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Fuzzy computational control for real Chua circuit

Marco A. Garms [a,*], Marco T.C. Andrade [a], Iberê L. Caldas [b]

[a] Escola Politécnica da Universidade de São Paulo, CP 05508–970, São Paulo, SP, Brazil
[b] Instituto de Física, Universidade de São Paulo, CP 66318, São Paulo, SP 05315–970, Brazil

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Abstract

We present a computational procedure to control an experimental chaotic system by applying the occasional proportional feedback (OPF) method. The method implementation uses the fuzzy theory to relate the variable correction to the necessary adjustment in the control parameter. As an application we control the chaotic attractors of the Chua circuit. We present the developed circuits and algorithms to implement this control in real time. To simplify the used procedure, we use a low resolution analog to digital converter compensated for a lowpass filter that facilitates similar applications to control other systems.

1. Introduction

Chaos control in mechanical and electrical engineering systems has been much investigated in the last years [1,2]. Among these systems are electric circuits with several applications as emission in lasers [3] or the demand of energy in electric power systems [4a].

Many of these applications are based on the method OGY (Ott, Grebogi and Yorke) of chaos control [5] that stabilizes unstable periodic orbits, immersed in the chaotic attractor, by small alterations of a control parameter.

A variant of the OGY method is the occasional proportional feedback – OPF that, instead of using the system dynamics to vary a parameter appropriately, calculates the correction in one of the variables to force it to pass through a small interval fixed in the phase space [6]. The OPF has been applied in several real situations by using analogical circuits [7]. However, digital implementations of this method were limited by the number of bits of the analogical digital conversion (AD) and by the time of this conversion [4b,8].

Even with those methods that prescribe the control variation, the sequence of the applied variations can still be improved by a learning or preliminary evaluation. For that, it is convenient to apply concepts of the fuzzy theory. In this way, actions can adjust the control parameter variations, even with imprecise information on the reference variable evaluation [9,10]. The fuzzy theory has been already applied to control chaotic systems [8,11].

The Chua circuit (CHC) [12] has been used to study the control of dissipative chaotic systems [13] due to the relative easiness of its implementation and due to its versatility in the generation of several kinds of attractors.

* Corresponding author.
E-mail address: marco.garms@poli.usp.br (M.A. Garms).

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Usually, the Chua circuit has been investigated by using analogical circuits or by simulations. Even so, to implement some experimental works, as in Ref. [4b] for chaos control and in Ref. [14] for circuits synchronization, AD interfaces have been connected to computers to read the signals.

This work uses computers to control an experimental Chua circuit by means of fuzzy techniques what has not yet been fully discussed in other studies. With this procedure, the use of computers allows us to follow the experimental circuit control besides propitiating a larger flexibility on the storage and on the orbits fuzzy treatment. Moreover, to treat the circuit analogical signals, the use of a low resolution (8 bits) converter AD was tested, compensated by a low-pass filter implemented by software.

The structure of this presentation is the following: the implementation is described in Sections 2 and 3 (and in Appendix A); the characterization is in Section 4. Section 5 contains the control of the Chua circuit followed by the carried through test conclusions. Final remarks are in Section 6.

2. Electric circuit

The circuit is a dissipative dynamical system, defined for the electric circuit of Fig. 1, composed by reactive elements (inductor L, capacitors C_1 and C_2), linear resistors (R and R'), and by a negative non-linear resistive element (R_{NN}).

The differential equation that describes this system is of third order, being the phase space defined by the variables v_{C_1} (tension on the capacitor C_1), v_{C_2} (tension on the capacitor C_2), and i_L (current for the inductor L) [12,15].

Depending on the parameters, related to the values of the elements that compose the circuit, solutions after transient correspond to periodic or chaotic attractors.

Fig. 2 shows the electronic circuit used to implement the experimental Chua circuit.

The element R_{NN} (non-linear negative resistor) shown in Fig. 2, and with the parameter values indicated there, possesses the characteristic curve $i \times v$ presented in Fig. 3.

To select one orbit the resistor R' value (Fig. 2) is modified during the OPF control [7] by a fuzzy error controller [10] according a software developed in C++ (DOS). These R' changes are implemented by hardware from outputs of the PC computer parallel interface (see Appendix A.1).

Varying R' (Fig. 2) changes the characteristic curve (Fig. 3) of the non-linear negative element (R_{NN}), and varying R changes the oscillations amplitude of the variables v_{C_1}, v_{C_2} and i_L. Moreover, adjusting R' or R it results in the attractor alteration. Thus, in this work, R value adjustments determine the attractor, while short time alterations of the R' values were used to control the chaotic attractor.

The inductor used in CHC was implemented by a gyrator circuit (block marked by L in Fig. 2 – see Appendix A.2) that allowed the simple adjustment of values in an extensive range. It was fixed $C_2' = C_2 = 3.3\mu F$. In this case, $C_1 = 33\mu F$ resulted in a characteristic time of about 100 ms (attractor loop).

The signals v_{C_1} and v_{C_2} sampling period (T_a) were fixed in 1 ms, what corresponds to about 100 samples for each attractor loop, being the resolutions of 256 levels for the excursions in each one of the coordinates axes, v_{C_1} and v_{C_2}. A low cost AD converter was developed, to read the dynamical variable signals v_{C_1} and v_{C_2}, being such readings carried through in sequence to each 1 ms and transferred to the computer through its parallel interface.

A low resolution (8 bits) AD converter was chosen despite the quantization noise level comparable to the system sensibility to small variations [2]. The following strategy was tested to treat the error: the signals v_{C_1} and v_{C_2} are filtered by a lowpass filter implemented by software with 10 Hz of cutoff frequency.

The used gyrator circuit and the interface with the computer are detailed in Appendix A.

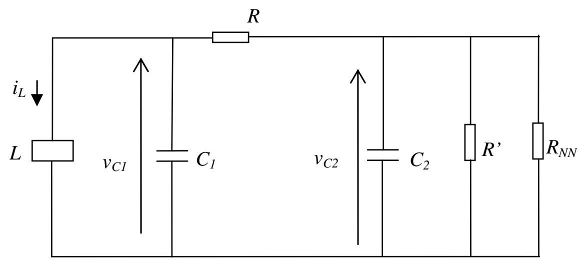

Fig. 1. Chua circuit.

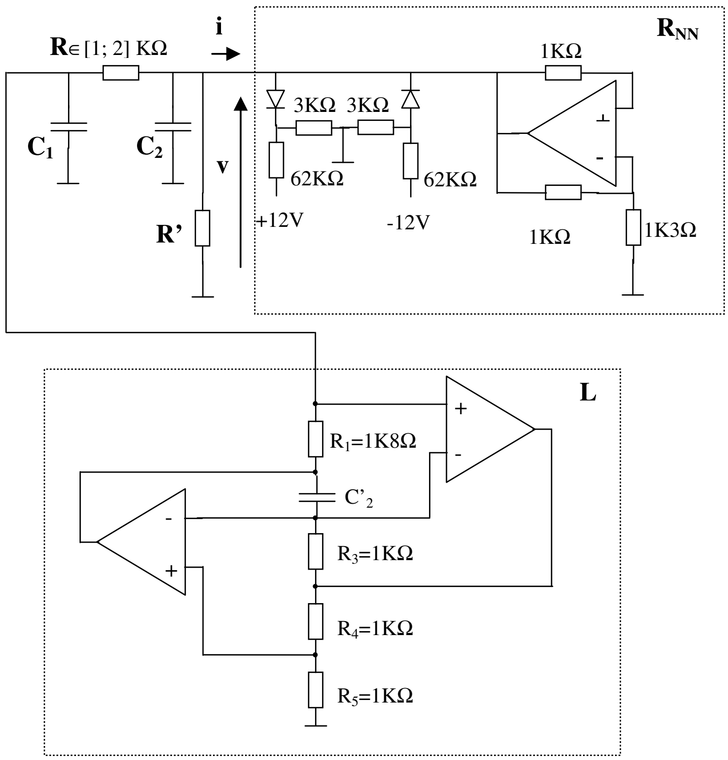

Fig. 2. Physical implementation of the Chua circuit.

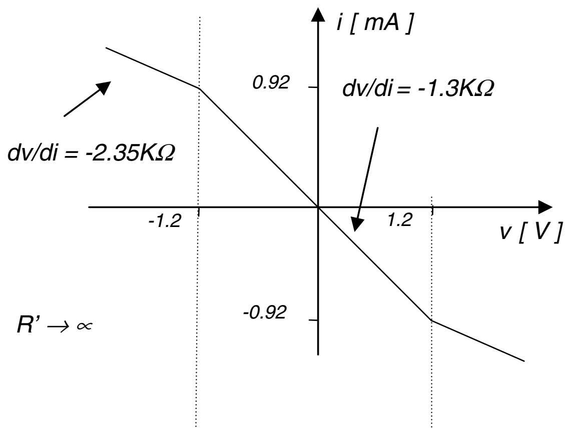

Fig. 3. Characteristic curve of element R_{NN}.

3. Lowpass filter

To reduce the discrete noise, the lowpass filter was implemented by software as indicated in Fig. 4. The samples are carried through to each sampling period T_a.

The filtered signal corresponds to the arithmetic average of the last m samples

$$y_n = \frac{x_n + x_{n+1} + \ldots + x_{n+(m-1)}}{m} = \frac{x_n + x_n z^{-1} + \ldots + x_n z^{-(m-1)}}{m}. \tag{1}$$

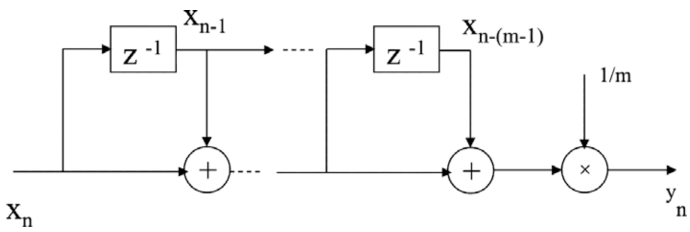

Fig. 4. Lowpass filter.

Applying the Discreet Fourier Transform [16] in expression (1) and using the following property of this transform

$$F\{xz^{-k}\} = F\{x\}e^{-jk\Omega} = X(\Omega)e^{-jk\Omega} \quad \text{with } \Omega = \omega T_a,$$

results

$$F\{y_n\} = Y(\Omega) = F\left\{\frac{x_n + x_n z^{-1} + \ldots + x_n z^{-(m-1)}}{m}\right\} == \frac{F\left\{x_n \sum_{k=0}^{m-1} z^{-k}\right\}}{m} = X(\Omega)\frac{\sum_{k=0}^{m-1} e^{-jk\Omega}}{m}.$$

From this last expression one concludes that the transfer function of the filter results in

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{\sum_{k=0}^{m-1} e^{-jk\Omega}}{m}. \tag{2}$$

The sum in Eq. (2) gives the value $\sum_{k=0}^{m-1} r^{-k} = (r^m - 1)/(r - 1)$ with $r = e^{-j\Omega}$. Thus, H can be written as

$$H(\Omega) = \frac{1}{m}\frac{e^{-jm\Omega} - 1}{e^{-j\Omega} - 1} = \frac{1}{m}\frac{e^{-jm\Omega/2}}{e^{-j\Omega/2}}\frac{e^{-jm\Omega/2} - e^{jm\Omega/2}}{e^{-j\Omega/2} - e^{j\Omega/2}} = \frac{1}{m}\frac{e^{-jm\Omega/2}}{e^{-j\Omega/2}}\frac{\sin(m\Omega/2)}{\sin(\Omega/2)}. \tag{3}$$

The transfer function absolute value is obtained from (3)

$$|H(\Omega)| = \frac{1}{m}\left|\frac{\sin(m\Omega/2)}{\sin(\Omega/2)}\right|. \tag{4}$$

The graph of this function is presented in Fig. 5 for $m = 24$, being $T_a = 1$ ms. In these conditions the bandpass is approximately 10 Hz.

The quantization noise of an AD conversion of n bits is given by [17]: SNR $\cong -6n$ dB equal to -48 dB for $n = 8$. On the other hand, the band of this noise is equal to [17]: $B \cong 1/2T_a = 500$ Hz. Therefore, one can write (considering the planed noise power spectrum in the range from 0 to 500 Hz):

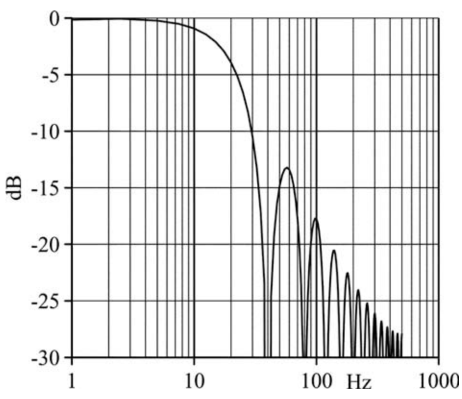

Fig. 5. Transfer function $|H(\Omega)| = \left|\frac{Y(\Omega)}{X(\Omega)}\right|$.

$$\mathrm{SNR}_{\mathrm{FPB}} \cong \mathrm{SNR} + 10\log\frac{10\ \mathrm{Hz}}{500\ \mathrm{Hz}} \cong -48 - 17 = -65\ \mathrm{dB}.$$

From that one concludes that the equivalent number of bits after the filtering is equal to

$$n_{\mathrm{eq}} = -(-65)/6 \cong 11\ \mathrm{bits}.$$

In agreement with [8], this resolution is within the typical limit of the sensibility of a chaotic system to small variations.

4. Chua circuit characterization

The Chua circuit can present chaotic behavior and, in this case, being a dissipative system, the trajectory in phase space tends to a trajectory limit that is a chaotic attractor [12].

Adjusting the value of R' (Fig. 1), one gets attractors in several forms. Fig. 6 presents the two kinds of chaotic attractors known as the Rössler and double scroll, resulting of the tests.

5. Control fuzzy of Chua circuit

Basically, a fuzzy error controller simulates human actions so that the error is minimized [10]. The application of the used control algorithm is represented in Fig. 7, whenever v_{C_1} crosses a given reference value v_{ref_1}, the R' value is altered to force v_{C_2} to reach a value near v_{ref_2}. This control is applied during a time period τ after which R' return to its original value.

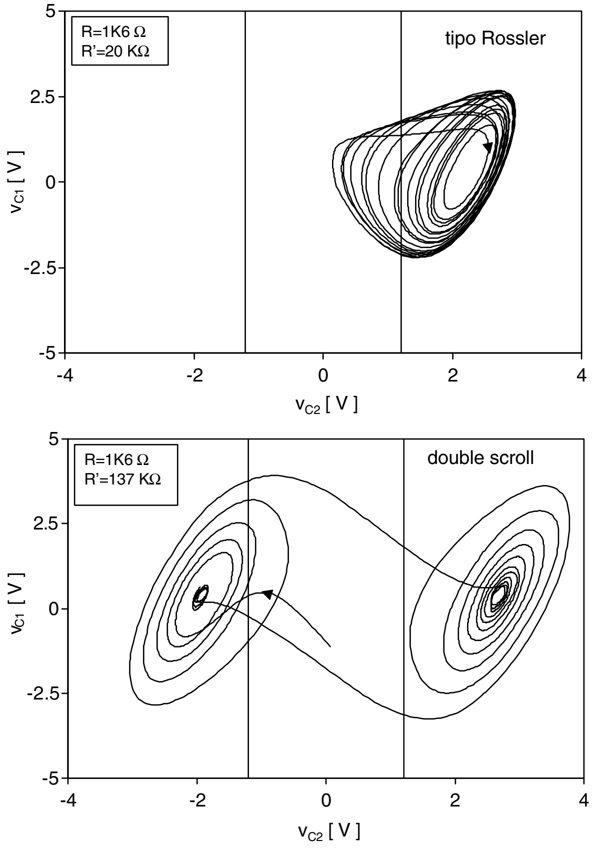

Fig. 6. Orbits of the Chua circuit.

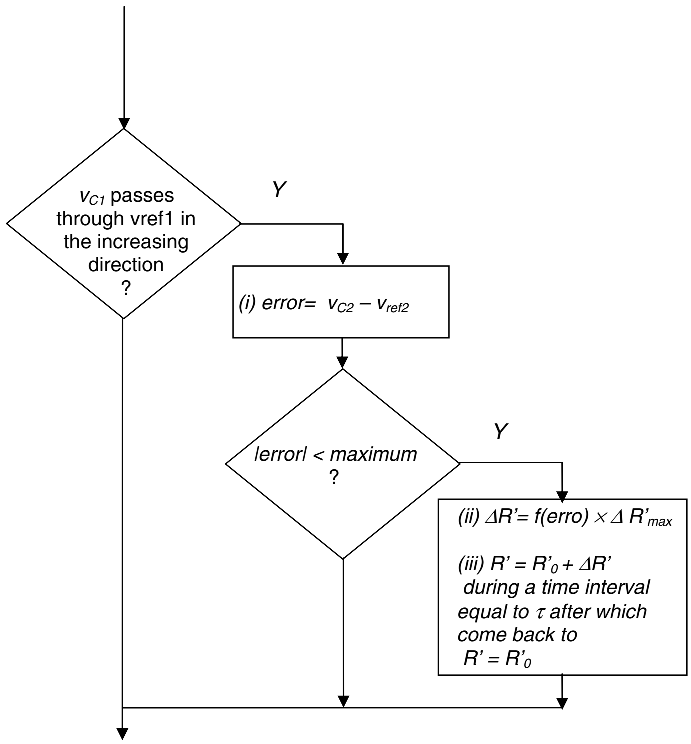

Fig. 7. The Chua circuit control.

The function f (error), used in the step (ii) of the algorithm indicated in Fig. 7, to calculate the control correction R' is implemented by using the concept of normalized fuzzy characteristic function (FCF) [18,19] presented in Fig. 8.

To stabilize a chaotic orbit, the FCF was defined experimentally, adjusting the pertinence function pair of the employed variables (center of the triangles in Fig. 8) through fuzzy rules.

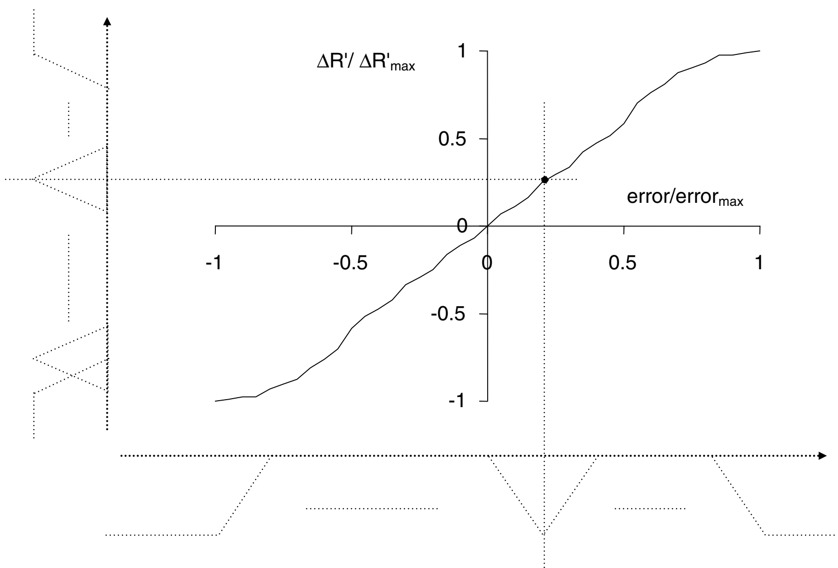

Fig. 8. Fuzzy characteristic function (normalized) for the control of the Chua circuit.

An example of this procedure is shown in Fig. 9 for a chaotic oscillation corresponding to the attractor of Fig. 10a. The periodic attractor obtained by controlling this oscillation is shown in Fig. 10b.

The oscillating voltages v_{C_1} and v_{C_2} are in Fig. 9a and b while in Fig. 9c is the sequence of the applied control parameter variations of R'. The R' pulses indicate the short time intervals when the control was applied. The control is rapidly

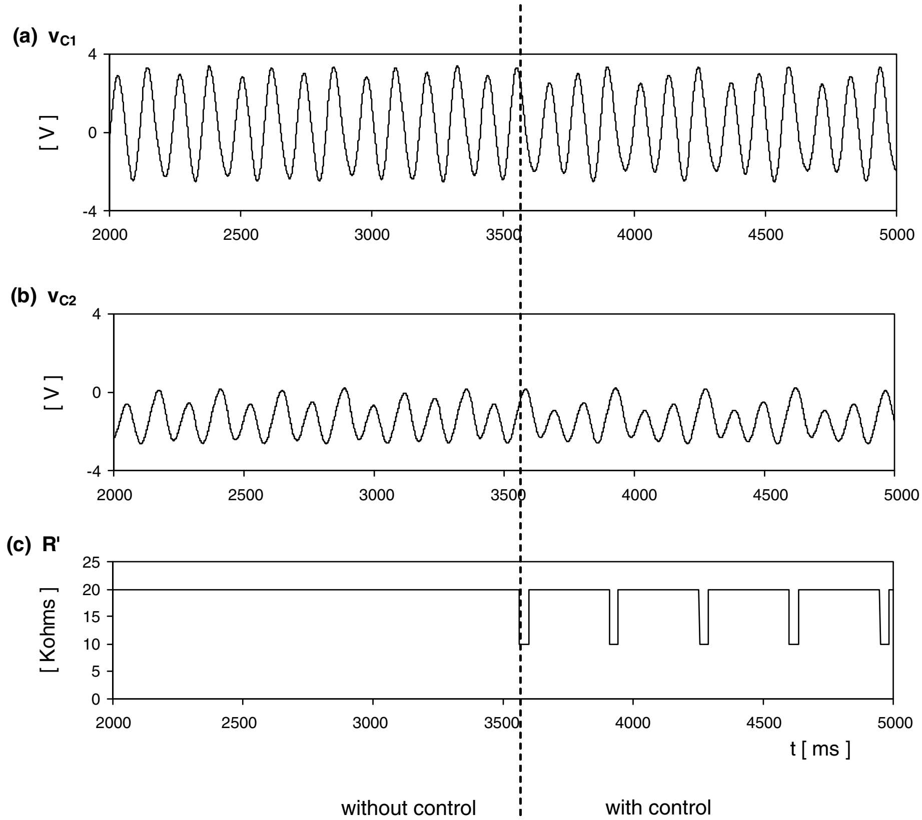

Fig. 9. OPF Control of Chua circuit: time signals.

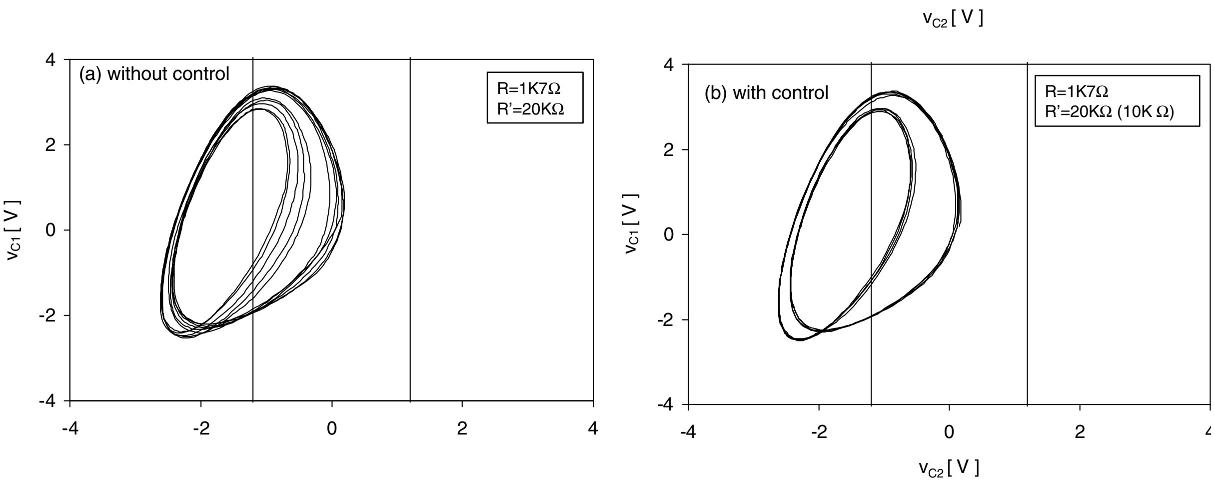

Fig. 10. OPF Control of the Chua circuit: diagrams in the phase space.

achieved as we can observe examining the real values indicated in Fig. 9. Similarly, the observed result of this fuzzy control has also been verified for several others examples.

6. Conclusions

Seeking a larger flexibility as well as the storage and to the treatment of orbits in chaotic systems, a computational fuzzy control of the Chua circuit was developed. The sampling was carried through by an AD converter of eight bits, following by a lowpass filter to compensate this low resolution.

Moreover, in this project integrated circuits CMOS were used to interconnect the Chua circuit to a PC compatible, through its parallel interface. The implemented program is executed in the operating system DOS, being developed in the language C++.

Due to its low cost, as also for not requiring any special software or hardware resources, it is feasible to implement this project for several applications or didactic purposes.

Acknowledgement

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Appendix A. Implementations

A.1. Interface for PC

Seeking the application of the tests related to the fuzzy control of the Chua circuit, in Fig. A1 the interface here developed is presented. The described elements and signals refer to this figure.

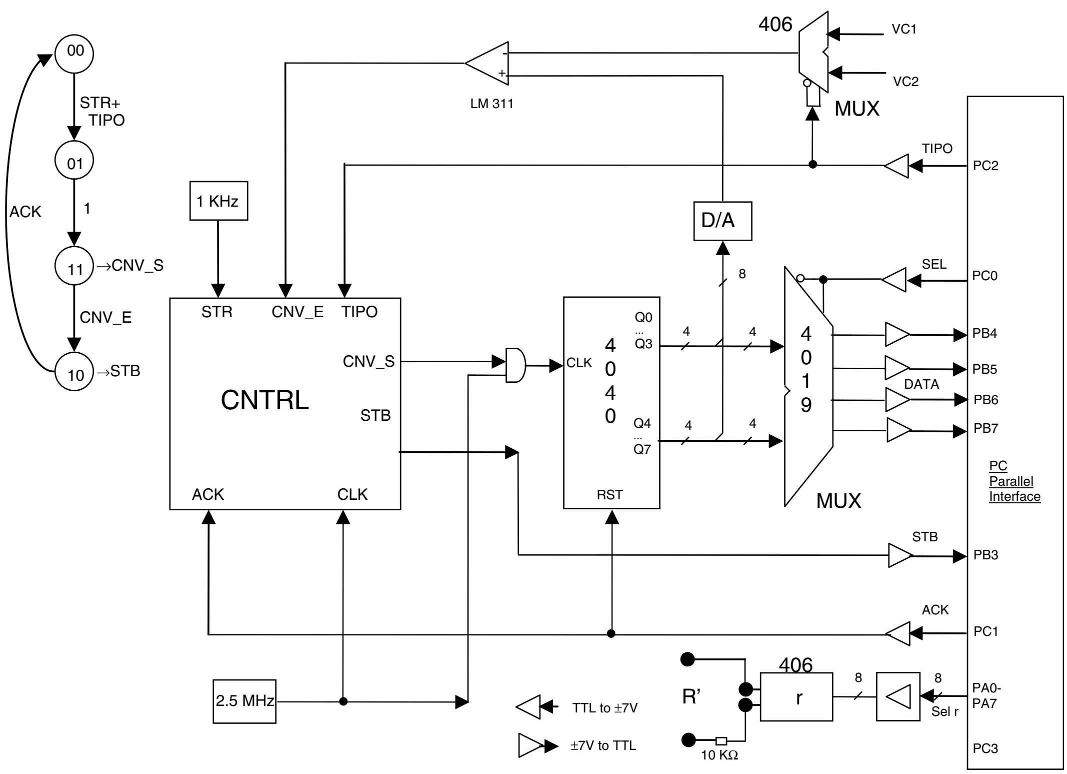


Fig. A1. PC interface circuit for Chua circuit.

Through this control interface it takes place the reading and generation of signals to sample the capacitors tensions v_{C_1} and v_{C_2}, as well as to impose the value of R' (see Fig. 2).

To read the signals v_{C_1} and v_{C_2} the following elements are used: counter (CI 4040), DA converter, Mux1 (CI 4066) and comparator (CI LM311). These readings are made to each 1 ms and transferred through Mux 2 (CI 4019). To do that, the signals STRB (strobe) and ACK (acknowledged) are changed by the control block (CNTRL) and the program executed in PC.

Using the PC parallel interface the signal TYPE is generated to select which of the signals v_{C_1} or v_{C_2} will be read. The circuit CNTRL controls these signals reading and transferring for PC. It also implements the finite automat, represented explicitly in Fig. A1 which generates the necessary processing.

The signals of the capacitors are represented, in the computer screen in real time, using cartesian graphs, as Figs. 6, 9 and 10, and the control takes place in agreement with the Algorithm described in Section 5.

The way of changing the resistor R', here proposed, is presented in Fig. A2: 4066 bilateral keys are used, controlled by binary signs generated by the program, and through the PC parallel interface, to obtain for this resistor a range variation of 10 kΩ to 137.5 kΩ with jumps of $\Delta R = 500\ \Omega$.

A.2. Circuit gyrator as inductor

It is presented in Fig. A3 the circuit gyrator that was used in the simulation of the inductor in the Chua circuit.

Considering the ideal operational amplifiers, the electrical currents in their input branches are null, as well as then electrical potential difference.

From this condition and starting from the Kirchoff laws, it can be written

$$\begin{aligned} &i = i_1, \quad i_1' = i_1 + i_2, \quad i_3' = i_3 + i_4, \quad i_4 = i_5, \\ &v_1 = v_2, \quad v_3 = v_4, \quad v_5 = v. \end{aligned} \tag{5}$$

Using expressions (5), of $v_5 = v$ we have $i_5 = \frac{v}{R_5} = i_4$ from where $v_4 = R_4 i_4 = R_4 \frac{v}{R_5} = v_3$ and $i_3 = \frac{v_3}{R_3} = \frac{R_4 v}{R_5 R_3} = i_2$. Therefore, $v_2 = \frac{i_2}{sC_2'} = \frac{R_4 v}{R_5 R_3}\frac{1}{sC_2} = v_1$ of which results: $i_1 = \frac{v_1}{R_1} = \frac{R_4 v}{R_5 R_3}\frac{1}{sC_2'}\frac{1}{R_1} = i$ and finally

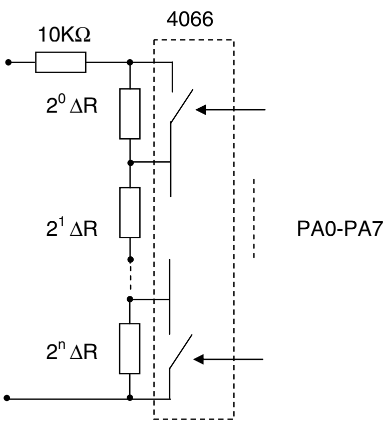
$$\frac{v}{si} = L = \frac{R_1 C_2' R_3 R_5}{R_4}. \tag{6}$$

One concludes that the electrical potential and current in the output of the operational amplifier A_1 are given by

$$v_{s1} = v_5 + v_4 - v_3 - v_2 = v_5 - v_2 = v\left(1 - \frac{1}{s}\frac{R_4}{R_5 R_3 C_2'}\right), \tag{7}$$

$$i_{s1} = -i_1' = -i_1 - i_2 = -v\frac{R_4}{R_3 R_5}\left(1 + \frac{1}{sR_1 C_2'}\right). \tag{8}$$

In similar way, the electrical potential and current in the output of the operational amplifier A_2 follow the expressions:



PA7-PA0	R’
11111111	0+10KΩ
11111110	ΔR+10KΩ
111111101	2ΔR+10KΩ
11111100	3ΔR+10KΩ
...	...
00000000	255ΔR+10KΩ

Fig. A2. PC Resistor adjustment.

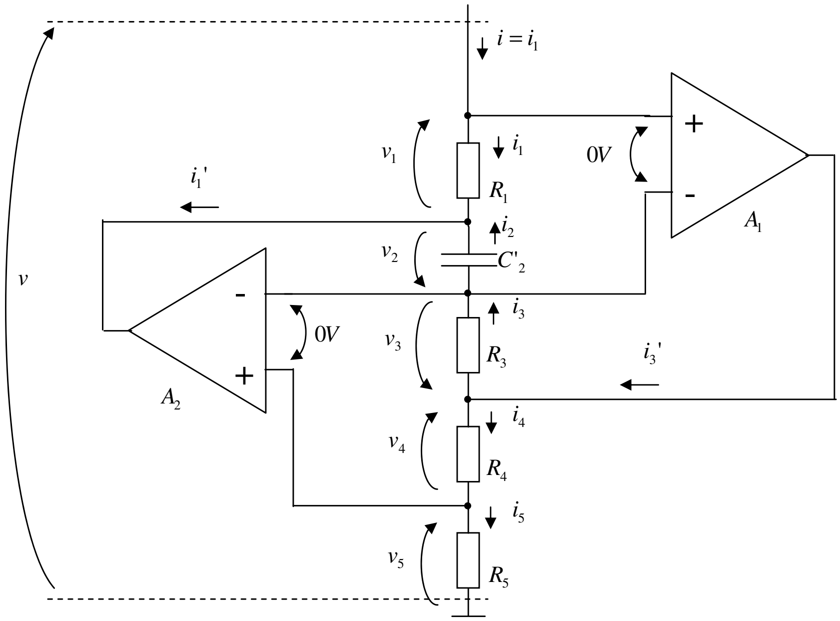


Fig. A3. Circuit gyrator for inductor simulation L of the Chua circuit (Fig. 2).

$$v_{s2} = v_5 + v_4 = v\left(1 + \frac{R_4}{R_5}\right), \tag{9}$$

$$i_{s2} = i'_3 = i_3 + i_4 = \frac{v}{R_5}\left(1 + \frac{R_4}{R_3}\right). \tag{10}$$

Using expressions (7) to(10) is possible to project the range of these signals in terms of the chosen amplifier limits.

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