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Conditional targeting for communication ☆

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Abstract

In this work we propose the use of a targeting method applied to chaotic systems in order to reach special trajectories that encode arbitrary sources of messages. One advantage of this procedure is to overcome dynamical constraints which impose limits in the amount of information that the chaotic trajectories can encode. Another advantage is the message decoding, practically instantaneous and independent of any special technique or algorithm. Furthermore, with this procedure, information can be transmitted with no errors due to bounded noise.

1. Introduction

Communication means the exchange of information between a transmitter and a receiver. As transmitter and receiver are physically apart from each other, the exchange of information has to be done through a physical medium. In order to transmit the information through this medium, one requires the use of some sort of signal that travels through it and that is able to encode the information. In addition to encoding the information, the signal has to be decoded after being transmitted through the medium. Thus, the signal should be robust to the noise present in the channel.

Chaotic signals seen to be promising in performing all these tasks required for an efficient communication. So, many works have proposed the use of chaotic signals to transmit information [1–12]. Particularly, in Ref. [12], it was shown that if the chaotic system that generate the signal is an optimal encoder its trajectory can efficiently encode the information.

As defined in [12], a chaotic system is called an optimal encoder when its topological entropy [13] is equal or greater than the Shannon entropy of the source that produces the message [14]. In the case such condition is not respected, the use of a chaotic trajectory to encode the source can only be done after a special encoding of the source, which is time demanding. This problem (I) should be found in chaos-based communication, because, in real communication systems, the message source can be chosen with any value for the Shannon entropy [15], while dynamical systems have dynamical constraints that imposes limitation into the upper value of the topological entropy.

It was also shown that the memory contained in the chaotic trajectory can be used to dynamically filter the noise [10,11], allowing good decoding of the information, after being transmitted through a noisy medium. This process deals with the dynamically filtering the noise out of a chaotic trajectory that have been transmitted through a channel, as reported in Refs. [10–12], such that the message can be fully recovered at the receiver. To solve this problem (II), basically, we have to discover out of a large number of trajectories, which one was really transmitted. This technique

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might consume much time, and the decoding might turns out to be inefficient. In the particular case where the noise can be considered to be bounded (or Gaussian with very small variance), a strategy was proposed in Ref. [16]. In a short, to encode the message, it chooses a selected set of trajectories from a dynamical system that are robust to certain bounded level of noise in the channel. The inconvenience in this strategy is that the topological entropy of the chosen set is smaller than the one produced by the chaotic system [12,16], what may limit this strategy applicability.

In this paper we want to show a targeting [17] technique that modifies the original chaotic dynamics in order to turn a chaotic system that is a non-optimal encoder to a system that is an optimal encoder to any type of sources, resolving *problem I*. That is done by creating new orbits. All these new orbits are created with the property of being robust to a bounded noise level present in the physical medium, resolving *problem II*.

We illustrate our proposed method using the Chua's circuit [18], a non-linear oscillator that can be thought as a prototype for any non-linear signal generator that creates the chaotic orbits to encode the information we want to transmit. Our paper is organized in the following way: In Section 2, we show the main characteristics of the Chua's circuit. In Section 3, we show how to encode a message into the trajectories of this circuit, by associating optimally symbols to the trajectory. In Section 4 we show how the message is extracted from the received signal when δ-bounded noise level is present in the channel. In Section 5, we present our targeting method to produce trajectories that are robust to noise in the channel, and that have the highest possible topological entropy a set can have, and in Section 6, we present an implementation of our communication scheme. Finally in Section 7 we present the conclusions.

2. The Chua's circuit

The Chua's circuit [18] is an electronic oscillator extensively studied because of the following characteristics: high complexity, few electronic components, and a rich variety of bifurcations and chaotic behavior. The circuit is composed by two capacitors, one inductor, one resistor, and a linear piecewise resistor. Fig. 1 is an schematic diagram of this circuit. The dynamical variables are the voltages V_{C1} and V_{C2} across the capacitors C_1 and C_2, respectively, and the current i_L through the inductor L. The dynamics of this circuit is described by three differential equations,

$$\begin{aligned} C_1\frac{\partial V_{C1}}{\partial t} &= \frac{1}{R}(V_{C2}-V_{C1}) - i_{\mathrm{NR}}(V_{C1}) \\ C_2\frac{\partial V_{C2}}{\partial t} &= \frac{1}{R}(V_{C1}-V_{C2}) + i_L \\ L\frac{\partial i_L}{\partial t} &= -V_{C2} \end{aligned} \tag{1}$$

The non-integrability of the circuit equations comes from a linear piecewise negative resistor whose characteristic curve is represented by:

$$i_{\mathrm{NR}}(V_{C1}) = m_0V_{C1} + 0.5(m_1-m_0)\mid V_{C1}+B_p\mid + 0.5(m_0-m_1)\mid V_{C1}-B_p\mid \tag{2}$$

In this work we rescale the variables and parameters which becomes dimensionless.

For doing numerical simulation of these differential equations we use the parameters:

$$C_1 = 10.0, \quad C_2 = 1.0, \quad G = 1/R = 0.5750, \quad L = 6.0 \tag{3}$$

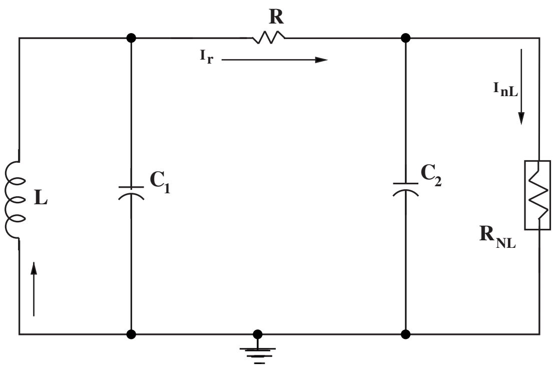

Fig. 1. Scheme of Chua's circuit.

The initial conditions are:

$$V_{C1} = -1.50, \quad V_{C2} = 1.034, \quad i_L = 0.845 \tag{4}$$

For the chosen set of parameters the unperturbed circuit trajectory is a Rössler-type attractor, which is a chaotic rotation around an unstable saddle-focus.

In our numerical simulations, we integrate the set of equations (1) using the fourth-order Runge–Kutta algorithm with integration step of $dt = 0.04$.

3. Encoding (decoding) the trajectory

Chaotic encoding is the process of representing the message to be sent in terms of a chaotic trajectory. To encode a chaotic trajectory the first step is to construct a partition in the phase space and associate symbols to the trajectory whenever it is located in one of the partitions. As the trajectory goes from one to another partition, a symbolic sequence is generated. The information contained in this symbolic sequence, measured by the Shannon entropy [14], in the case the partition is generating [19], is maxima, and it represents the maximum amount of information that a chaotic trajectory encodes in the absence of any control in the system. In the case one allows the control of the chaotic dynamics, the amount of information a chaotic trajectory encodes is given by the topological entropy [12], the information capacity of the system, that is, the ability a chaotic system has of generating a given amount of symbolic sequences.

So, by controlling the chaotic trajectory, as done in Refs. [1,2], we make the dynamical system to respond with a trajectory whose symbolic sequence has not only some desired topological entropy but also it is the message to be transmitted. So, transmitting this trajectory through a channel means transmitting the message.

Due to the fact that the Rössler-type attractor has a fractal dimension close to 2, a 1-D first return mapping gives a very good description of its dynamics. So, as done in [1,2], the partition we encode the trajectories can be well defined in this mapping. This mapping is a 1-D discrete first return mapping, a 1-D projection of the 2-D Poincaré section [20] of this flow. The Poincaré section is positioned at $V_{C2}(t) = 0$, and whenever the trajectory crosses this plane from a positive $V_{C2}(t)$ to a negative $V_{C2}(t)$, we keep the values of $V_{C1}(t)$ as the discrete variable x. As the trajectory crosses this plane, we construct the first return mapping, Fig. 2, of the variable V_{C1} on this section

$$x_{n+1} = F(x_n) \tag{5}$$

where n describes the nth crossing of this trajectory in that Poincaré section that happens for a time $t = t_n$.

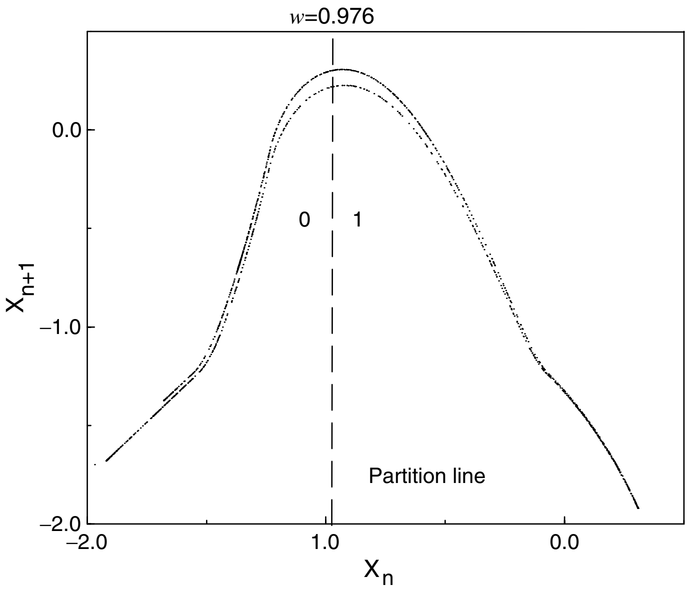

Fig. 2. First return map for the unperturbed Rössler-type attractor.

So, as done in [1,2], the symbolic dynamics of this circuit is obtained by associating symbols with intervals of the domain of the variable x [21]. As explained in [12], the partition of the domain in x should be done through an optimization of an entropy function, of the symbolic sequence generated by a trajectory passing by that partition. Given a point x_p, we define that a trajectory passing (for $V_{C2} = 0.0$) in $x_n < x_p$ represents a symbol 0, and 1 otherwise. Thus, a trajectory $x = x_n, x_{n+1}, \ldots$, has the symbolic sequence $b = b_i, b_{i+1}, \ldots$, with b_i equal to either 0 or 1.

This symbolic sequence is the signature of the chaotic dynamics if x_p is chosen such that the entropy of that sequence is maxima. So, by the entropy function

$$W(x_p) = \lim_{N\to\infty} \frac{\ln E(N)}{N} \tag{6}$$

where $E(N)$ is the number of allowed symbol sequences of length N. In practice, for the calculation of W, we consider $N = 10$. The generating partition, $x_p = w$, as well as, the topological entropy, H_{T}, is obtained by

$$H_{\mathrm{T}} = \sup_w W(w) \tag{7}$$

where the supremum of the function W is obtained for $x_p = w = 0.976$, resulting in $H_{\mathrm{T}} = 0.66$. In Fig. 2 we represent by a dashed line the partition. This entropy is smaller than $\ln(2) \cong 0.693$, which is the entropy of a binary symbolic sequence that contains all possible sequences of length $N\left(\frac{\ln 2^N}{N}\right)$. Therefore, the considered dynamical system, even with control of the type proposed in [1,2], can only encode source messages with Shannon entropy lower than 0.693. As a way to visualize the possible binary symbolic sequences that the Chua's circuit can encode, we plot the symbol plane. For that, we group the symbolic sequences in sequences of 10 symbols (length $N = 10$). Thus, we represent a particular length-10 symbolic sequence in a decimal number r_j given by:

$$r_j = \sum_{i=Nj}^{N(j+1)-1} b_i 2^{(N(j+1)-i-1)} \tag{8}$$

where r_1 is the decimal associated with the symbolic sequence composed by the first 10 symbols $b_1 b_2 \cdots b_{10}$, r_2 is the decimal associated with the symbolic sequence $b_{11} b_{12} \cdots b_{20}$, and so on.

The symbol plane, r_j versus r_{j+1}, can be seen in Fig. 3. The importance of this plane is that the transformation that places the pair of points r_j, r_{j+1} to the pair of points r_{j+1}, r_{j+2} is equivalent to the transformation F that places the point x_n into the point x_{n+1} [22]. The gaps in the symbol plane represents non-permitted transitions of symbolic sequences. Also, this plane will help us in understanding the action of our proposed targeting in creating new orbits.

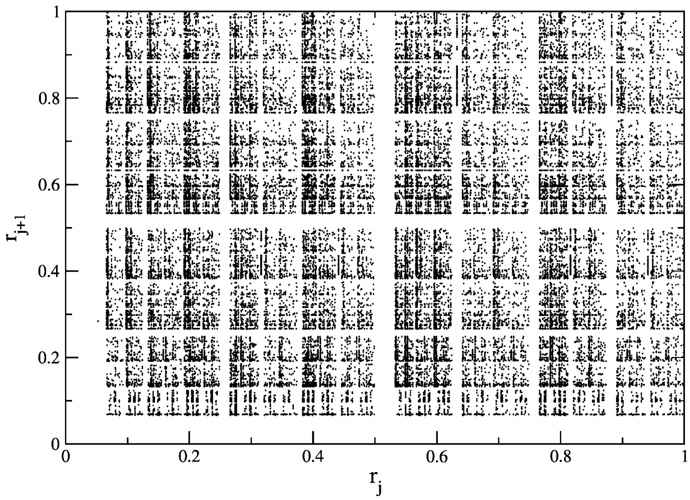

Fig. 3. Symbol plane for the non-perturbed Chua's circuit considering symbolic sequences of length $N = 10$. The gaps represent non-permitted transitions of symbolic sequences.

4. Decoding with δ-bounded noise level

The Chua's circuit has three dynamical variables: $V_{C1}(t)$, $V_{C2}(t)$, and $i_L(t)$. In order for the receiver to decode the information using the encoding suggested in the previous section, the transmitter would have to transmit not only the variable $V_{C1}(t)$, but also the variable $V_{C2}(t)$, with which one can construct the return map $(x_n \times x_{n+1})$, where the partition is defined. For efficient purposes, one does not want to use two signals to transmit a message. So, what one could do, in practice, is to transmit only one signal, V_{C1} for example, and the return map where the partition function is calculated should be a return map of a Poincaré section of the trajectory embedded in a time-delay coordinate system, done with the variable $V_{C1}(t)$.

However, for the sake of simplicity, we assume that what is sent to the receiver are the values x_n, and the decoding is performed just by checking whether x_n is smaller or bigger than the partition value w. Due to the presence of bounded noise level in the channel, what arrives to the receiver it is not x_n, but $x_n \pm \delta\eta_n$, where η represents a uniform random variable with a domain in the interval [0,1], and δ is the maximum amplitude of the noise present in the channel.

Decoding could be efficiently done just by using a subset x'_n that never falls in the interval $I = [w - \delta, w + \delta]$. The reason is that doing this, if x'_n encodes a symbol b (0 or 1), $x'_n \pm \delta\eta_n$ will necessarily be decoded as the symbol b. However, as demonstrated in [12], the topological entropy of the subset of trajectories x'_n is smaller than the entropy of the set x_n. Therefore, the subset x'_n will only be able to encode messages whose Shannon entropy is equal to or less than $H_T(x'_n)$. Let us assume that the message to be transmitted is modeled by a random binary source, that is, any sequence of 0 and 1 is permitted. In this case, the set x'_n would not be able to encode this message. Thus, what we do is to perturb the dynamical system in order to create an orbit represented by x'_n for which none of its elements falls inside the interval $[w - \delta, w + \delta]$, having the property that $H_T(x'_n) = H_S(M)$, where $H_S(M)$ represents the Shannon entropy of the source message.

Optimal decoding means that given a binary message M represented by $m_1, m_2, m_3, \ldots, m_l$, where m_i can be either 0 or 1, the transmitted trajectory $x'_n = x'_1, x'_2, x'_3, \ldots, x'_l$ has to be such that $x'_1 + \delta\eta_1, x'_2 + \delta\eta'_2, x_3 + \delta\eta'_3, \ldots, x'_l + \delta\eta_l$ decodes the symbolic sequence $B = b_1, b_2, b_3, \ldots, b_l$, such that $M = B$, that is, $m_1 = b_1$, $m_2 = b_2$, and so on.

5. Targeting method

Given a point x'_n that does not belong to the interval I $(x'_n \cap I = \emptyset)$ and its next iteration x'_{n+1}, the targeting method is applied if one of the following conditions are verified:

- (I) $x'_{n+1} \cap I \neq \emptyset$.
- (II) x'_{n+1} encodes 1 (or 0) while the element of the message b_{n+1} is equal to 0 (or 1).

In the case one of these two conditions are verified, we perturb $i_L(t = t_n)$, replacing i_L to $i_L + \Delta P$ at the time $t = t_n$, such that not only $x'_{n+1} \cap I = \emptyset$, but also that x'_{n+1} is decoded to the element of the message b_{n+1}. We choose to perturb i_L, instead other variable, because the i_L coordinate is oriented along the most contracting direction of the vector field (2). That means that an initial condition perturbed in the i_L coordinate, when iterated by the non-perturbed dynamics goes very fast to the invariant dynamics, the non-perturbed attractor.

In practice, the perturbation is calculated in order to have the orbit targeted from the point with coordinates x'_n to a point with coordinates x'_{n+1} in the ϵ vicinity of either one of the two points located in the non-perturbed attractor: the point T_0 with coordinate $V_{C1} = -1.6$ (and $V_{C2} = 0$), and the point T_1 with coordinate $V_{C1} = -0.1$ (and $V_{C2} = 0$), where $\epsilon = 0.005$. If a **0** is the desired symbol to be transmitted, the chaotic orbit is targeted to the vicinity of T_0, otherwise, if the desired symbol is **1**, the orbit is target toward T_1. These two target points T_0 and T_1 can assume arbitrarily values on the non-perturbed set x_n, such that $T_i \cap I = \emptyset$.

Given a particular x'_n value, two values of perturbations can be applied. The perturbation ΔP_0, to drive the orbit to a point x'_{n+1}, within the vicinity of the target point T_0, and the perturbation ΔP_1, to drive the orbit to a point x'_{n+1}, within the vicinity of the target point T_1. The perturbation ΔP_0 is applied if the symbol **0** is to be transmitted, and the perturbation ΔP_1 is applied if the symbol **1** is to be transmitted.

The amplitudes ΔP_b, with b representing either **0** or **1**, are calculated through a learning process (off-line). For each initial condition x'_n, we calculate the two sets of control amplitudes, ΔP_0 and ΔP_1 applied into i_L, in order to drive the point x'_{n+1} to the vicinity of either T_0, or T_1. The calculated values of perturbations are shown in Fig. 4.

Note that the values for which $\Delta P_b = 0$ in these two strips in Fig. 4 represent values of x'_n close to the stable manifolds of the targets T_0 (top strip) and T_1 (down strip). Note that since the perturbation ΔP_b can be arbitrary, it can have a component out of the chaotic attractor. Therefore, after the perturbation is applied, the orbit is placed in an

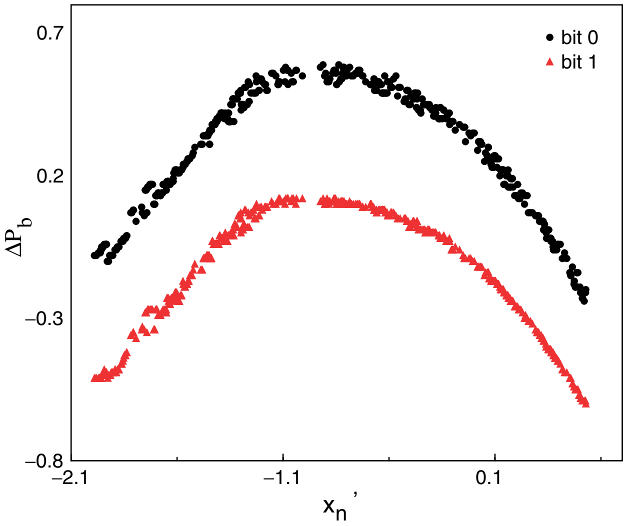

Fig. 4. The control ΔP_b applied into i_L to direct the initial conditions x_n to the vicinity of T_0 (ΔP_0) or to the vicinity of T_1 (ΔP_1).

initial condition on the basin of attraction of the chaotic attractor, however, away from it. Even though the perturbation is not very small, the iteration of this out-of-the-attractor initial condition, until it reaches the section on $V_{C2} = 0$, is sufficient to place the orbit close to the original non-perturbed attractor.

6. Communicating

We first demonstrate the communication method to transmit a random generated sequence of zeros and ones. We show in Fig. 5a the temporal evolution of the variable x_n for the Rössler-type attractor of Eqs. (1) and (2). When the targeting method is used to generate the controlled orbit x'_n, such that it encodes the desired message, all the points x'_n

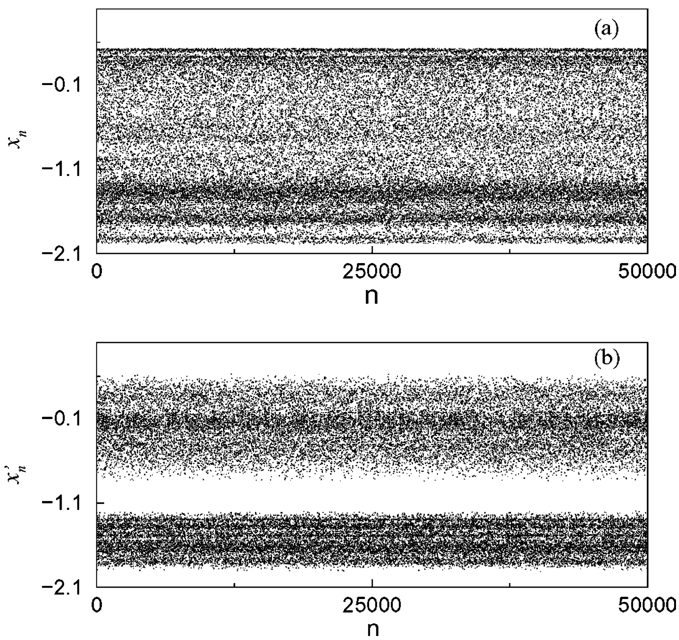

Fig. 5. (a) Evolution of the non-perturbed trajectory x_n. (b) Evolution of the targeted variable x'_n.

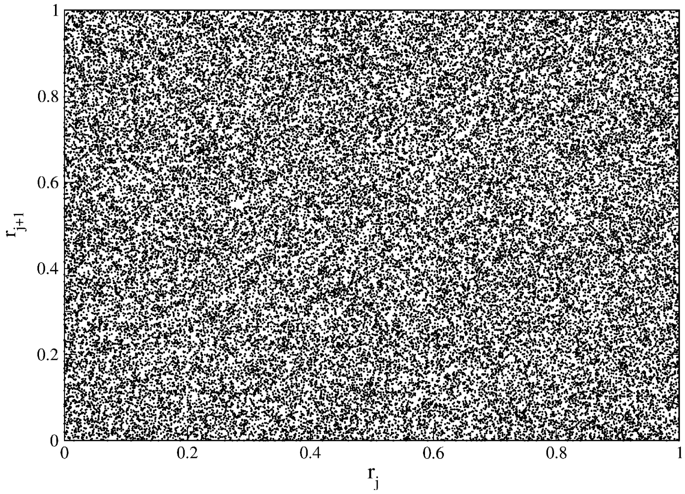

Fig. 6. Symbol plane for the controlled Chua's circuit, considering symbolic sequences of length $N = 10$, calculated using the driven trajectory x'_n. The gaps represent non-permitted transitions of symbolic sequences.

falls outside the interval I as can be seen in Fig. 5b. In addition, the probability distribution of x'_n is more concentrated in the vicinity of the target points $T_0 = -1.6$ and $T_1 = -0.1$. To show that the driven trajectory x'_n can encode any sequence of zeros and ones, we make the symbol plane of it, as shown in Fig. 6. In this figure, if the message had an infinity length, and the considered symbolic sequences were very long in length, no gaps would be seen, meaning that all the plane space would be filled out. As a consequence, the topological entropy of the targeted trajectory is $H_T(x'_n) = \ln(2)$ [also calculated by Eq. (6)].

When a perturbation does not need to be applied, i.e., neither condition (I) nor condition (II) is fulfilled, the targeted trajectory x'_n gets very close to the non-perturbed trajectory x_n. In fact, in the case the message is a random binary source, the average number of cases for which control pulses are not applied tends to values close to 50%. This means that the non-perturbed set x_n and the targeted set x'_n have many neighbor points, as can be seen in the return map x'_n versus x'_{n+1} of Fig. 7, which can be compared to the return map of Fig. 2.

Therefore, when perturbation is applied, transient dynamics is used. On the other hand, when no perturbation is applied, the original dynamics on the attractor of the circuit is used. However, as the perturbation is being applied to

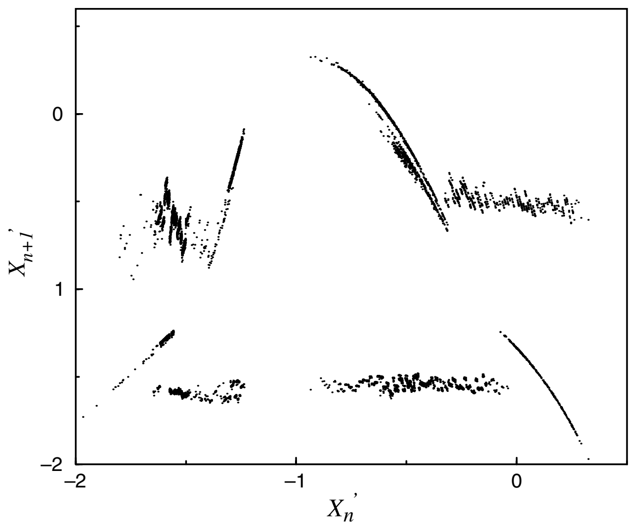

Fig. 7. Return map of the driven trajectory x'_n versus x'_{n+1}.

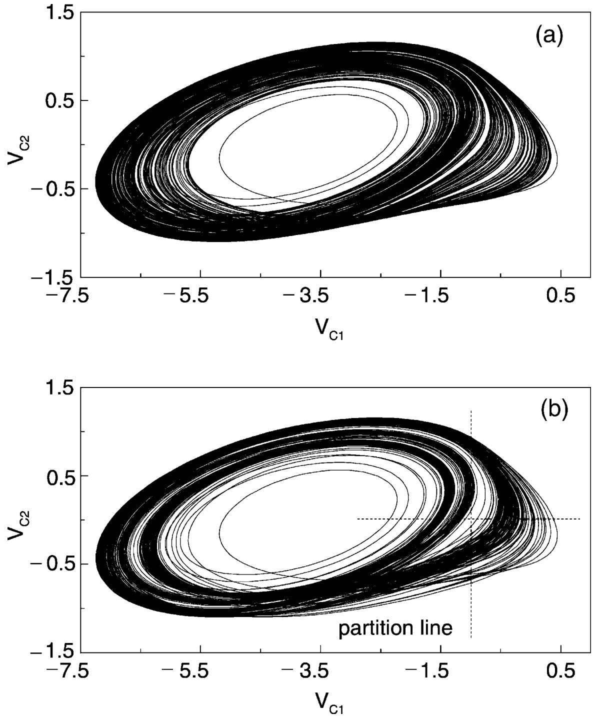

Fig. 8. (a) Projection of the non-perturbed trajectory of Eqs. (1) on the $V_{C1} \times V_{C2}$ plane. (b) Projection of the driven trajectory on the same plane.

the variable i_L, evolution of these perturbed initial condition goes very fast to the invariant dynamics, and therefore, the driven trajectory is similar to the non-perturbed trajectory as shown in the attractor projections of Fig. 8, where we show.

To better understand the consequences of the targeting method, we calculate the power spectra of the evolution of V_{C1}, from which we obtained x_n and x'_n, shown in Fig. 5a and b, respectively. These spectra are shown respectively in Fig. 9a and b, and they are very similar indicating that the targeted trajectory is similar to the non-perturbed trajectory. Note, however, that there are new peaks in the targeted trajectory. This is a consequence of the new trajectories, created by the targeting method, consequence of a transient dynamics.

Next, we apply our proposed method to transmit the name of the famous soccer player *Pelé* coded in a binary-ASCII format. Thus, we transmit the following binary sequence: "01010000 01100101 01101100 11101001".

In Fig. 10(a) in squares we show the pulse perturbations ΔP_0 and in circles ΔP_1. The targeted trajectory that encodes the name Pelé is shown in Fig. 10b.

7. Conclusions

We propose a targeting method applied to chaotic systems to create an optimal dynamical system to communicate, that is, a dynamical system that generates trajectories that can not only encode arbitrary source messages, but can also be transmitted through a noisy channel. This proposed targeting preserves part of the original dynamics and creates a new, transient dynamics. The use of a set of transient orbits to communicate, overcomes the inability of a dynamical system to generate arbitrary demanded symbolic sequences. In the case the noise is bounded and has an amplitude smaller than δ, instantaneous decoding is done, without the need of any chaotic filtering method. Experimentally, such decoding could be simply done using a comparator.

In the case there is Gaussian noise in the channel, or a non-bounded noise level whose amplitude is bigger than the gap δ, successful decoding, i.e., recovering of the message can still be performed if the received point $x'_n + \delta\eta_n$ has x'_n close to some point of the non-perturbed set x_n. Also, in the case a dropout occur, i.e., there is the interruption of the transmission, and part of the trajectory is not sent, this lost information can still be recovered, by the methods presented in Refs. [11,12], if the dropout occurred while the dynamical system is not being targeted.

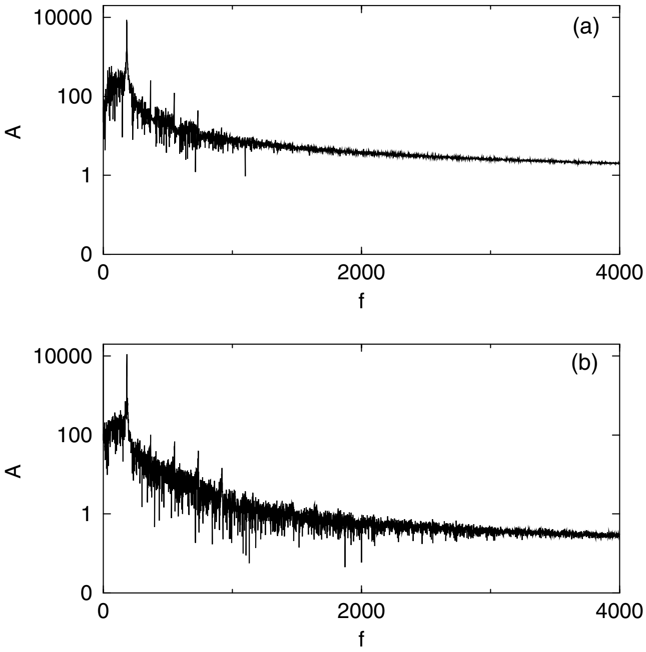

Fig. 9. (a) Power spectra of the evolution shown in Fig. 5(a). (b) The same for the perturbed evolution of Fig. 5(b).

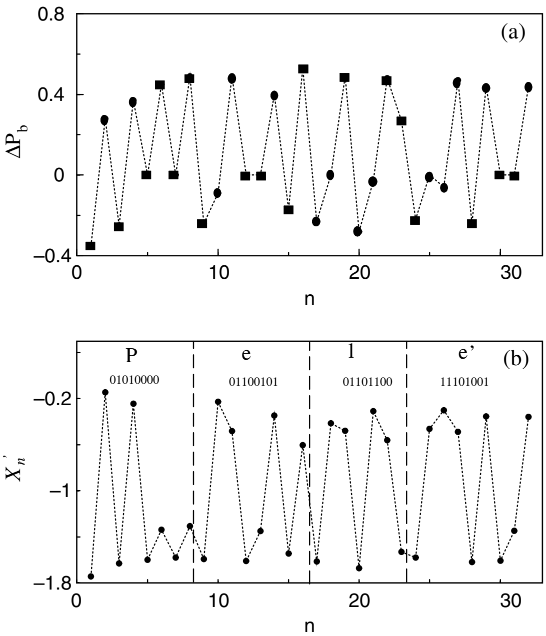

Fig. 10. (a) Pulse perturbations applied into the Matsumoto–Chua's circuit to transmit the name Pelé using our propose scheme of communication. (b) The driving trajectory x_n'.

We have not been concerned with the security of the transmission in this work. However, if security is needed, one could send to the receiver the series of perturbations ΔP_b. Being the initial condition a secret information, only who knows it (the receiver) can recover the message, applying the received perturbations to its initial condition. In that case, the dynamical system [the Chua's circuit, Eq. (2)] should be considered a public information. The use of the perturbation would at least fill up one requirement of security. It is a non-inversible function with respect to x'_n, as one can see in Fig. 4. So, if an eavesdroper has only the knowledge of ΔP_b, there are at least two values of x'_n, and without complete knowledge of x'_n there would be no ways to find out about x'_n.

References

[1] Hayes S, Grebogi C, Ott E. Communicating with chaos. Phys Rev Lett 1993;70:3031–4.

[2] Hayes S, Grebogi C, Ott E, Mark A. Experimental control of chaos for communication. Phys Rev Lett 1994;73:1781–4.

[3] Cuomo KM, Oppenheim AV. Circuit implementation of synchronized chaos with applications to communication. Phys Rev Lett 1993;71:65–8.

[4] Ogorzalek MJ. Taming Chaos. 2. Control. IEEE Trans Circuits Syst I 1993;40:700–6.

[5] Sato A, Endo T. Experiments of secure communications via chaotic synchronization of phase locked loops. IEEE Trans Fundament 1995;E78A:1286–90.

[6] Pecora LK, Carroll TL. Synchronization in chaotic systems. Phys Rev Lett 1990;64:821–4.

[7] Pecora LM, Carroll TL. Synchronizing nonautonomous chaotic circuits. IEEE Trans Circuits Syst 1993;40:646–50.

[8] Stojanovski T, Kocarev L, Parlitz U. Digital coding via chaotic systems. IEEE Trans Circuits Syst—I 1997;44:562–5.

[9] Dedieu H, Kennedy MP, Hassler M. Chaos shift keying: modulation and demodulation of a carrier using self-synchronizing Chua's circuits. IEEE Trans Circuits Syst—II 1993;40:634–42.

[10] Rosa Jr E, Hayes S, Grebogi C. Noise filtering in communication with chaos. Phys Rev Lett 1997;78:1247–50.

[11] Baptista MS. Cryptography with Chaos. Phys Lett A 1998;50:240–4;
Baptista MS, Rosa Jr E, Grebogi C. Communication through chaotic modeling of languages. Phys Rev E 2000;61:3590–600;
Baptista MS, Macau EE, Grebogi C, Lai Y-C, Rosa E. Integrated communication using chaos. Phys Rev E 2000;62:4835–45;
Baptista MS, Lopez L. Information transfer in chaos-based communication. Phys Rev E 2002;65:055201–5;
Santos EP, Baptista MS, Caldas IL. Dealing with final state sensitivity for synchronous communication. Physica A 2002;308:101–12;
Baptista MS, Rosa Jr E, Reyes MB, Sartorelli JC, Grebogi C. Communication based on topology preservation of chaotic dynamics. Int J Bifurc Chaos 2003;13:2551–60.

[12] Baptista MS, Macau EE, Grebogi C. Conditions for efficient chaos-based communication. Chaos 2003;13:145–50.

[13] Adler RC, Konheim AC, McAndrew MH. Topological entropy. Trans Am Math Soc 1965;114:309.

[14] Shannon CE, Weaver W. The mathematical theory of communication. Illinois: The University of Illinois Press; 1949.

[15] Haykin S. Communication systems. New York: John Wiley & Sons; 1994.

[16] Bollt E, Lai Y-C, Grebogi C. Coding, channel capacity, and noise resistance in communicating with chaos. Phys Rev Lett 1997;79:3787–90.

[17] Shimbrot T, Grebogi C, Ott E, Yorke JA. Using small perturbations to control chaos. Nature 1993;363:411–7.

[18] Matsumoto T, Chua LC. The double scroll. IEEE Trans Circuits Syst 1985;CAS-32:797–818.

[19] Wiggins S. Introduction to applied nonlinear dynamical systems and chaos. New York: Springer; 1996.

[20] Lin HB. Elementary symbolic dynamics. New York: World Scientific Press; 1987.

[21] Schweizer J, Kennedy MP. Predictive Poincaré control: a control theory for chaotic systems. Phys Rev E 1995;52:4865–76.

[22] Cvitanovic P, Gunaratne GH, Procaccia I. Topological and metric properties of Henon-type strange attractors. Phys Rev A 1988;38:1503–20.