Control of trajectories of the kicked logistic map

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ABSTRACT

The main topological changes of the logistic map attractors, caused by a sequence of periodic kicks, are reported. This procedure brings up a three-parameter kicked logistic map with distinct dynamic features. Thus, its parameter space structure exhibts highly interleaved sets with different attractors, and complex basins of attraction are created. Consequently, the logistic map attractors can be modified or suppresed by these perturbations. Furthermore, additional roads to chaos, and abrupt attractor changes are identified in the new bifurcation diagrams.

1. INTRODUCTION

In the last years, much work has been done in many scientific disciplines such as Physics, Engineering, Chemistry, etc. in order to investigate the response of periodic and chaotic systems to aplied periodic forces[1,2]. Significant knowledge has been obtained from this work : chaos can be suppressed by external forcing[3], and striking changes in the dynamics occur even for weak perturbations[4]. However, despite all the recent remarkable progress, this subject is still far from being fully explored.

Although familiar concepts such as bifurcation diagrams, attractors, basin of attractors, etc. are fundamental to investigate the dynamical systems in terms of the variables governing the dynamics, the structure of parameter space can indicate some important characteristics of these systems[5,6]. Thus, it seems worthwhile to look for sets of parameters for which dynamical systems present similar characteristics such as periodicity, topology of attractors, etc.. Such analysis has been applied to unidimensional maps with three parameters[7,8]. A particular cut in the parameter space, fixing two parameters, leads to bifurcation diagrams by plotting the corresponding variable values as a function of the third parameter. In fact, this procedure is a particular case of a more general useful tool, isodiagrams, recently introduced to extract information from the parameter space of dynamical systems[5].

In the treatment of nonlinear dynamics, usually described by differential equations, nonlinear problems are often reduced to discrete maps by the Poincaré section. In particular, unidimensional maps have been the object of increasing interest, both due to their intrinsic mathematical richness and to the large number of dynamical systems which experimentally display transitions into chaos via the universal bifurcation scenario[2].

The most extensively studied of these maps is the logistic map[2] :

$$X_{n+1} = bX_n(1 - X_n), \tag{1}$$

where $n \geq 0$ and $0 \leq b \leq 4$.

Other similar unidimensional maps have been used to investigate relevant characteristics of nonlinear systems as, for example, the new roads to chaos[9], the creation and destruction of periodic orbits[10], the control of chaos[11], the periodic entrainment[12], and the coupling between maps[13].

In this paper we consider the logistic map perturbed by a sequence of kicks with a costant amplitude q and a period t:

$$X_{n+1} = bX_n(1 - X_n) + q\delta_{n,t} \tag{2}$$

where $\delta_{n,t} = 1$ if $\frac{n}{t}$ is an integer and 0 if not. This kicked logistic map exibts a new specific dynamics[7].

The aim of the present work is to study numerically the control of the logistic map oscillations through these kicks, and to investigate the dynamical alterations introduced by these pertubations. Thus, we present, for fixed kick periods, the structure of the two-dimensional parameter space, $b \times q$, with components corresponding to periodic or chaotic attractors. Furthermore, suppression of chaos, periodic entrainment, roads to chaos, and other dynamical properties are shown in bifurcation diagrams.

2. NUMERICAL ANALYSIS

For the quantification of chaos and order, the Lyapunov esponent[2]

$$\lambda = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \mid \frac{\partial X_{n+1}}{\partial X_n} \mid \tag{3}$$

is calculated for the kicked logistic map trajectories. Unless stated otherwise, the first value of the iteration is set to $X_0 = 0.2$ in this work. The order (i.e. predictability) is indicated by $\lambda < 0$. The chaos (i.e. sensitive dependence on initial conditions) is indicated by $\lambda > 0$.

A double precision arithmetic was used throughout the work. Before applying Eq.(3), 1000 iterations were performed in order to allow transient to die away; after that satisfactory convergence of λ was achieved by setting $N = 40000$. Furthermore, an initial transient of 100000 iterations has been left out of considerations of the bifurcation diagrams, which have been done with 200 consecutive points for each value of the varying control parameter.

3. STRUCTURE OF THE PARAMETER SPACE

In this section we discuss the structure of sets of parameters b, q, and t, characterized by chaotic or periodic oscillations obtained from Eq.(2).

Examples of these structures are schown in Figs. 1 and 2. In these figures the points with $q = 0$ stand for the logistic map; the others illustrate modifications, caused by the periodic kick perturbations, on the umperturbed orbits.

For each of these figures, the Lyapunov coefficients of 90000 points (300×300) in the plane $b \times q$ were calculated. Points with $\lambda > 0$ (in black) and $\lambda < 0$ (in white) form complex interconnected sets. This can be better seen in Fig.3 , which shows magnifications of two small areas indicated in Fig.2.

For $q > 1 - \frac{b}{4}$ the system is not, in general, limited to X values in the interval $0 < X < 1$, i.e., no finite attractor exist, and X is driven to infinity. Despite this, even in these parameter space region, there are sparse thin sets of points standing for chaotic or periodic attractors (Fig. 3A). For these figures, λ was recalculated, considering a transient of 3000 iterations and $N = 100000$, without any change in this observed structure. Nevertheless, besides their kick periodic dependence, the position and the structures of these sets depend also on the initial conditions X_0.

The basins of attraction, typical of the previous mentioned region, can be illustrated by peculiar broken biffurcation diagrams, with sparse periodic and chaotic attractors separated by white parameter regions without chaotic attractors. Fig. 5 shows two of these diagrams, corresponding to a fixed q line in the Fig. 3A, obtained for two different initial conditions X_0. The slight differences observed in these figures are examples of the oscillatory mode alterations and of the modifications in the basins of attraction as X_0 is changed.

To parameters satisfying the condition $q > 1 - \frac{b}{4}$ correspond orbits that diverge after visiting points in the interval

$$\frac{1}{2}(1 + \sqrt{1 + \frac{4}{b}(q-1)}) > X > \frac{1}{2}(1 - \sqrt{1 + \frac{4}{b}(q-1)}). \qquad (4)$$

Thus, attractors exist for initial conditions if their transitory orbits do not intercept any points inside the interval determined by the condition (4).

4. ROADS TO CHAOS

As it can be seen from the parameter space structures, the introduction of the new two control parameters, q and t, can alter the logistic map attractor. In this section we describe alterations, caused by the kick sequences, on the well known period-doubling road to chaos observed on the logistic map orbits[2].

In order to investigate the roads to chaos, magnifications of bifurcation diagrams in appropiate parameter sets have been examined.

Figs. 6,7, and 8 show typical bifurcation diagrams obtained for t=2, and b or q as the second fixed parameter. Since the parameters in these figures satisfy the condition $q < 1 - \frac{b}{4}$, their observed attractors do not depend on the initial conditions X_0.

Figs. 6 and 8 show period doubling cascades to chaos, with sequences of periodic orbits with periods 2^j (j=0,1,2,$\cdots$). These orbit periods increase geometrically inside each window, as for the logistic map[2]. Long period non chaotic orbits, observed in descontinuous maps[13], were not found.

Fig. 7 shows inverse cascades, as it has been observed for descontinuous unidimensional maps[9,13,14,15]. However, the observed orbit periods decresase only geometrically, with no arithmetically period increase (by adding the first element to the previous window), as in descontinuous maps[9,13]. On the other hand, inverse cascades were not observed, for fixed t and q, in the bifurcation diagrams with b as the growing parameter.

The finite attractor disappearance[9] and abrupt entrances into chaos were observed in growing q bifurcation diagrams.

In the investigated diagrams, the period p stable orbits, $p = Nt$ (N=1,2,3,$\cdots$) ,appear as a result of period-doubling bifurcations or they proceed away from chaos (in this last case p can also be odd).

5. CONCLUSIONS

The investigated kicked logistic map exhibts, among other properties, the suppression of chaos observed in the umperturbed logistic map, the periodic entrainment, and the creation and destruction of periodic orbits. This map has highly interleaved basins of attraction on the phase space, and highly interleaved regions with different attractors in the parameter space. Besides the transitions into chaos via Feigenbaum scenario, the obtained bifurcation diagrams show, for specific regions in the threedimensional parameter space, other roads to chaos.

As it was seen in this paper, the control of the logistic map orbits, through a sequence of periodic kicks, alters the dynamics of the umperturbed system. However, it is also possible to control these orbits, without altering the dynamics, by applying the method developed by Ott, Grebogi, and York[11,16], which converts a chaotic attractor to any one of a large number of possible attracting periodic motion by making only small time-dependent parameter perturbation.

Intermitency has been also investigated in the kicked logistic map, and its occurrance is useful to control the orbits, since in this regime very small b or q variations affect significantly the attractor topology.

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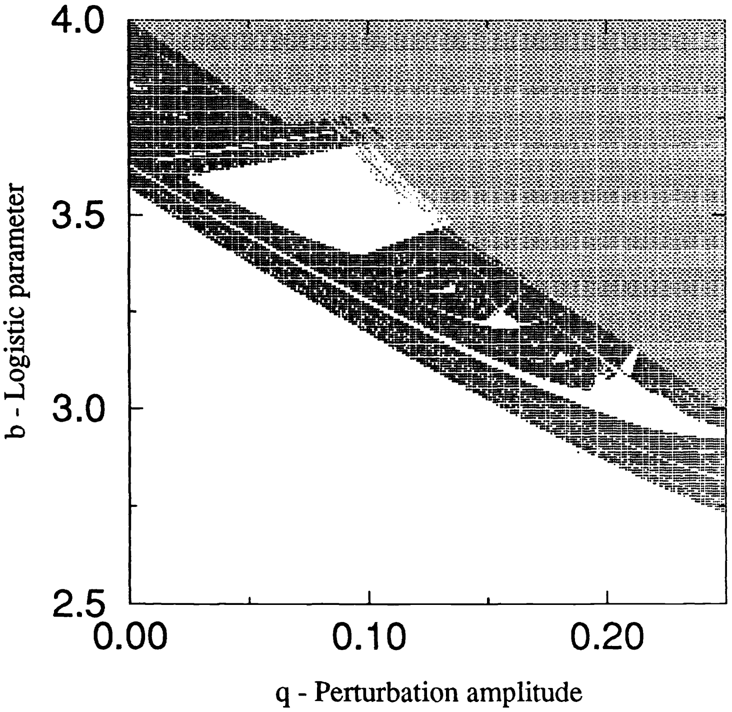


Fig.1 Attractor regions in the parameter space for kick period t = 2. Black(white) pixels represent points with $\lambda > 0$ ($\lambda < 0$), and gray pixels represent points without limited attractor.

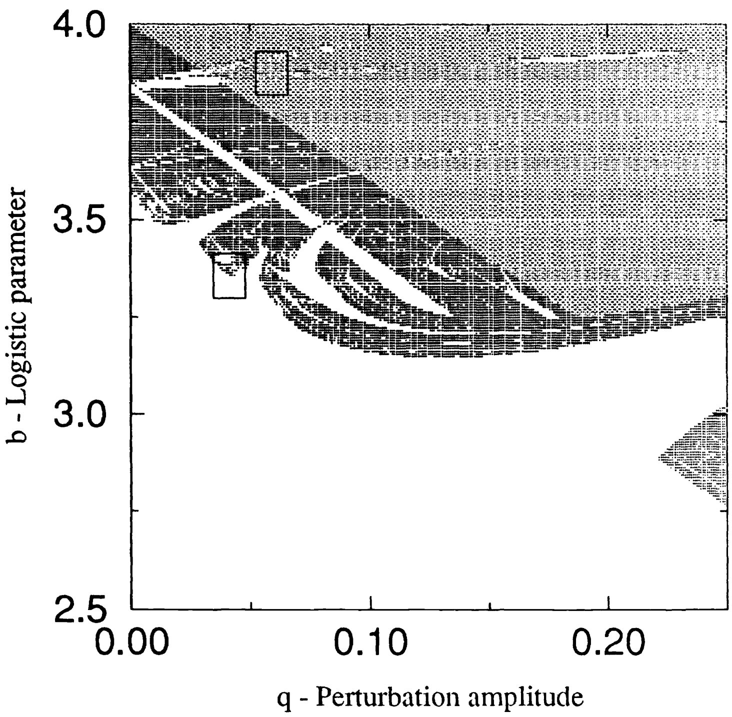


Fig.2 Attractor regions in the parameter space for kick period t = 3. Black(white) pixels represent points with $\lambda > 0$ ($\lambda < 0$), and gray pixels represent points without limited attractor.

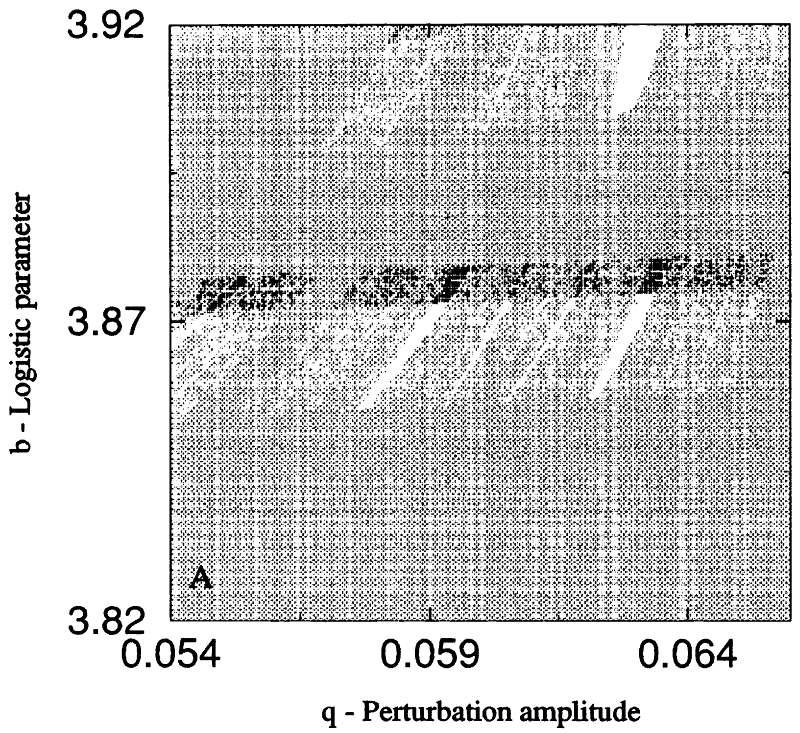
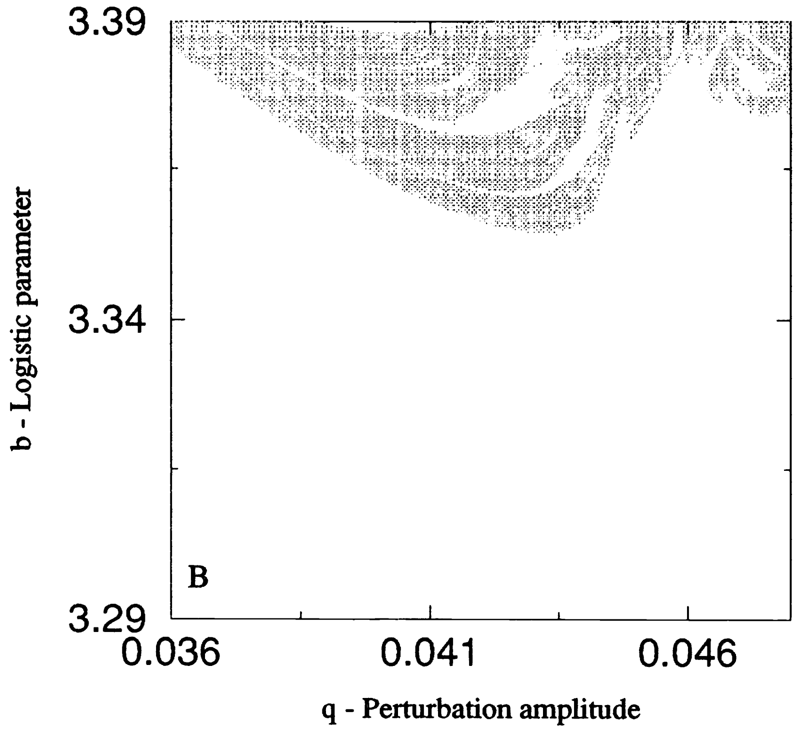


Fig.3 Magnifications of two regions indicated by squares in the Fig.2 , for Xo=0.2 and t=3.



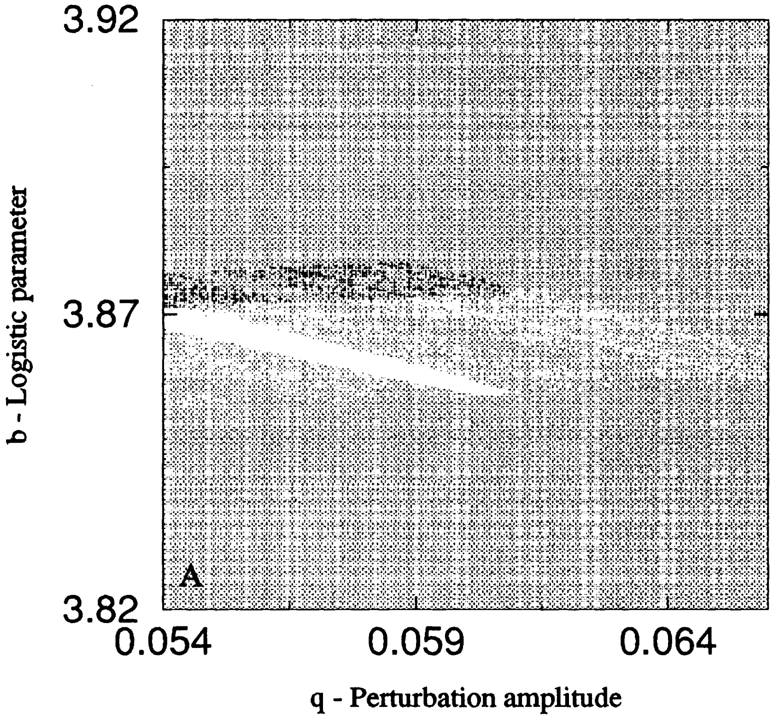
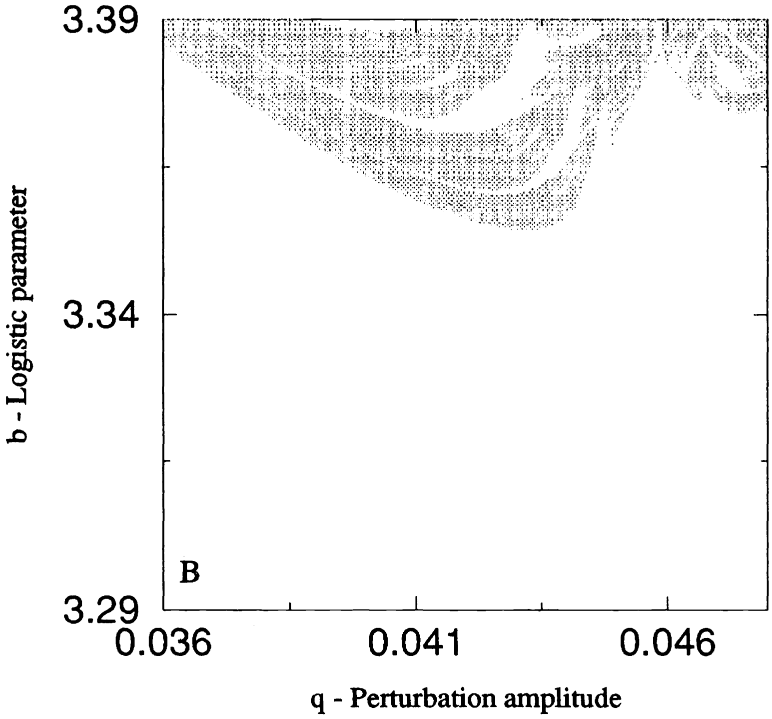


Fig.4 Magnifications of two regions indicated by squares in the Fig.2 , for Xo=0.3 and t=3.



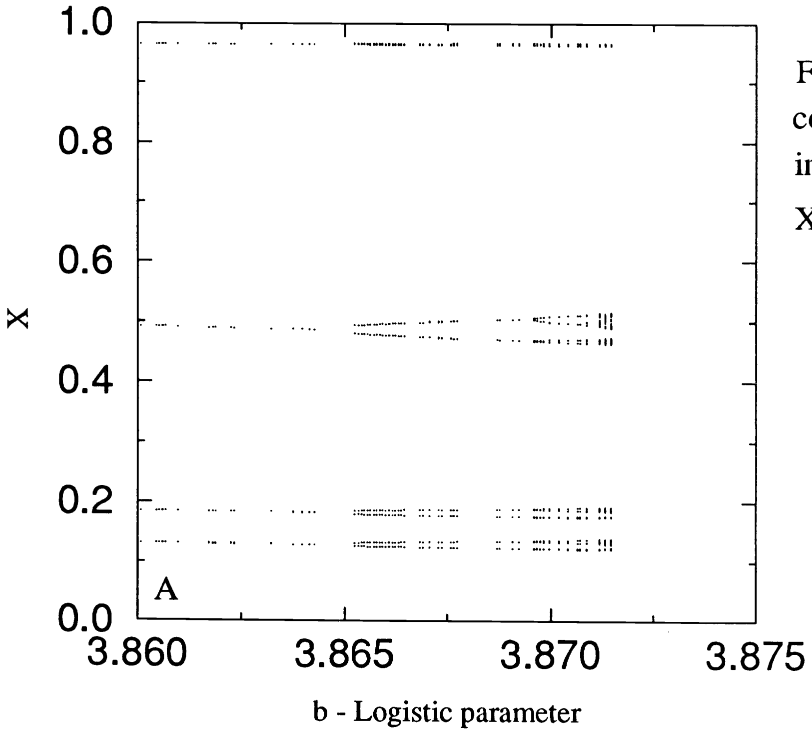
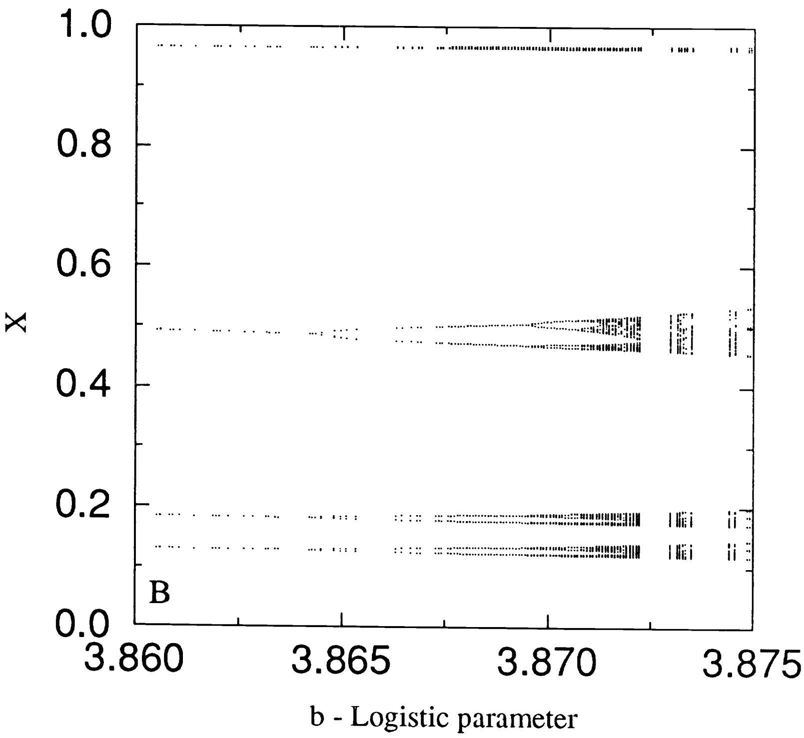


Fig.5 Broken bifurcation diagrams corresponding to q=0.054 indicated in Figs. 3A and 4A for Xo = 0.2 (A) Xo = 0.3 (B) and t = 3.



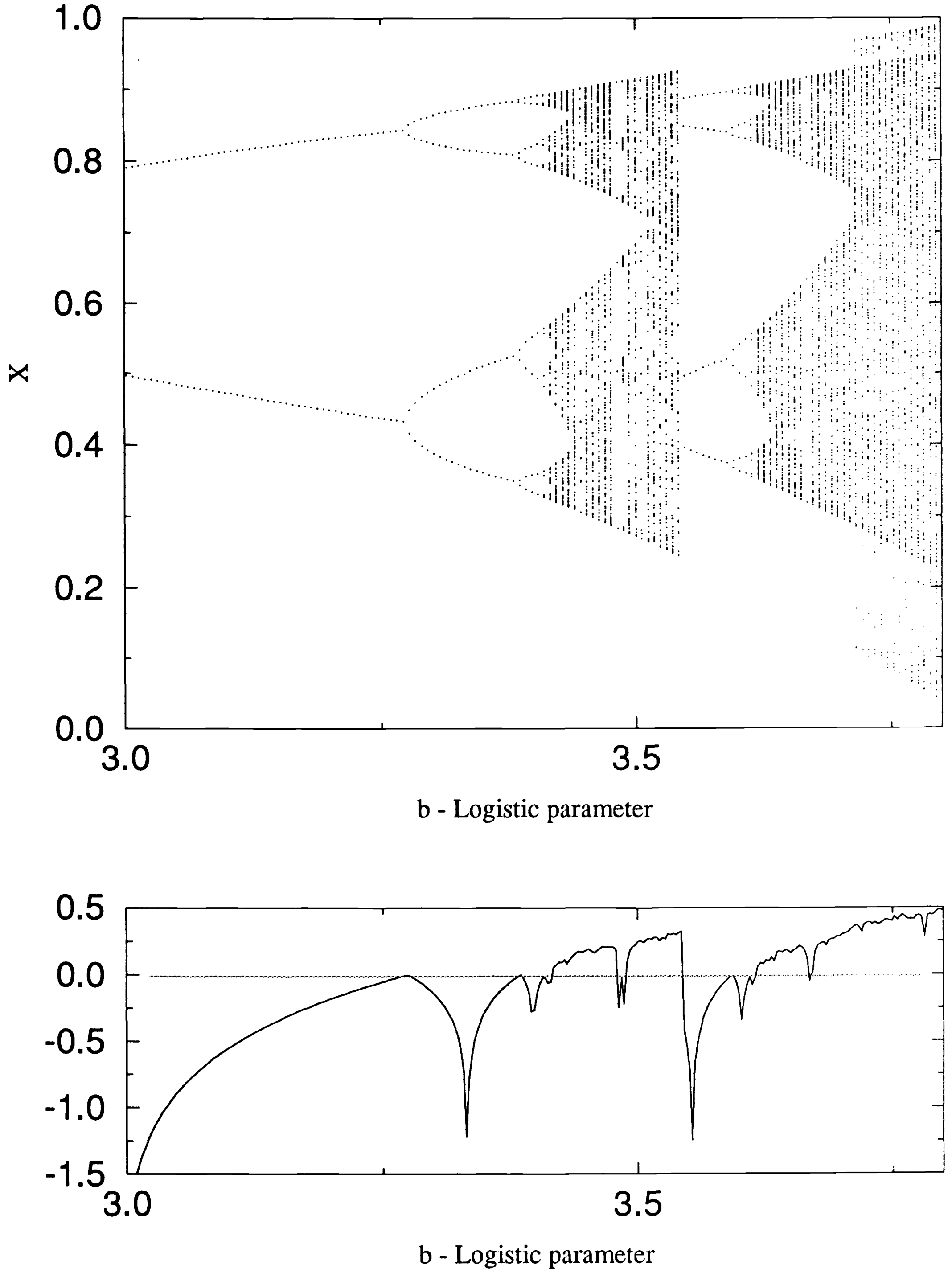


Fig.6 Bifurcation diagram and Lyapunov coefficient,λ,for q=0.04,t=2.

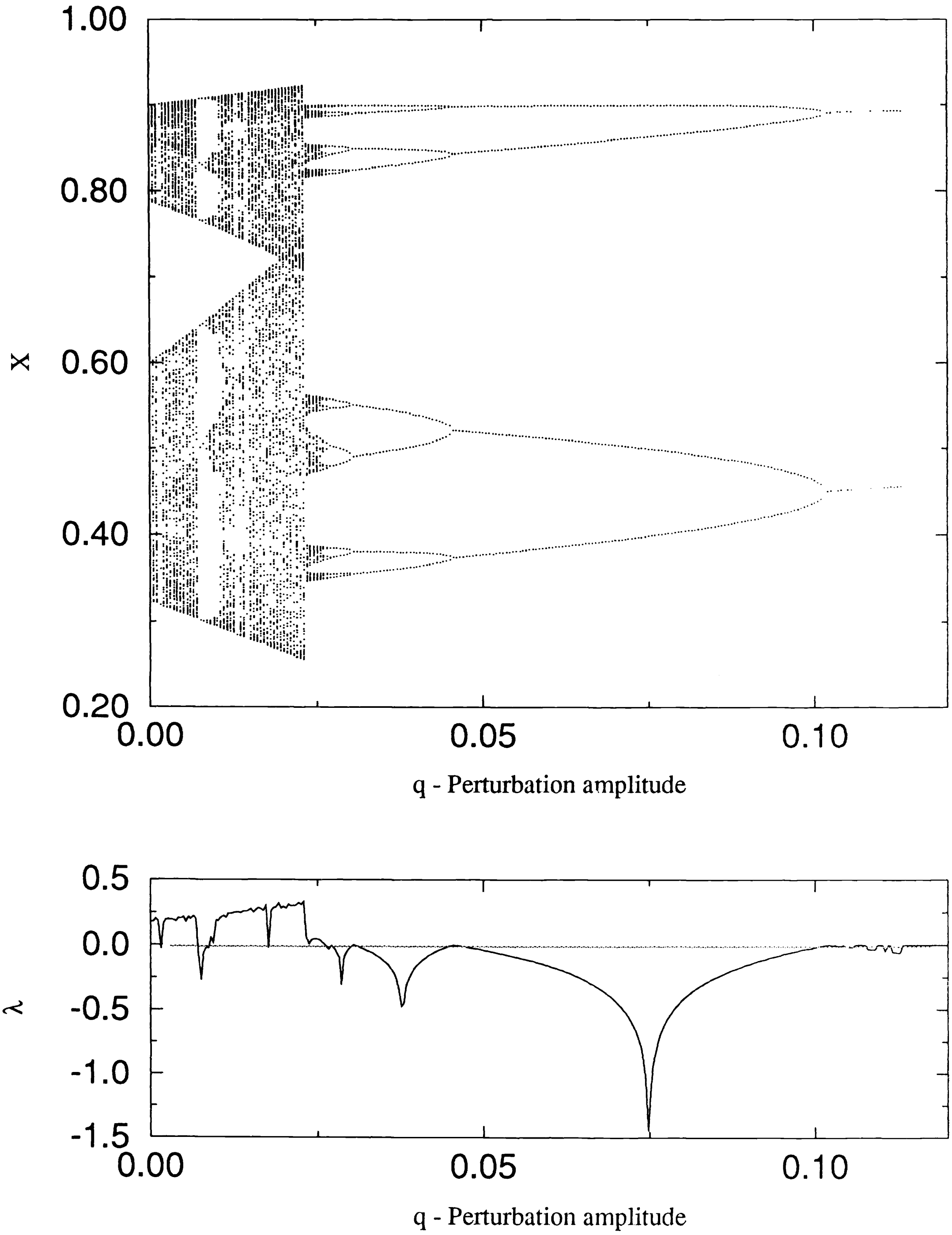


Fig.7 Bifurcation diagram and Lyapunov coefficient for b=3.6, and t=2, showing inverse cascades.

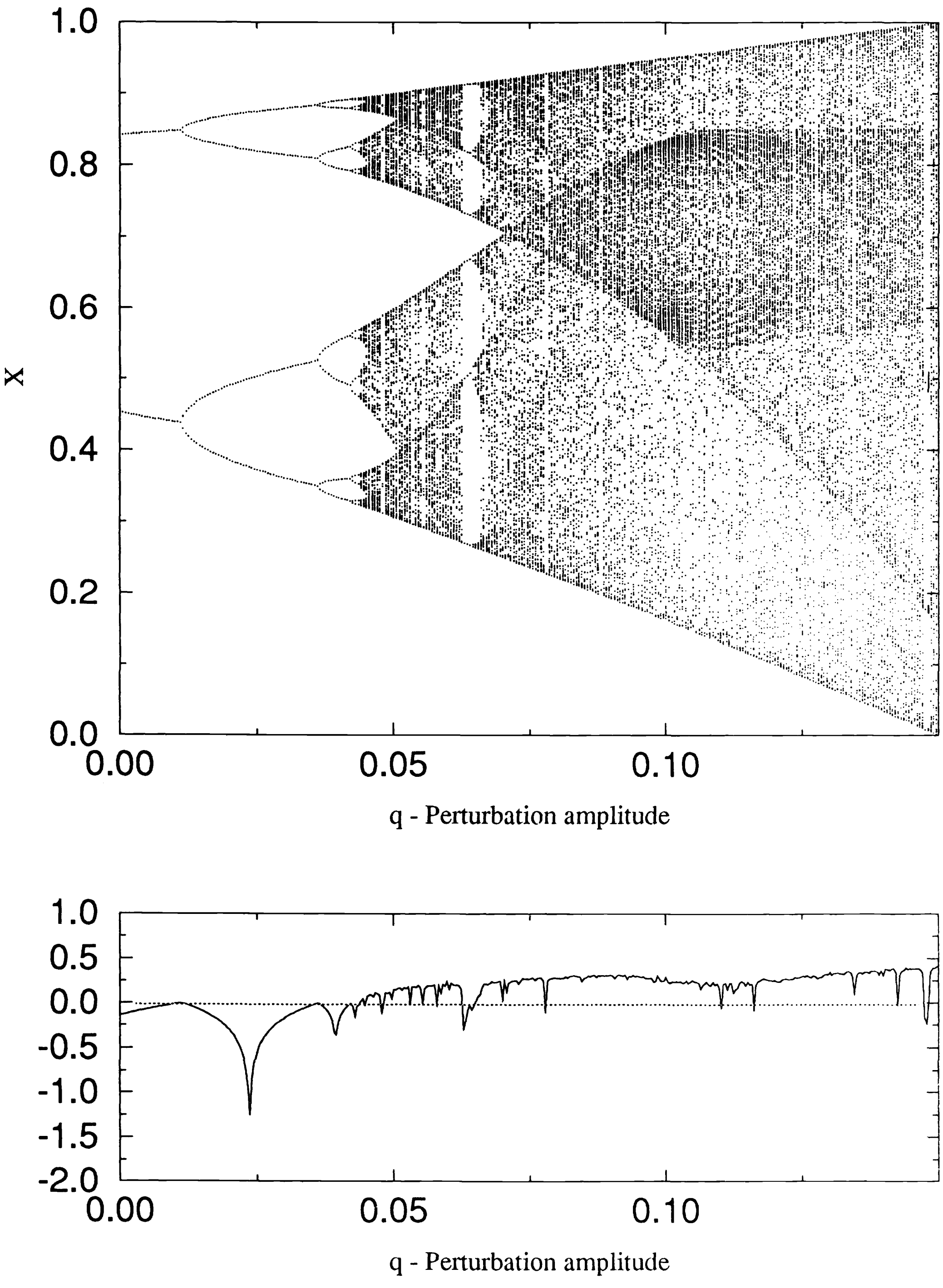


Fig.8 Bifuraction diagram and Lyapunov coefficient, λ, for b=3.4, and t=2, showing routes to chaos.