac{3 \cdot (2\ell+1)}{4\pi(2\ell'+1)} \right)^{1/2} \langle \ell 010 | \ell' 0 \rangle \, \frac{(2\ell'+1)}{4\pi} \, P_{\ell'}\left(\hat{k}' \cdot \hat{k}\right) \tag{39}$$

which, upon substitution, yields

$$\sum_{\nu} \left\langle \ell \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \vec{\sigma} \cdot \hat{k} \left\langle \hat{k}' \middle| \ell' \, \frac{1}{2} \, j\nu \right\rangle$$

$$= -\frac{(2j+1)}{\sqrt{4\pi}} \sqrt{6(2\ell+1)} \; W\left(\ell j 1 \frac{1}{2} ; \frac{1}{2} \, \ell'\right) \langle \ell 010 | \ell' 0 \rangle \, P_{\ell'}\left(\hat{k}' \cdot \hat{k}\right) \tag{40}$$

We note in passing that the Clebsch-Gordon coefficient is zero for the diagonal terms, i.e. $\ell = \ell'$.

For the off-diagonal terms, $\ell' = \ell \pm 1$, we can reduce the expression further, using the identity

$$W\left(\ell j 1 \frac{1}{2} ; \frac{1}{2} \, \ell'\right) \langle \ell 010 | \ell' 0 \rangle = \frac{(-)^{\ell+j-\frac{1}{2}}}{\sqrt{3(2\ell+1)}} \left\langle j \, - \frac{1}{2} \, \frac{1}{2} \, \frac{1}{2} \middle| \ell' 0 \right\rangle \quad . \tag{41}$$

We obtain

$$\sum_{\nu} \left\langle \ell \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \vec{\sigma} \cdot \hat{k} \left\langle \hat{k}' \middle| \ell' \, \frac{1}{2} \, j\nu \right\rangle = (-)^{\ell+j+\frac{1}{2}} \, \frac{(2j+1)}{4\pi} \sqrt{2} \left\langle j \, - \frac{1}{2} \, \frac{1}{2} \, \frac{1}{2} \middle| \ell' 0 \right\rangle P_{\ell'} \left(\hat{k}' \cdot \hat{k}\right) . \tag{42}$$

Evaluating explicitly the two possibilities, we have

$$\sum_{\nu} \left\langle \ell \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \vec{\sigma} \cdot \hat{k} \left\langle \hat{k}' \middle| \ell+1 \, \frac{1}{2} \, j\nu \right\rangle = -\frac{(2j+1)}{4\pi} \, P_{\ell+1}\left(\hat{k}' \cdot \hat{k}\right) \tag{43}$$

$$\sum_{\nu} \left\langle \ell+1 \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \vec{\sigma} \cdot \hat{k} \left\langle \hat{k}' \middle| \ell \, \frac{1}{2} \, j\nu \right\rangle = -\frac{(2j+1)}{4\pi} \, P_{\ell+1}\left(\hat{k}' \cdot \hat{k}\right) \tag{44}$$

We thus have

$$\mathrm{Tr}\left(\vec{\sigma} \cdot \hat{k} \, F\right) = -2\left(1 + \hat{k}' \cdot \hat{k}\right) h\left(\hat{k}' \cdot \hat{k}\right)$$

$$= -\frac{1}{k} \sum_{j} (2j+1) \, t^j_{\ell+1\ell} \left(P_\ell\left(\hat{k}' \cdot \hat{k}\right) + P_{\ell+1}\left(\hat{k}' \cdot \hat{k}\right)\right) \quad . \tag{45}$$

Recapitulating, we can write the scattering amplitude as

$$F(\theta,\phi) = \frac{4\pi}{k} \sum_{j\nu} \left(\left\langle \hat{k}' \middle| \ell \, \frac{1}{2} \, j\nu \right\rangle , \left\langle \hat{k}' \middle| \ell+1 \, \frac{1}{2} \, j\nu \right\rangle \right) \begin{pmatrix} t^j_\ell & t^j_{\ell+1\ell} \\ t^j_{\ell+1\ell} & t^j_{\ell+1} \end{pmatrix} \begin{pmatrix} \left\langle \ell \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \\ \left\langle \ell+1 \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \end{pmatrix}$$

$$= f\left(\hat{k}' \cdot \hat{k}\right) + i \, \vec{\sigma} \cdot \hat{k} \times \hat{k}' \, g\left(\hat{k}' \cdot \hat{k}\right) - \vec{\sigma} \cdot \left(\hat{k} + \hat{k}'\right) h\left(\hat{k}' \cdot \hat{k}\right) \tag{46}$$

where

$$f\left(\hat{k}'\cdot\hat{k}\right) = \frac{1}{k}\sum_{j}\frac{(2j+1)}{2}\left[t_\ell^j P_\ell\left(\hat{k}'\cdot\hat{k}\right) + t_{\ell+1}^j P_{\ell+1}\left(\hat{k}'\cdot\hat{k}\right)\right]$$

$$= \frac{1}{k}\sum_{\ell}\left[(\ell+1)\, t_\ell^{j=\ell+\frac{1}{2}} + \ell\, t_\ell^{j=\ell-\frac{1}{2}}\right] P_\ell\left(\hat{k}'\cdot\hat{k}\right) \quad , \tag{47}$$

$$g\left(\hat{k}'\cdot\hat{k}\right) = \frac{1}{k}\sum_{\ell}\left[t_\ell^{j=\ell+\frac{1}{2}} - t_\ell^{j=\ell-\frac{1}{2}}\right] P'_\ell\left(\hat{k}'\cdot\hat{k}\right) \tag{48}$$

and

$$h\left(\hat{k}'\cdot\hat{k}\right) = \frac{1}{k}\,\frac{1}{1+\hat{k}\cdot\hat{k}'}\sum_{j}\frac{(2j+1)}{2}\, t^j_{\ell+1\,\ell}\left(P_\ell\left(\hat{k}'\cdot\hat{k}\right) + P_{\ell+1}\left(\hat{k}'\cdot\hat{k}\right)\right) \quad . \tag{49}$$

III. THE CROSS-SECTIONS

The resulting spin-averaged elastic angular distribution is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\,\mathrm{Tr}(FF^+) = |f(\cos\theta)|^2 + \sin^2\theta\,|g(\cos\theta)|^2$$

$$+ \; 2(1+\cos\theta)\,|h(\cos\theta)|^2 \tag{50}$$

while the spin polarization is

$$\vec{P}(\theta,\phi) = \mathrm{Tr}\left(F\,\vec{\sigma}\,F^+\right)/\mathrm{Tr}(FF^+)$$

$$= 2\left[\hat{k}\times\hat{k}'\,\mathrm{Im}(f(\cos\theta)\,g^*(\cos\theta)) - \left(\hat{k}+\hat{k}'\right)\,\mathrm{Re}(f(\cos\theta)\,h^*(\cos\theta\right.$$

$$\left. - \left(\hat{k}-\hat{k}'\right)(1+\cos\theta)\,\mathrm{Re}(g(\cos\theta)\,h^*(\cos\theta))\right]/\frac{d\sigma}{d\Omega} \tag{51a}$$

and the spin rotation

$$\vec{Q}(\theta,\phi) = 2\left[\hat{k}\times\hat{k}'\,\mathrm{Re}(f(\cos\theta)\,g^*(\cos\theta)) + \left(\hat{k}+\hat{k}'\right)\,\mathrm{Im}(f(\cos\theta)\,h^*(\cos\theta\right. \tag{51b}$$

We can now use the partial wave expansion to calculate cross-sections directly For the elastic cross-section, we have

$$\sigma_E = \int d\Omega \, F^+ F$$

$$= \left(\frac{4\pi}{k}\right)^2 \sum_{j\nu} \left(\left\langle \hat{k} \middle| \ell \, \frac{1}{2} \; j\nu \right\rangle , \left\langle \hat{k} \middle| \ell+1 \, \frac{1}{2} \; j\nu \right\rangle \right)$$

$$\begin{pmatrix} t_\ell^{j*} & t_{\ell\ell+1}^{j*} \\ t_{\ell\ell+1}^{j*} & t_{\ell+1}^{j*} \end{pmatrix} \begin{pmatrix} t_\ell^{j} & t_{\ell\ell+1}^{j} \\ t_{\ell\ell+1}^{j} & t_{\ell+1}^{j} \end{pmatrix} \begin{pmatrix} \left\langle \ell \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \\ \left\langle \ell+1 \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \end{pmatrix}$$

$$= \frac{4\pi}{k^2} \sum_j \frac{(2j+1)}{2} \left[\left|t_\ell^j\right|^2 + \left|t_{\ell\ell+1}^j\right|^2 + \left|t_{\ell+1}^j\right|^2 + \left|t_{\ell\ell+1}^j\right|^2 \right]$$

$$- \frac{4\pi}{k^2} \, \vec{\sigma} \cdot \hat{k} \sum_j \frac{(2j+1)}{2} \left[2\mathrm{Re}\left(t_{\ell\ell+1}^{j*} \left(t_\ell^j + t_{\ell+1}^j \right) \right) \right] \quad . \tag{52}$$

Defining the S-matrix as

$$\begin{pmatrix} S_\ell^j & S_{\ell\ell+1}^j \\ S_{\ell\ell+1}^j & S_{\ell+1}^j \end{pmatrix} = \begin{pmatrix} 1 + 2i\, t_\ell^j & 2i\, t_{\ell\ell+1}^j \\ 2i\, t_{\ell\ell+1}^j & 1 + 2i\, t_{\ell+1}^j \end{pmatrix} \quad , \tag{53}$$

we can rewrite the elastic cross-section as

$$\sigma_E = \frac{\pi}{k^2} \sum_j \frac{(2j+1)}{2} \left[\left|S_\ell^j - 1\right|^2 + \left|S_{\ell\ell+1}^j\right|^2 + \left|S_{\ell+1}^j - 1\right|^2 + \left|S_{\ell\ell+1}^j\right|^2 \right]$$

$$- \frac{\pi}{k^2} \, \vec{\sigma} \cdot \hat{k} \sum_j \frac{(2j+1)}{2} \cdot 2 \cdot \mathrm{Re}\left(S_{\ell\ell+1}^{j\,*} \left(S_\ell^j - 1 + S_{\ell+1}^j - 1 \right) \right) \quad . \tag{54}$$

Similarly, we find the absorption cross-section to be

$$\sigma_{\mathrm{ABS}} = \frac{4\pi^2}{k^2} \sum_{j\nu} \left(\left\langle \hat{k} \middle| \ell \, \frac{1}{2} \; j\nu \right\rangle , \left\langle \hat{k} \middle| \ell+1 \, \frac{1}{2} \; j\nu \right\rangle \right)$$

$$\left[\mathbf{1} - \begin{pmatrix} S_\ell^j & S_{\ell\ell+1}^j \\ S_{\ell\ell+1}^j & S_{\ell+1}^j \end{pmatrix}^+ \begin{pmatrix} S_\ell^j & t_{\ell\ell+1}^j \\ S_{\ell\ell+1}^j & S_{\ell+1}^j \end{pmatrix} \right] \begin{pmatrix} \left\langle \ell \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \\ \left\langle \ell+1 \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \end{pmatrix}$$

$$= \frac{\pi}{k^2} \sum_j \frac{(2j+1)}{2} \left[1 - \left|S_\ell^j\right|^2 - \left|S_{\ell\ell+1}^j\right|^2 + 1 - \left|S_{\ell+1}^j\right|^2 - \left|S_{\ell\ell+1}^j\right|^2 \right]$$

$$+ \frac{\pi}{k^2} \, \vec{\sigma} \cdot \hat{k} \sum_j \frac{(2j+1)}{2} \cdot 2 \cdot \mathrm{Re}\left(S_{\ell\ell+1}^{j\,*} \left(S_\ell^j + S_{\ell+1}^j \right) \right) \quad . \tag{55}$$

Adding the two, we obtain the expected expressions for the total cross-section

$$\sigma_T = \sigma_E + \sigma_{\mathrm{ABS}}$$

$$\frac{4\pi^2}{k^2} \sum_{j\nu} \left(\left\langle \hat{k} \middle| \ell \, \frac{1}{2} \; j\nu \right\rangle , \left\langle \hat{k} \middle| \ell+1 \, \frac{1}{2} \; j\nu \right\rangle \right)$$

$$\left[2\,\mathbf{1} - 2\mathrm{Re} \left[\begin{pmatrix} S_\ell^j & S_{\ell\ell+1}^j \\ S_{\ell\ell+1}^j & S_{\ell+1}^j \end{pmatrix} \right] \right] \begin{pmatrix} \left\langle \ell \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \\ \left\langle \ell+1 \, \frac{1}{2} \, j\nu \middle| \hat{k} \right\rangle \end{pmatrix}$$

$$= \frac{2\pi}{k^2} \sum_j \frac{2j+1}{2} \left[1 - \mathrm{Re}\left(S_\ell^j\right) + 1 - \mathrm{Re}\left(S_{\ell+1}^j\right) \right]$$

$$+ \frac{2\pi}{k^2} \, \vec{\sigma} \cdot \hat{k} \sum_j \frac{(2j+1)}{2} \cdot 2 \cdot \mathrm{Re}\left(S_{\ell\ell+1}^j \right) \quad . \tag{56}$$

The spin-averaged total cross-section is

$$\overline{\sigma}_T = \frac{2\pi}{k^2} \sum_j \frac{2j+1}{2} \left[1 - \mathrm{Re}\left(S_\ell^j\right) + 1 - \mathrm{Re}\left(S_{\ell+1}^j\right) \right] \quad . \tag{57}$$

The longitudinal asymmetry coefficient,

$$\varepsilon \equiv \frac{\sigma_{T+} - \sigma_{T-}}{\sigma_{T+} + \sigma_{T-}}$$

is

$$\varepsilon = \frac{2\pi}{k^2} \sum_j (2j+1)\ \mathrm{Re}\left(S^j_{\ell\ell+1}\right) / \bar{\sigma}_T \quad . \tag{58}$$

In the next section, we shall use the above expressions for σ_E, σ_{ABS}, σ_T and ε to analyze the system $n+^{232}$Th at thermal neutron energies, recently studied by the TRIPLE collaboration.

IV. OPTICAL MODEL DESCRIPTION OF PNC IN THE SYSTEM ı $+^{232}$Th AT THERMAL NEUTRON ENERGIES

In this section we apply the theory developed in Section III to the scattering ‹ neutron from ^{232}Th at the low energies. We use for strong PC interaction the Madla‧ ł-Young optical potential(11) which describes very well neutron scattering from actinidc ·uclei at $E_n < 10$ MeV. It is given by

$$\begin{aligned}
U(r) &= -V_0\, f_r(r) - i\, W_0\, f_I'(r) \\
f_i(r) &= \left(1 + \exp\frac{r - R_i}{a_i}\right)^{-1} \\
V_0 &= 50.378 - 27.073\left(\frac{N-Z}{A}\right) - 0.354\, E_{\mathrm{Lab}} \ \ (\mathrm{MeV}) \\
R_r &= 1.264\, A^{1/3}\,\mathrm{fm} \quad , \quad a_r = 0.612\ \mathrm{fm} \\
W_0 &= 9.265 - 12.666\left(\frac{N-Z}{A}\right) - 0.232\, E_{\mathrm{Lab}} + 0.003318\, E^2_{\mathrm{Lab}} \ \ (\mathrm{Me\backslash} \\
R_I &= 1.256\, A^{1/3} \quad , \quad a_I = 0.553 + 0.0144\, E_{\mathrm{Lab}} \ \ (\mathrm{MeV}) \\
V_{\mathrm{s.o.}}(r) &= \frac{\hbar}{m_\pi c^2}\, V^{(0)}_{\mathrm{s.o.}}\, \frac{1}{r}\, f'_{\mathrm{s.o.}}(r) \\
V_{\mathrm{s.o.}} &= 6.2 \ \ (\mathrm{MeV}) \\
R_{\mathrm{s.o.}} &= 1.01\, A^{1/3}\ \mathrm{fm} \\
a_{\mathrm{s.o.}} &= 1.75\ \mathrm{fm}
\end{aligned} \tag{59}$$

We have calculated, σ_E, σ_{ABS}, σ_T and ε for $n+^{232}$Th in energy region 1(MeV < $E < 10$ MeV, using the Madland-Young8) (M-Y) optical potential and a PNC ›tential, Eq. 3, with a form factor $\nu(r)$ given by

$$\nu(r) = \frac{1}{2}\,\varepsilon_7\,\hbar c\,10^{-7}\left[1 + \exp\left(\frac{r - r_0\,A^{1/3}}{a}\right)\right]^{-1} \quad , \tag{60}$$

$$r_0 = 1.25\ \mathrm{fm} \quad , \quad a = 0.6\ \mathrm{fm} \quad .$$

The parameter ε_7 is properly adjusted to account for the experimental value of the longitudinal asymmetry $\varepsilon\left(P\frac{1}{2}\right)$.

The cross-sections are shown in figure 1 vs. E_n in the interval 10^5 MeV $< E_n <$ 10 MeV. We see that in the region $E_n < 10^{-3}$ MeV, σ_E is negligible compared to σ_{ABS}, whereas in the interval 10^{-3} MeV $< E_n < 1$ MeV it dominates. At $E_n > 1$ MeV both cross-sections are almost of equal magnitude.

In figure 2 we show the transmission coefficient for $S\frac{1}{2}$, $P\frac{1}{2}$ and $P\frac{3}{2}$ waves vs. E_n. At $E_n < 0.1$ MeV the S-wave transmission coefficient is several orders of magnitude larger than either the $P\frac{1}{2}$ or $P\frac{3}{2}$ ones, which are almost equal in magnitude. At $E_n > 1$ MeV the P-wave transmission coefficient becomes almost twice as large as the S-wave one.

From the transmission coefficients, we can evaluate the S- and P-wave strength functions S_0 and S_1, which results (at $E_n = 1$ eV) in

$$S_0 = \frac{T\left(S\frac{1}{2}\right)}{2\pi\sqrt{E_{\mathrm{Lab}}}\,(\mathrm{eV})} = 1.2\times10^{-4} \tag{61}$$

$$S_1 = \left[\frac{1}{3}\,\frac{T\left(P\frac{1}{2}\right)}{2\pi\sqrt{E_{\mathrm{Lab}}}\,(\mathrm{eV})} + \frac{2}{3}\,\frac{T\left(P\frac{3}{2}\right)}{2\pi\sqrt{E_{\mathrm{Lab}}}\,(\mathrm{eV})}\right] \Big/ \left(\frac{k^2R^2}{1+k^2R^2}\right) = 2.0\times10^{-4} \tag{62}$$

In Eqs. (28) and (29) T refers to the transmission coefficient and R in Eq. (29) is taken to be 1.25 fm. The above values of S_0 and S_1 are in reasonable agreement with the experimental ones[12)] given, respectively, by $0.84 \pm 0.07\times10^{-4}$ and $1.48 \pm 0.07\times10^{-4}$. The value of S_1 obtained in Ref. 9) with the Perey-Buck potential[13)], or equivalently the Wilmore-Hodgson potential[14)] is 5.29×10^{-4}. The difference between the Wilmore-Hodgson or Perey-Buck potential and the Madland-Young potential is clearly exhibited in Fig. 3 which shows the corresponding strength functions vs. the mass number.

We have also calculated the longitudinal asymmetry, ε, defined in Eq. (58). As in the cross-section, ε can also be divided into compound elastic, ε_E, absorption, $\varepsilon_{\mathrm{ABS}}$, and total

$\varepsilon_T = \varepsilon_E + \varepsilon_{\mathrm{ABS}}$ components. The results are shown in Figs. 4a and 4b vs. E_n. The values ε_T at small E_n are shown in Fig. 4c. The values of $\varepsilon_T\left(P\frac{1}{2}\right)$ at $E_n = 1$ eV is $\varepsilon_T\left(P\frac{1}{2}\right) = 6.7\times10^{-4}$ $(\varepsilon_7 = 1)$. Since the resonances in the $n+^{232}$Th system start at $E_n = 8$ eV, Fig. 5, (Ref. 15), we have to take the value of ε_T at this energy. From figure 4c we have $\varepsilon_T\left(P\frac{1}{2}\right) = 2.37\times10^{-4}$ $(E_n = 8,\ \varepsilon_7 = 1)$. Thus, to account for the experimental value of $\varepsilon\left(P\frac{1}{2}\right)$ in the resonance region $(E_n \geq 8$ eV), which is 8 ± 6% [5)], we have taken for $\varepsilon_7 = 307 \pm 240$. This value is more than three orders of magnitude larger than estimates based on meson-exchange models.

For the purpose of completeness, we show in figure 6, the PNC S-matrix elements $S_0^{j=1/2},1$ and $S_0^{j=3/2},1$ vs. E_n for $\varepsilon_7 = 1$. We see clearly that at $E_n < 1$ MeV, $S_{0,1}^{j+1/2}$ is completely dominant, as anticipated.

In Ref. 9), Koonin et al., obtained for ε_7 the value of 27. Their result was based on using the Perey-Buck[13)] potential as already mentioned earlier. To exhibit the difference between our result and theirs, we show in figure 7, the longitudinal asymmetry ε, for $\varepsilon_7 = 1$ and $E_n = 1$ eV vs. the mass number. We see clearly that whereas for $A < 100$, the P-B and M-Y potentials give practically identical results, they differ appreciably in the heavy target region. For $A+232$, the P-B yields $\varepsilon\left(P\frac{1}{2}\right) = 2.7\times10^{-3}$ $(E_n = 1$ eV, $\varepsilon_7 = 1)$. This is about four times bigger than our M-Y value. Note the conspicuous giant shape resonances present in both calculations.

The variation of ε with E_n is shown in figure 8. The $E_n^{-1/2}$ trend is clearly seen in both potentials. Koonin et al. took the value of ε of the P-B potential at $E_n = 1$ eV and compared it with the experimental value. We, however, took the M-Y value of ε at $E_n = 8$ eV and compared with data. This is the origin of the difference in our extracted value of ε_7.

V. DISCUSSION AND CONCLUSIONS

In this paper we have treated the problem of parity-nonconservation in thermal neutron scattering within the optical model. Taking for the OM potential a sum of a strong parity conserving and a weak parity non-conserving (PNC) interactions, we have treated the PNC piece within the distorted wave Born approximation. We have then calculated the average cross-section and the longitudinal asymmetry for the system $n+^{232}$ Th, recently studied by the TRIPLE collaboration[5]. We have found that the PNC interaction is more than 1000 times bigger than estimates based on meson-exchange models. In this respect we completely agree with the conclusions of Koonin, Johnson and Vogel[9], who found a factor 100 enhancement using a different optical potential.

The origin of this large discrepancy is not fully understood. At most, we conjecture a combined nuclear structure effects involving a doorway mechanism of the type discussed by Auerbach[8] and a strong modification of the PNC interaction inside heavy nuclei. Since the PNC interaction involves products of strong and weak meson-nucleon couplings, medium changes of both, following the reasonings of Brown and Rho[16], could give rise to rather large overall medium renormalization. Clearly, only detailed calculation would settle the question.

As a final remark, we mention that the longitudinal asymmetry exhibits giant shape resonances both in energy and in mass number. This should help plan future experiments when ε could be larger. Furthermore, a careful measurement of both the spin polarization and spin rotation at very small angles, where the fh^* term in Eqs. (51a) and (51b) become appreciable, may supply more information about the PNC interaction.

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FIGURE CAPTIONS

Fig. 1. The average cross-sections vs. E_n for $n+^{232}$Th, calculated with the Madland-Young potential. a) Regular energy scale, b) logarithmic energy scale.

Fig. 2. The transmission coefficients $T\left(S\frac{1}{2}\right)$, $T\left(P\frac{1}{2}\right)$, and $T\left(P\frac{3}{2}\right)$ vs. E_n.

Fig. 3. The singlet, S_0 (1a), and triplet, S_1 (1b) strength functions vs. the mass number A. The full curve is obtained with the Wilmore-Hodgson (Perey-Buck) potential while the dashed curve is obtained with the Madland-Young potential. The neutron Lab. energy is $E_n = 1$ eV.

Fig. 4. The longitudinal asymmetry coefficient $\varepsilon\left(P\frac{1}{2}\right)$ vs. E_n. (See text for details). a)+b) 1 eV $< E_n <$ 10 MeV, c) 1 eV $< E_n <$ 1 KeV.

Fig. 5. The $n+^{232}$Th resonances. a) S-resonances, b) P-resonances, c) all resonances. (From Ref. 14).

Fig. 6. The PNC S-matrix elements, $\left|S_{0,1}^{j=1/2}\right|$ and $\left|S_{1,2}^{j=3/2}\right|$ vs. E_n.

Fig. 7. The asymmetry coefficient $\varepsilon\left(P\frac{1}{2}\right)$ for $\varepsilon_7 = 1$, $E_n = 1$ eV vs. A. Details of curves are the same as in Fig. 3.

Fig. 8. The longitudinal asymmetry coefficient $\varepsilon\left(P\frac{1}{2}\right)$ for $\varepsilon_7 = 1$, $A = 232$ vs. the neutron Lab. energy. Details of curves are the same as in Fig. 1.

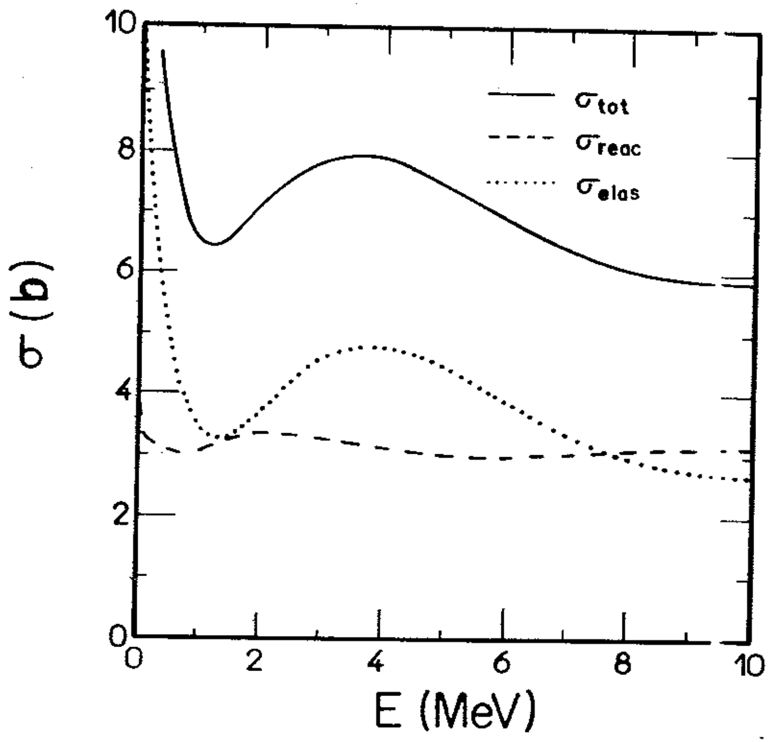
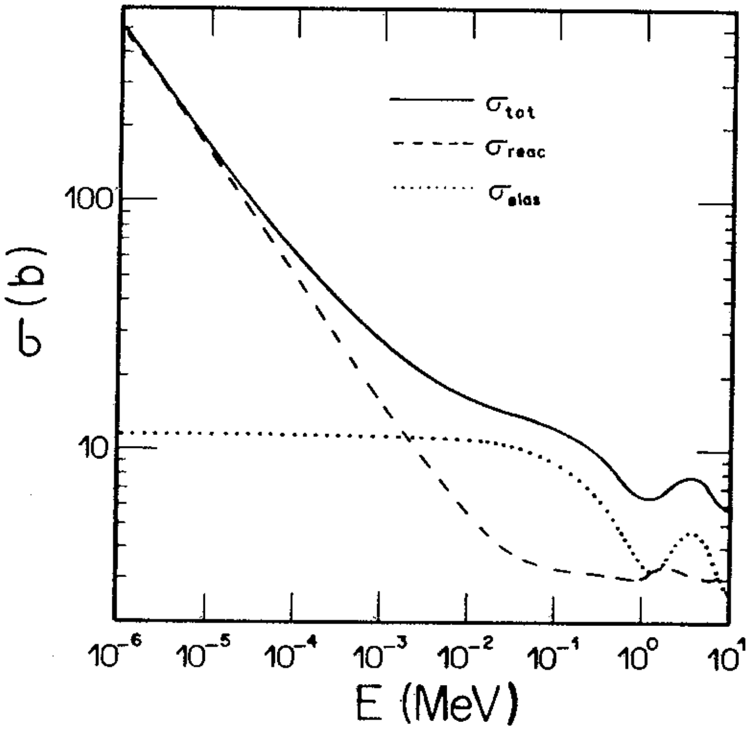
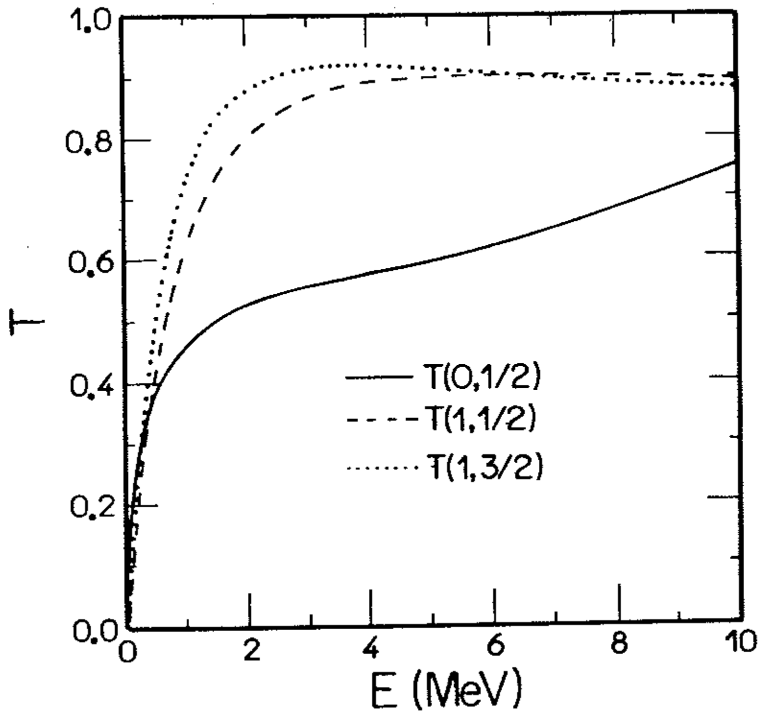


Fig. 1a



σ_{tot}
σ_{reac}
σ_{elas}
100
10
σ (b)
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}
E (MeV)

Fig. 1 b



1.0
0.8
0.6
0.4
0.2
0.0
T
T(0,1/2)
T(1,1/2)
T(1,3/2)
0 2 4 6 8 10
E (MeV)

Fig. 2a

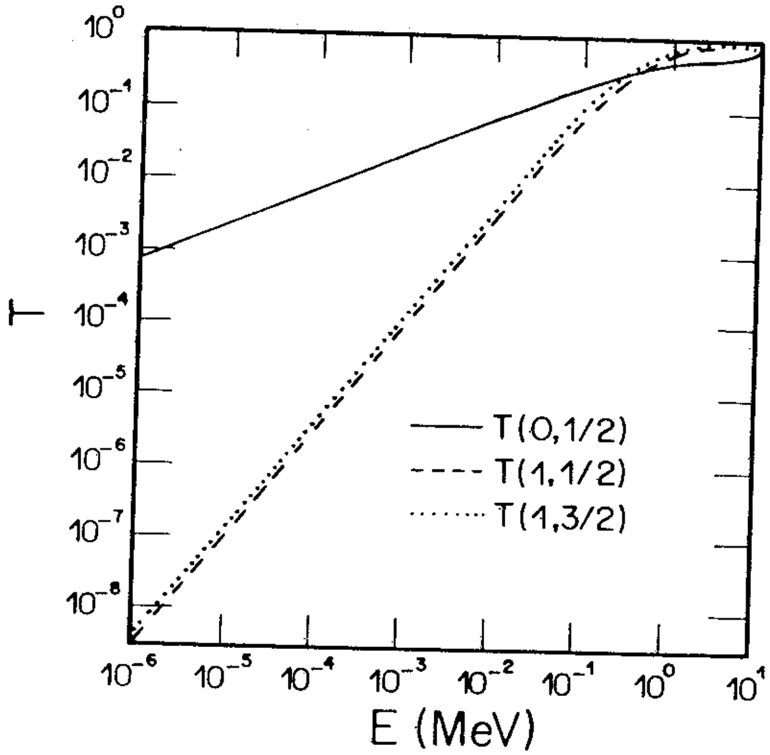


Fig. 2b

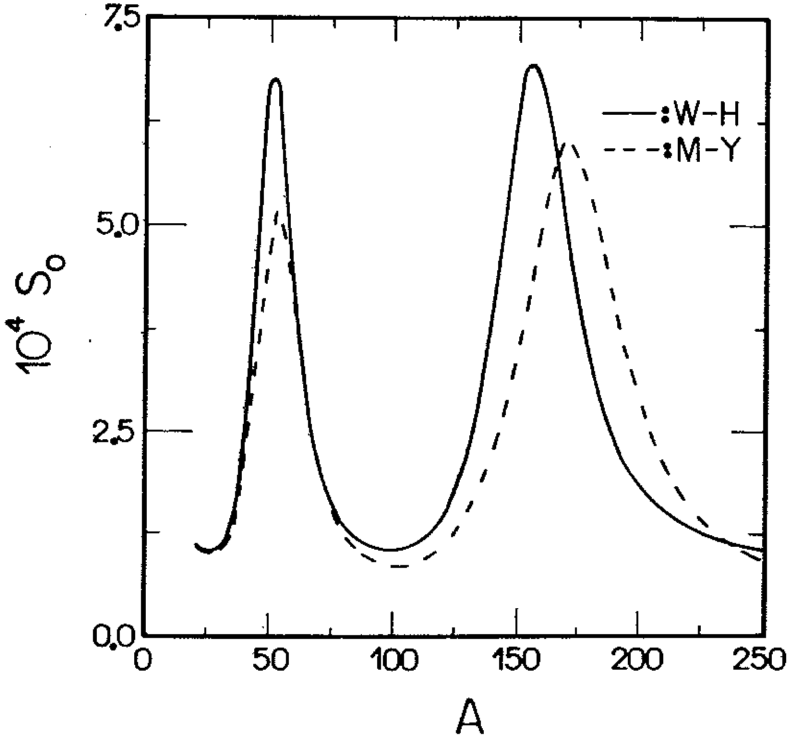
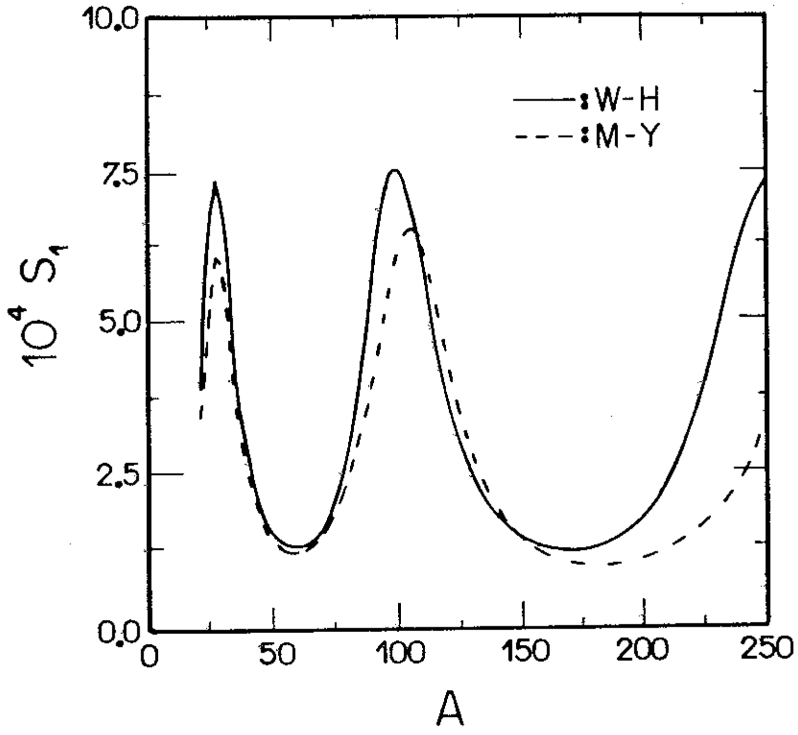
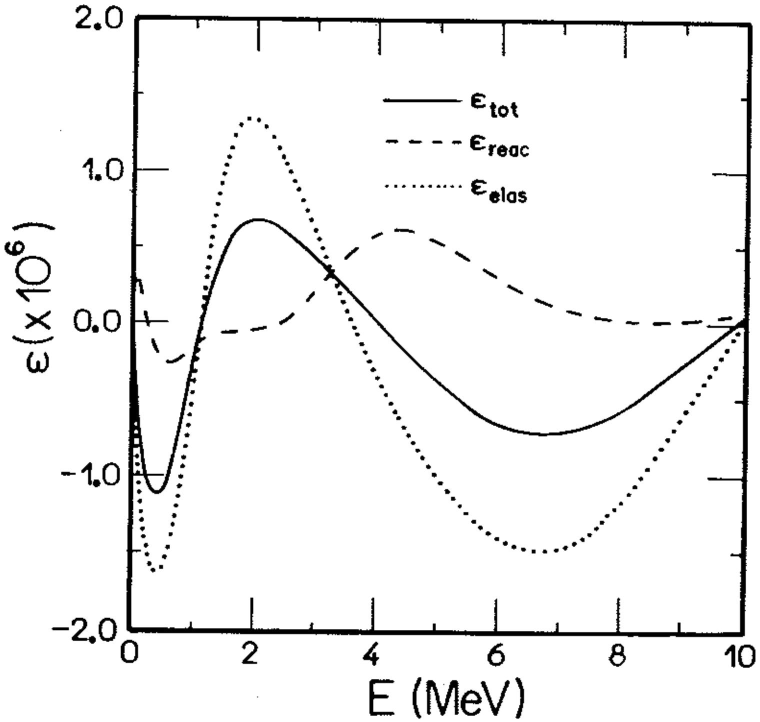


Fig. 3a



10.0
7.5
5.0
2.5
0.0
0
50
100
150
200
250
A
$10^4 S_1$
: W-H
: M-Y

Fig. 3b



2.0
1.0
0.0
-1.0
-2.0
0
2
4
6
8
10
E (MeV)
$\varepsilon (\times 10^6)$
ε_{tot}
ε_{reac}
ε_{elas}

Fig. 4a

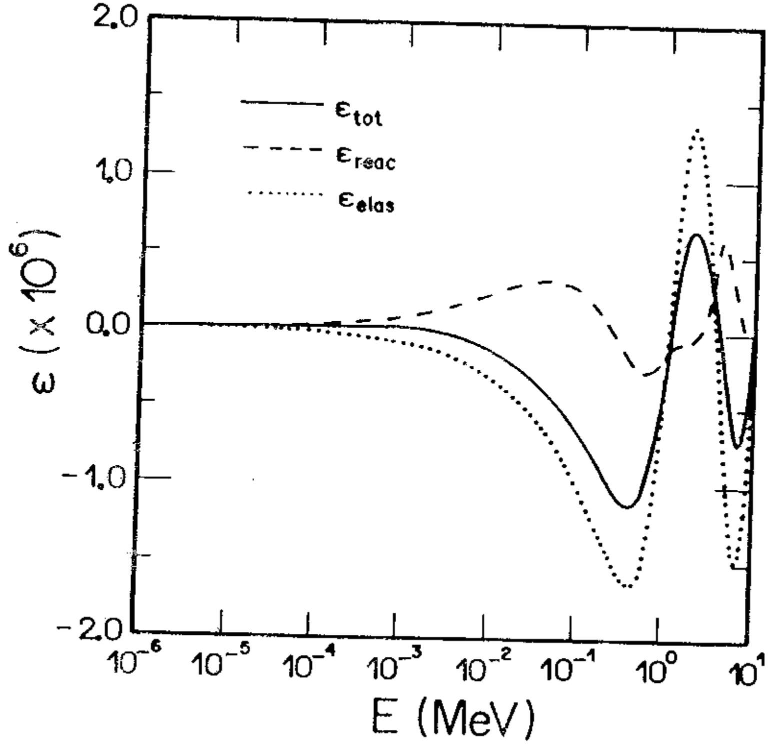

2.0
1.0
0.0
-1.0
-2.0
ε_{tot}
ε_{reac}
ε_{elas}
ω ($\times 10^6$)
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}
E (MeV)

Fig. 4b

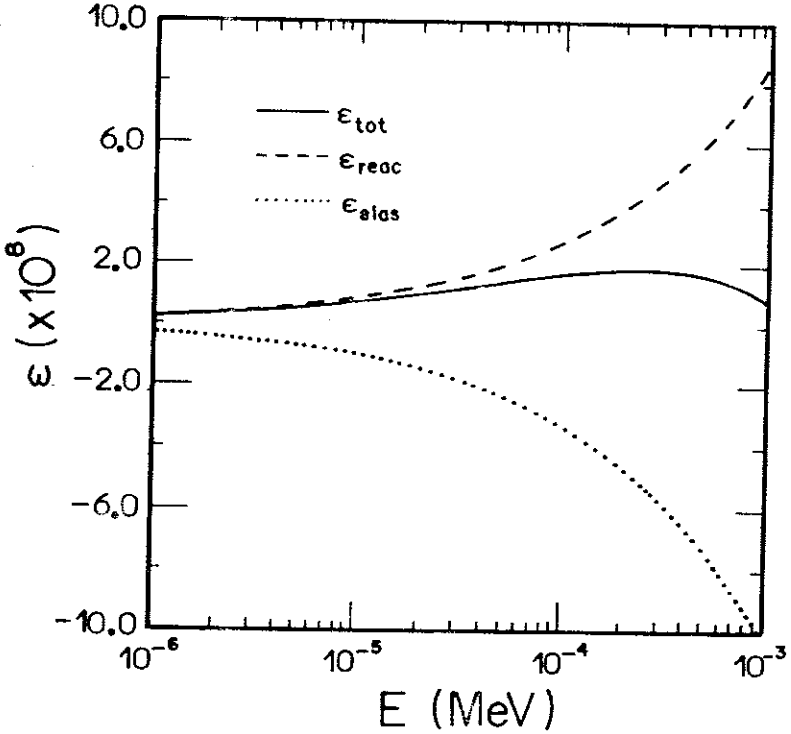

10.0
6.0
2.0
-2.0
-6.0
-10.0
ε_{tot}
ε_{reac}
ε_{elas}
ω ($\times 10^8$)
10^{-6} 10^{-5} 10^{-4} 10^{-3}
E (MeV)

Fig. 4c

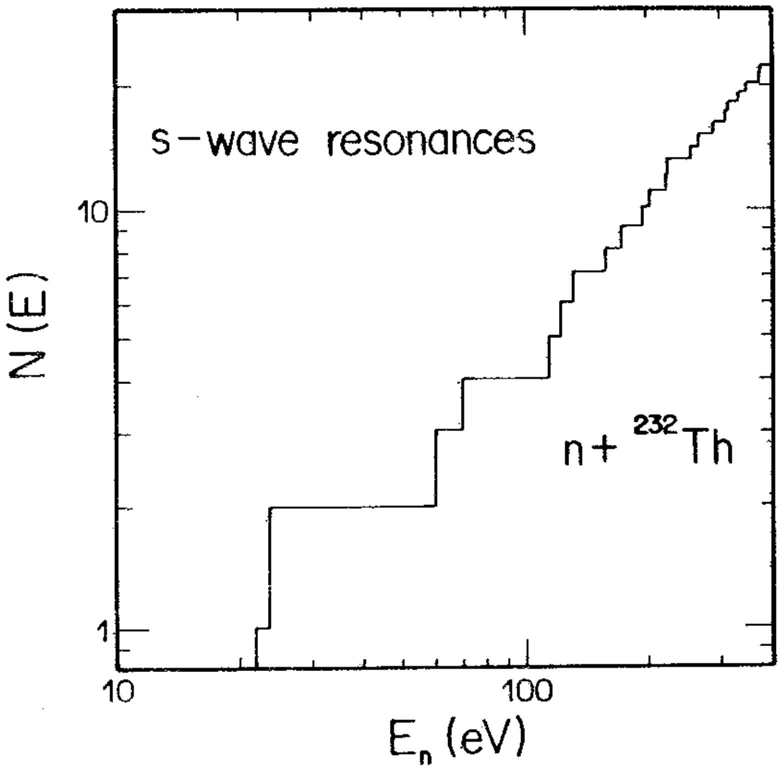

s-wave resonances
n+ 232Th
N (E)
10
1
10
100
En (eV)

Fig. 5a

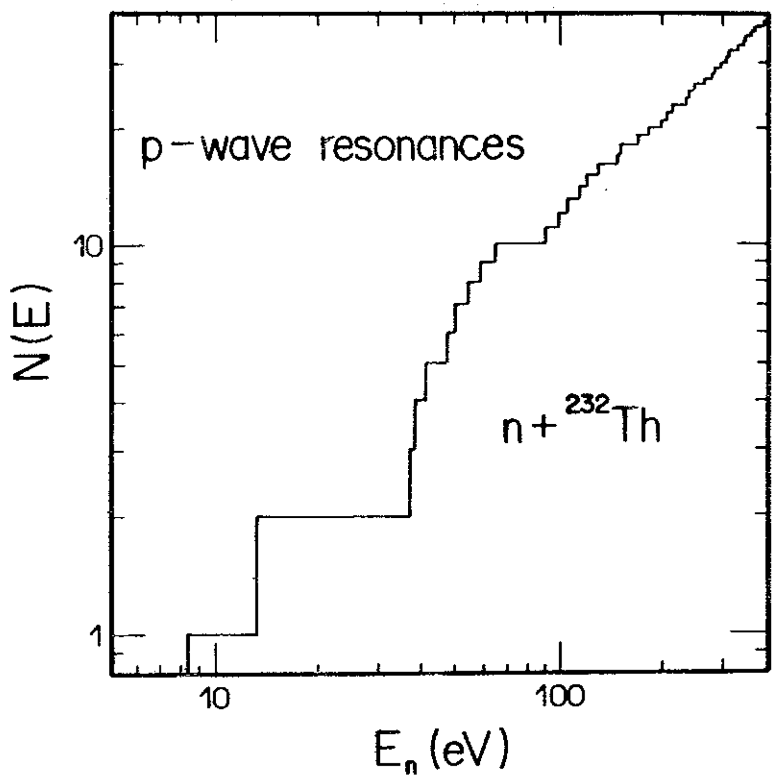

p-wave resonances
n+ 232Th
N(E)
10
1
10
100
En (eV)

Fig. 5b

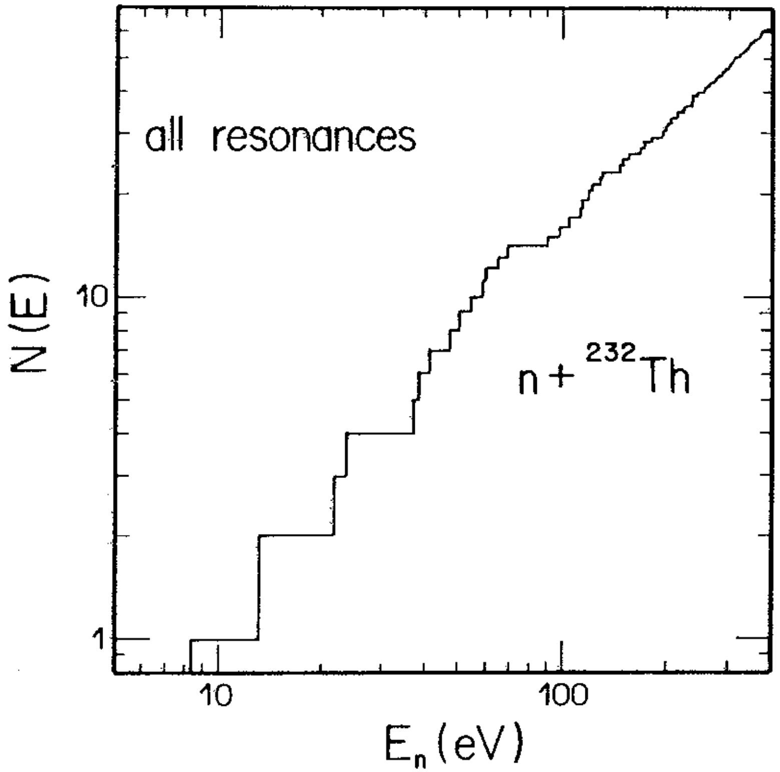

all resonances
n + 232Th
N (E)
10
1
10
100
E_n (eV)

Fig. 5c

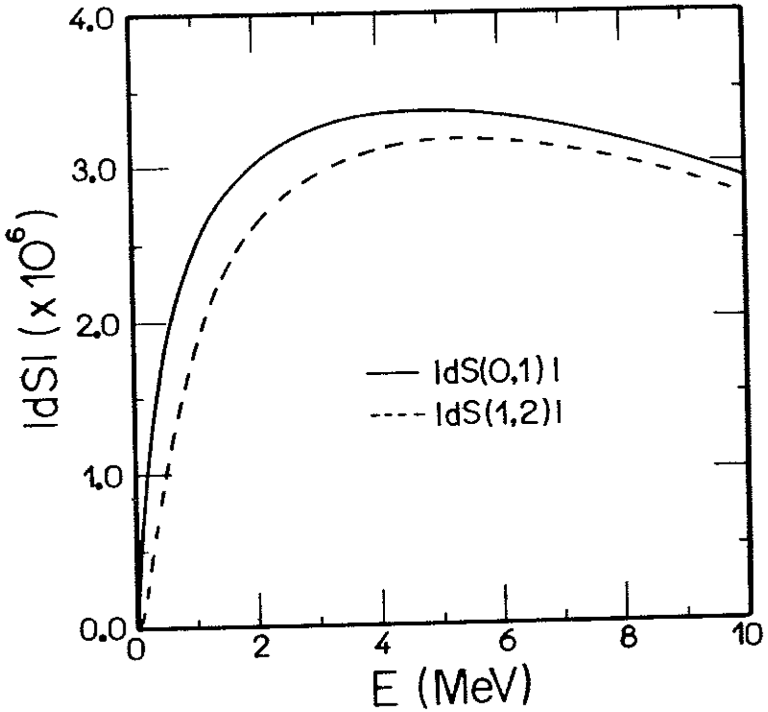

4.0
3.0
2.0
1.0
0.0
0
2
4
6
8
10
|dS| (x 10^6)
|dS(0,1)|
|dS(1,2)|
E (MeV)

Fig. 6a

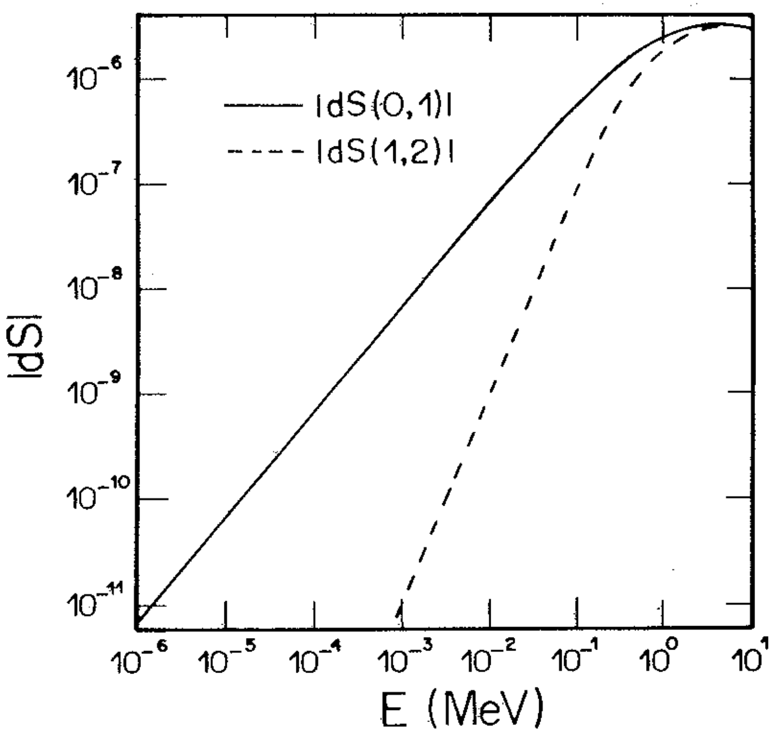


Fig. 6b

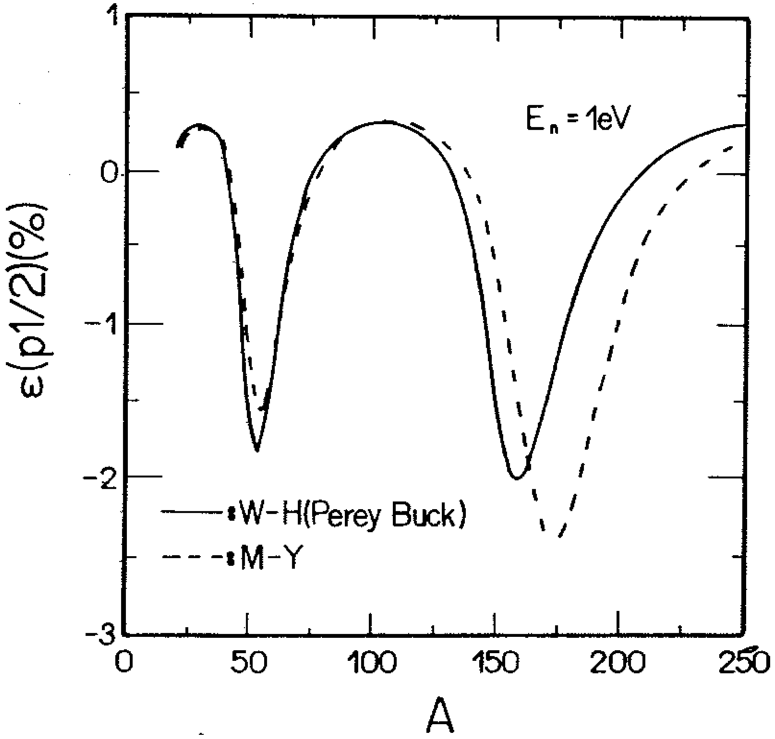


Fig. 7

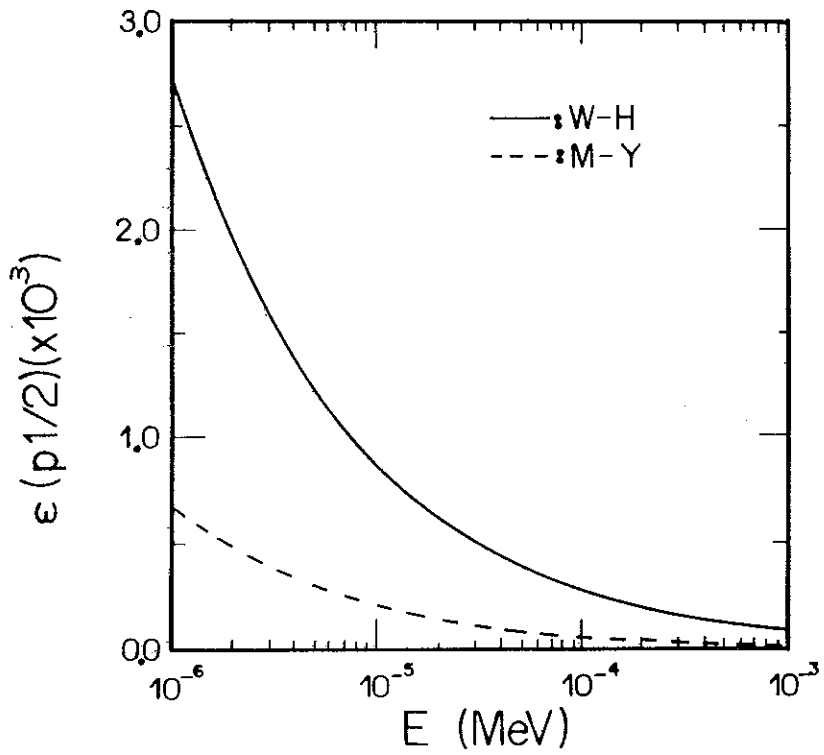


Fig. 8